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1) Let $X_1, X_2, ..., X_n$ be a random sample of size n($\geq$ 11) from a population with continuous and strictly increasing cumulative distribution function $F(.)$ with an unknown median M. To test $H_0 : M = 10$ against $H_1 : M > 10$ at level α, let the statistic T denote the number of observations larger than 10. Let t_0 be the observed value of the test statistic T. Consider the test which rejects H_0 if $T \geq c$. Then the p-value of the test is
   a) $\sum_{i=t_0}^{n} \frac{n!}{i!(n-i)!} (0.5)^n$
   b) $\sum_{i=10}^{n} \frac{n!}{i!(n-i)!} (0.5)^n$
   c) $\sum_{i=0}^{10} \frac{n!}{i!(n-i)!} (0.5)^n$
   d) $\sum_{i=0}^{t_0} \frac{n!}{i!(n-i)!} (0.5)^n$
2) Consider the simple linear regression model
$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,\ i = 1, 2, ..., n,$$
   where β_0 and β_1 are unknown parameters, ϵ_i's are uncorrelated random errors with mean 0 and finite variance $\sigma^2 > 0$. Let $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$ and $\hat{\beta_1}$ be the least squares estimator of β_1. Then which of the following statements is true?
   a) The covariance between $\bar{y}$ and $\hat{\beta_1}$ is less than 0
   b) The covariance between $\bar{y}$ and $\hat{\beta_1}$ is greater than 0
   c) The covariance between $\bar{y}$ and $\hat{\beta_1}$ is equal 0
   d) The covariance between $\bar{y}$ and $\hat{\beta_1}$ does not exist
3) Consider the simple linear regression model
$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,\ i = 1, 2, ..., n,$$
   where β_0 and β_1 are unknown parameters, ϵ_i's are uncorrelated random errors with mean 0 and finite variance $\sigma^2 > 0$. Let $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$ and $\hat{y}_i = \hat{\beta_0} + \hat{\beta_1} x_i$,$i = 1, 2, ..., n$, where $\hat{\beta_0}$ and $\hat{\beta_1}$ represent least squares estimators of β_0 and β_1, respectivly. Let $T_1 = \frac{1}{n-2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$ and $T_2 = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$. Then which one of the following statements is true?
   a) Both T_1 and T_2 are unbiased estimators of σ^2
   b) T_1 is an unbiased estimator of σ^2, but T_2 is not an unbiased estimator of σ^2
   c) T_1 is not an unbiased estimator of σ^2, but T_2 is an unbiased estimator of σ^2
   d) Neither T_1 and T_2 is an unbiased estimators of σ^2
4) Consider the power series $\sum_{n=0}^{\infty} a_n x^n$, where $a_{2n+1} = \frac{1}{2^{2n+1}}$ and $a_2 n = \frac{1}{3^{2n}}$ for $n = 0, 1, 2, ...$. Then radius of convergence of the power series equals _______($integer$).
5) Let X be a random variable having Poisson distribution with mean $\lambda > 0$ such that $P(X = 4) = 2P(X = 5)$. If $p_k = P(X = k)$, $k = 0, 1, 2, ...,$ and $p_\alpha = max p_k$, then α equals _______($integer$).
6) Let X_1, X_2, X_3 be independent and identically distributed random variables with common probability density function $f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$
   Then $P(\min\{X_1, X_2, X_3\} \geq E(X_1))$ equals _______ (rounded off to two decimal places).
7) Let (X, Y) have a bivariate normal distribution with $E(X) = E(Y) = 0$. Denote the conditional variance of X given $Y = 1$ by $Var(X|Y = 1)$ and the conditional variance of Y given $X = 2$ by $Var(Y|X = 2)$. If $\frac{E(Y|X=2)}{E(X|Y=1)} = 8$ then $\frac{Var(Y|X=2)}{Var(X|Y=1)}$ equals _______($integer$).
8) Let X be a random sample of size one from a population having $N\left(0, \sigma^2\right)$ distributing, where $\sigma > 0$ is an unknown parameter. Let $\Phi(.)$ denote the cumulative distribution function of a standard normal

random variable and let $\chi^2_{\nu,\alpha}$ denote the $(1-\alpha)$-th quantile of the central chi-square distribution with ν degrees of freedom. It is given that $\Phi(1.96) = 0.975$, $\Phi(1.64) = 0.95, \chi^2_{1,0.05}, \chi^2_{2,0.05} = 5.991$. To test $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 2$, using the Neyman-Pearson most powerful test of size 0.05, the critical region is given by $\lambda(X) > c$, where $c \geq 0$ is a constant and $\lambda(x) = \frac{f(x;\sigma^2=2)}{f(x;\sigma^2=1)}$, where $f\left(x;\sigma^2\right)$ is the probability density function of a $\mathrm{N}\left(0,\sigma^2\right)$ distribution. Then the value of c equal to _______

9) Consider the simple linear regression model

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,\ i = 1, 2, ..., n,$$

where β_1 are unknown parameter, $\epsilon_i' s$ are uncorreleted random errors with mean 0 and finite variance $\sigma^2 > 0$. The five data points $(x_1, y_1) = (2,5)$, $(x_2, y_2) = (1,6)$, $(x_3, y_3) = (3,4)$, $(x_4, y_4) = (2,3)$, and $(x_5, y_5) = (4,6)$ yield the least squares estimate of β_1 equal to _______.

10) Consider the function $f : R^2 \to R$ defined by//

$$f(x,y) = 108xy - 2x^2y - 2xy^2.$$

Which one of the following statements is NOT true?

a) f has four critical points
b) f has a local minimum at $(0,0)$
c) f has a local maximum at $(18,18)$
d) f has two or more saddle points

Consider the function $f : R^2 \to R$ defined by $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{otherwise} \end{cases}$

If f_x denotes the partial derivative of f with respect to x and f_y denotes the partial derivative of f with respect to y, then which one of the following statements is NOT true?

a) f is continuous ai $(0,0)$
b) $f_x(0,0) \neq f_y(0,0)$
c) f_x is continuous at $(0,0)$
d) f_y is not continuous at $(0,0)$

11) Let X be a random variable with probability density function $f(x) = \begin{cases} \frac{3}{8}(x+1)^2 & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

If $Y = 1 - X^2$, then $\mathrm{P}\left(Y \geq \frac{3}{4}\right)$ equals

a) $\frac{19}{32}$
b) $\frac{9}{16}$
c) $\frac{15}{32}$
d) $\frac{5}{8}$

12) Let X be random variable with probability density function $f(x) = \begin{cases} \frac{c_1}{\sqrt{x}} & \text{if } 0 < x \leq 1 \\ \frac{c_2}{x^2} & \text{if } 1 < x < \infty \\ 0 & \text{otherwise} \end{cases}$ where c_1 and c_2 are appropriate real constants. If $P\left(X \in \frac{1}{4}, 4]\right) = \frac{5}{8}$, then consider the following statements:

(I) $P(X \in [3,5]) = \frac{1}{12}$

(II) Both X as well as $\frac{1}{X}$ do not have finite expectations.

Which of the above statements is/are true?

a) Only (I)
b) Only (II)
c) Both (I) and (II)
d) Neither (I) nor (II)