

# Definite Integrals and Applications of Integrals

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## I. COMPREHENSION BASED QUESTIONS

### PASSAGE-1

Let the definite integral be defined by the formula  $\int_a^b f(x) dx = \frac{b-a}{2}(f(a) + f(b))$ . For a more accurate result for  $c \in (a, b)$ , we can use  $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx = F(c)$  so that for  $c = \frac{a+b}{2}$ , we get  $\int_a^b f(x) dx = \frac{b-a}{4}(f(a) + f(b) + 2f(c))$ .

1)  $\int_0^{\frac{\pi}{2}} \sin x dx =$  **(2006- 5M -2)**

- (a)  $\frac{\pi}{8}(1 + \sqrt{2})$
- (b)  $\frac{\pi}{4}(1 + \sqrt{2})$
- (c)  $\frac{\pi}{8\sqrt{2}}$
- (d)  $\frac{\pi}{4\sqrt{2}}$

2)  $\lim_{x \rightarrow a} \frac{\int_a^x f(t) dt - (\frac{x-a}{2})(f(x) + f(a))}{(x-a)^3} = 0$ , then  $f(x)$  is of maximum degree **(2006- 5M -2)**

- (a) 4
- (b) 3
- (c) 2
- (d) 1

3) If  $f''(x) < 0 \forall x \in (a, b)$  and  $c$  is a point such that  $a < c < b$ , and  $(c, f(c))$  is the point lying on the curve for which  $F(c)$  is maximum, then  $f'(c)$  is equal to **(2006- 5M -2)**

- (a)  $\frac{f(b)-f(a)}{b-a}$
- (b)  $\frac{2(f(b)-f(a))}{b-a}$
- (c)  $\frac{2f(b)-f(a)}{2b-a}$
- (d) 0

### PASSAGE - 2

Consider the functions defined implicitly by the equation  $y^3 - 3y + x = 0$  on various intervals in the real line. If  $x \in (-\infty, -2) \cup (2, \infty)$ , the equation implicitly defines a unique real-valued differentiable function  $y = f(x)$ . If  $x \in (-2, 2)$ , the equation implicitly defines a unique real valued differentiable function  $y = g(x)$  satisfying  $g(0) = 0$ .

4) If  $f(-10\sqrt{2}) = 2\sqrt{2}$ , then  $f''(-10\sqrt{2}) =$  **2008**

- a)  $\frac{4\sqrt{2}}{7^3 3^2}$
- b)  $\frac{-4\sqrt{2}}{7^3 3^2}$

- c)  $\frac{4\sqrt{2}}{7^3 3}$
- d)  $\frac{-4\sqrt{2}}{7^3 3}$

5) The area of the region bounded by the curve  $y = f(x)$ , the x-axis, and the lines  $x = a$  and  $x = b$ , where  $-\infty < a < b < -2$ , is **2008**

- a)  $\int_a^b \frac{x}{3((f(x))^2-1)} dx + bf(b) - af(a)$
- b)  $-\int_a^b \frac{x}{3((f(x))^2-1)} dx + bf(b) - af(a)$
- c)  $\int_a^b \frac{x}{3((f(x))^2-1)} dx - bf(b) + af(a)$
- d)  $-\int_a^b \frac{x}{3((f(x))^2-1)} dx - bf(b) + af(a)$

6)  $\int_{-1}^1 g'(x) dx =$  **2008**

- a)  $2g(-1)$
- b) 0
- c)  $-2g(1)$
- d)  $2g(1)$

**PASSAGE-3** Consider the function  $f: (-\infty, \infty) \rightarrow (-\infty, \infty)$  defined by  $f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}$ ,  $0 < a < 2$ .

7) Which of the following is true? **2008**

- a)  $(2+a)^2 f''(1) + (2-a)^2 f''(1)$
- b)  $(2+a)^2 f''(1) - (2-a)^2 f''(1)$
- c)  $f'(1)f'(-1) = (2-a)^2$
- d)  $f'(1)f'(-1) = -(2+a)^2$

8) Which of the following is true? **2008**

- a)  $f(x)$  is decreasing on  $(-1, 1)$  and has a local minimum at  $x=1$
- b)  $f(x)$  is increasing on  $(-1, 1)$  and has a local minimum at  $x=1$
- c)  $f(x)$  is increasing on  $(-1, 1)$  but has neither a local maximum nor a local minimum at  $x=1$
- d)  $f(x)$  is decreasing on  $(-1, 1)$  but has neither a local maximum nor a local minimum at  $x=1$

9) Let  $g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2} dt$ . Which of the following is true?

- a)  $g'(x)$  is positive on  $(-\infty, 0)$  and negative on  $(0, \infty)$
- b)  $g'(x)$  is negative on  $(-\infty, 0)$  and positive on  $(0, \infty)$

- c)  $g'(x)$  changes sign on both  $(-\infty, 0)$  and  $(0, \infty)$   
 d)  $g'(x)$  does not change sign on  $(-\infty, \infty)$

**PASSAGE-4**

Consider the polynomial  $f(x) = 1 + 2x + 3x^2 + 4x^3$ . Let  $s$  be the sum of all distinct real roots of  $f(x)$  and let  $t = |s|$ . **2010**

- 10) The real numbers lies in the interval  
 a)  $(-\frac{1}{4}, 0)$   
 b)  $(-11, -\frac{3}{4})$   
 c)  $(-\frac{3}{4}, -\frac{1}{2})$   
 d)  $(0, \frac{1}{4})$
- 11) The area bounded by the curve  $y = f(x)$  and the lines  $x = 0$ ,  $y = 0$  and  $x = t$ , lies in the interval  
 a)  $(\frac{3}{4}, 3)$   
 b)  $(\frac{21}{64}, \frac{11}{16})$   
 c)  $(9, 10)$   
 d)  $(0, \frac{21}{64})$
- 12) The function  $f'(x)$  is  
 a) increasing in  $(-t, -\frac{1}{4})$  and decreasing in  $(-\frac{1}{4}, t)$   
 b) decreasing in  $(-t, -\frac{1}{4})$  and increasing in  $(-\frac{1}{4}, t)$   
 c) increasing in  $(-t, t)$   
 d) decreasing in  $(-t, t)$

**PASSAGE-5**

Given that for each  $a \in (0, 1)$ ,

$\lim_{h \rightarrow 0^+} \int_h^{1-h} t^{-a} (1-t)^{a-1} dt$  exists. Let this limit be  $g(a)$ . —

In addition, it is given that the function  $g(a)$  is differentiable on  $(0, 1)$ . **JEE Adv.2014**

- 13) The value of  $g(\frac{1}{2})$  is  
 a)  $\pi$   
 b)  $2\pi$   
 c)  $\frac{\pi}{2}$   
 d)  $\frac{\pi}{4}$
- 14) The value of  $g'(\frac{1}{2})$  is  
 a)  $\frac{\pi}{2}$   
 b)  $\pi$   
 c)  $-\frac{\pi}{2}$   
 d) 0