Definite Integrals and Applications of Integrals

AI24BTECH11013-Geetha charani

I. Comprehension Based Questions

PASSAGE-1

Let the definite integral be defined by the formula $\int_a^b f(x)\,dx = \frac{b-a}{2}(f(a)+f(b))$. For a more accurate result for $c \in (a,b)$, we can use $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = F(c)$ so that for $c = \frac{a+b}{2}$, we get $\int_a^b f(x)\,dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$.

1) $\int_0^{\frac{\pi}{2}} \sin x\,dx =$ **(2006- 5M -2)**
   (a) $\frac{\pi}{8}(1+\sqrt{2})$
   (b) $\frac{\pi}{4}(1+\sqrt{2})$
   (c) $\frac{\pi}{8\sqrt{2}}$
   (d) $\frac{\pi}{4\sqrt{2}}$
2) $\lim_{x\to a} \frac{\int_a^x f(t)dt-(\frac{x-a}{2})(f(x)+f(a))}{(x-a)^3} = 0$, then f(x) is of maximum degree **(2006- 5M -2)**
   (a) 4
   (b) 3
   (c) 2
   (d) 1
3) If $f''(x) < 0\ \forall\ x \in (a,b)$ and c is a point such that $a < c < b$, and $(c, f(c))$ is the point lying on the curve for which $F(c)$ is maximum, then $f'(c)$ is equal to **(2006- 5M -2)**
   (a) $\frac{f(b)-f(a)}{b-a}$
   (b) $\frac{2(f(b)-f(a))}{b-a}$
   (c) $\frac{2f(b)-f(a)}{2b-a}$
   (d) 0

PASSAGE - 2

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty,-2) \cup (2,\infty)$, the equation implicitly defines a unique real-valued differentiable function $y = f(x)$. If $x \in (-2,2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

4) If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2})=$ **2008**
   a) $\frac{4\sqrt{2}}{7^3 3^2}$
   b) $\frac{-4\sqrt{2}}{7^3 3^2}$
   c) $\frac{4\sqrt{2}}{7^3 3}$
   d) $\frac{-4\sqrt{2}}{7^3 3}$
5) The area of the region bounded by the curve $y = f(x)$, the x-axis, and the lines x = a and x = b, where $-\infty < a < b < -2$, is **2008**
   a) $\int_a^b \frac{x}{3((f(x))^2-1)}dx$ $+bf(b)$-$af(a)$
   b) -$\int_a^b \frac{x}{3((f(x))^2-1)}dx$ $+bf(b)$-$af(a)$
   c) $\int_a^b \frac{x}{3((f(x))^2-1)}dx$ -$bf(b)$+$af(a)$
   d) -$\int_a^b \frac{x}{3((f(x))^2-1)}dx$ -$bf(b)$+$af(a)$
6) $\int_{-1}^{1} g'(x)dx =$ **2008**
   a) $2g(-1)$
   b) 0
   c) $-2g(1)$
   d) $2g(1)$

   PASSAGE-3 Consider the function f: $(-\infty,\infty) \to (-\infty,\infty)$ defined by $f(x) = \frac{x^2-ax+1}{x^2+ax+1}$, $0 < a < 2$.
7) Which of the following is true? **2008**
   a) $(2+a)^2 f''(1) + (2-a)^2 f''(1)$
   b) $(2+a)^2 f''(1) - (2-a)^2 f''(1)$
   c) $f'(1)f'(-1) = (2-a)^2$
   d) $f'(1)f'(-1) = -(2+a)^2$
8) Which of the following is true? **2008**
   a) $f(x)$ is decreasing on $(-1,1)$ and has a local minimum at x=1
   b) $f(x)$ is increasing on $(-1,1)$ and has a local minimum at x=1
   c) $f(x)$ is increasing on $(-1,1)$ but has neither a local maximum nor a local minimum at x=1
   d) $f(x)$ is decreasing on $(-1,1)$ but has neither a local maximum nor a local minimum at x=1
9) Let $g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2}dt$. Which of the following is true?
   a) $g'(x)$ is positive on $(-\infty,0)$ and negative on $(0,\infty)$
   b) $g'(x)$ is negative on $(-\infty,0)$ and positive on $(0,\infty)$

c) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$
d) $g'(x)$ does not change sign on $(-\infty, \infty)$

PASSAGE-4

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$. **2010**

10) The real numbers lies in the interval
a) $(-\frac{1}{4}, 0)$
b) $(-11, -\frac{3}{4})$
c) $(-\frac{3}{4}, -\frac{1}{2})$
d) $(0, \frac{1}{4})$

11) The area bounded by the curve $y = f(x)$ and the lines $x = 0$, $y = 0$ and $x = t$, lies in the interval
a) $(\frac{3}{4}, 3)$
b) $(\frac{21}{64}, \frac{11}{16})$
c) $(9, 10)$
d) $(0, \frac{21}{64})$

12) The function f'(x) is
a) increasing in $(-t, -\frac{1}{4})$ and decreasing in $(-\frac{1}{4}, t)$
b) decreasing in $(-t, -\frac{1}{4})$ and increasing in $(-\frac{1}{4}, t)$
c) increasing in $(-t, t)$
d) decreasing in $(-t, t)$

PASSAGE-5

Given that for each $a \in (0, 1)$, $\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1 - t)^{a-1}\, dt$ exists. Let this limit be $g(a)$.
In addition, it is given that the function $g(a)$ is differentiable on $(0, 1)$. **JEE Adv.2014**

13) The value of $g(\frac{1}{2})$ is
a) π
b) 2π
c) $\frac{\pi}{2}$
d) $\frac{\pi}{4}$

14) The value of $g'(\frac{1}{2})$ is
a) $\frac{\pi}{2}$
b) π
c) $-\frac{\pi}{2}$
d) 0