Matrix theory

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6) If $z = x + iy$ and $\omega = \frac{1-iz}{z-i}$, then $|\omega| = 1$ implies that,in the complex plane (1983 - 1 Mark)
   a) z lies on the imaginary axis
   b) z lies on the real axis
   c) z lies on unit circle
   d) None of these

7) The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if (1983 - 1 Mark)

   a) $z_1 + z_4 = z_2 + z_3$
   b) $z_1 + z_3 = z_2 + z_4$
   c) $z_1 + z_2 = z_3 + z_4$
   d) None of these

8) If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c = (1 - r)a + rb$ and $w = (1 - r)u + rv$,where r is a complex number,then the two triangles (1985 - 2 Marks)

   a) have the same area
   b) are similar
   c) are congruent
   d) none of these

9) If $\omega\,(\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$ then A and B are respectively (1995S)

   a) $0, 1$ b) $2, 1$ c) $1, 0$ d) $-1, 1$

10) Let z and ω be two non zero complex numbers such that $|z| = |\omega|$ and $\text{Arg}\, z + \text{Arg}\omega = \pi$, then z equals (1995S)

   a) ω b) $-\omega$ c) $\overline{\omega}$ d) $-\overline{\omega}$

11) let z and ω be two complex numbers such that $|z| \leq 1$, $|\omega| \leq 1$ and $|z + i\omega| = |z - i\overline{\omega}| = 2$ then z equals (1995S)

   a) 1 or i b) i or -1 c) 1 or -1 d) i or -1

12) For positive numbers n_1, n_2 the value of the expression $(1 + i)^{n_1} + \left(1 + i^3\right)^{n_1} + \left(1 + i^5\right)^{n_2} + \left(1 + i^7\right)^{n_2}$, where $i = \sqrt{-1}$ is a real number if and only if (1996 - 2 Marks)

a) $n_1 = n_2 + 1$
b) $n_1 = n_2 - 1$
c) $n_1 = n_2$
d) $n_1 > 0, n_2 > 0$

13) If $i = \sqrt{-1}$ then $4 + 5\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is a real number if and only if (1999 - 2 Marks)

a) $1 - i\sqrt{3}$ b) $-1 + i\sqrt{3}$ c) $i\sqrt{3}$ d) $-i\sqrt{3}$

14) If arg $(z) < 0$,then arg $(-z)$ − arg (z) = (2000S)

a) π b) $-\pi$ c) $\frac{-\pi}{2}$ d) $\frac{\pi}{2}$

15) If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is (2000S)

a) equal to 1
b) less than 1
c) greater than 3
d) equal to 3

16) Let z_1 and z_2 be n^{th} roots of unity which substend a right angle at the origin.Then n must be of the form (2001S)

a) $4k + 1$ b) $4k + 2$ c) $4k + 3$ d) $4k$

17) The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is (2001S)

a) of area zero
b) right angled triangle
c) equilateral
d) obtuse-angled triangle

18) For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - i| = 5$, the minimum value of $|z_1 - z_2|$ (2002S)

a) 0 b) 2 c) 7 d) 17

19) If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq 1$), then $\mathrm{Re}(\omega)$ is (2003S)

a) 0
b) $\frac{-1}{|z+1|^2}$
c) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$
d) $\frac{\sqrt{2}}{|z+1|^2}$

20) If $\omega (\neq 1)$ be a cube root of unity and $\left(1 + \omega^2\right)^n = \left(1 + \omega^4\right)^n$, then the least positive value of n is (2004S)

a) 2 b) 3 c) 5 d) 6