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1)
$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n^2+k^2}$$
is equal to

a) $\frac{e}{3}$ b) $\frac{5}{6}$ c) $\frac{3}{4}$ d) $\frac{\pi}{4}$

2) Let $\mathbf{F}=(x-y+z)(\mathbf{i}+\mathbf{j})$ be a vector field on $\mathbb{R}^3$. The line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the triangle with vertices $(0,0,0)$, $(5,0,0)$, and $(5,5,0)$ traversed in that order.

a) -25 b) 25 c) 50 d) 5

3) Let $\{1,2,3,4\}$ represent the outcomes of a random experiment, and $P(\{1\})=P(\{2\})=P(\{3\})=P(\{4\})=\frac{1}{4}$. Suppose that $A_1=\{1,2\}$, $A_2=\{2,3\}$, $A_3=\{3,4\}$, and $A_4=\{1,2,3\}$.Then which of the following statements is true?

a) A_1 and A_2 are not independent.
b) A_3 and A_4 are independent.
c) A_1 and A_4 are independent
d) A_2 and A_4 are independent

4) A fair die is rolled two times independently. Given that the outcome on the first roll is 1, the expected value of the sum of the two outcomes is

a) 4 b) 4.5 c) 3 d) 5.5

5) The dimension of the vector space of 7×7 real symmetric matrices with trace zero and the sum of the off-diagonal elements zero is

a) 47 b) 28 c) 27 d) 26

6) Let A be a 6×6 complex matrix with $A^3\neq 0$ and $A^4=0$.Then the number of Jordan blocks of A is 2.

a) 1 or 6 b) 2 or 3 c) 4 d) 5

7) Let $X_1,\ \ldots,\ X_n$ be a random sample from a uniform distribution defined over $(0,\theta)$,where $\theta>0$ and $n\geq 2.X_{(1)}=\min\{X_1,\ldots,X_n\}$ and $X_{(n)}=\max\{X_1,\ldots,X_n\}$.Then the covariance between $X_{(n)}$ and $\frac{X_{(1)}}{X_{(n)}}$ is

a) 0 b) $n(n+1)\theta$ c) $n\theta$ d) $n^2(n+1)\theta$

8) $X_1, \ldots, X_n$ be a random sample drawn from a population with probability density function $f(x;\theta) = \theta x^{\theta-1},\ 0 \le x \le 1,\ \theta > 0$.Then the maximum likelihood estimator of θ is given by

a) $\frac{-n}{\sum_{i=1}^{n} \log_e X_i}$
b) $\frac{-\sum_{i=1}^{n} \log_e X_i}{n}$
c) $\left(\prod_{i=1}^{n} X_i\right)^{\frac{1}{n}}$
d) $\frac{\prod_{i=1}^{n} X_i}{n}$

9) $Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$,for $i = 1, \ldots, 10$,where x_{1i} and x_{2i} are fixed covariates, and ϵ_i are uncorrelated random variables with mean 0 and unknown variance σ^2. Here β_0, β_1, and β_2 are unknown parameters. Further, define $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \hat{\beta}_2 x_{2i}$, where $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$ is the unbiased least squares estimator of $(\beta_0, \beta_1, \beta_2)$. Then an unbiased estimator of σ^2 is given by

a) $\frac{\left(\sum_{i=1}^{10}(Y_i-\hat{Y}_i)^2\right)}{10}$ b) $\frac{\left(\sum_{i=1}^{10}(Y_i-\hat{Y}_i)^2\right)}{7}$ c) $\frac{\left(\sum_{i=1}^{10}(Y_i-\hat{Y}_i)^2\right)}{8}$ d) $\frac{\left(\sum_{i=1}^{10}(Y_i-\hat{Y}_i)^2\right)}{9}$

10) For i = 1, 2, 3, let $Y_i = \alpha + \beta x_i + \epsilon_i$,where x_i are fixed covariates, and ϵ_i are independent and identically distributed standard normal random variables. Here, α and β are unknown parameters. Given the following observations the best linear

Y_i	0.5	2.5	0.5
x_i	1	1	-2

unbiased estimate of $\alpha + \beta$ is equal to

a) 1.5 b) 1 c) 1.8 d) 2.1

11) Consider a discrete time Markov chain on the state space $\{1, 2, 3\}$ with one-step transition probability matrix $\begin{pmatrix} & 1 & 2 & 3 \\ 1 & 0.7 & 0.3 & 0.0 \\ 2 & 0.0 & 0.6 & 0.4 \\ 3 & 0.0 & 0.0 & 1.0 \end{pmatrix}$.Which of the following statements is true?

a) States 1, 3 are recurrent and state 2 is transient.
b) State 3 is recurrent and states 1, 2 are transient.
c) States 1, 2, 3 are recurrent.
d) States 1, 2 are recurrent and state 3 is transient.

12) The minimal polynomial of the matrix $\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ is

a) $(x-1)(x-2)$
b) $(x-1)^2(x-2)$
c) $(x-1)(x-2)^2$

d) $(x-1)^2(x-2)^2$

13) Let $\{X_1, X_2, X_3\}$ be a trivariate normal random vector with mean vector $(-3, 1, 4)$ and variance-covariance matrix $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 3 & -3 \\ 0 & -3 & 4 \end{pmatrix}$.Which of the following statements are true?

(i) X_2 and X_3 are independent.

(ii) $X_1 + X_3$ and X_2 are independent.

(iii) (X_2, X_3) and X_1 are independent.

(iv) $\frac{1}{2}(X_2 + X_3)$ and X_1 are independent.

a) (i) and (iii) b) (ii) and (iii) c) (i) and (iv) d) (iii) and (iv)