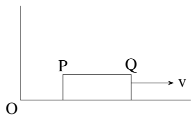
2023 Ph 40-52

ai24btech11035 - V.Preethika

1) A rod PQ of proper length L lies along the x-axis and moves towards the positive x direction with speed $v = \frac{3c}{5}$ with respect to the ground, where c is the speed of light in vacuum. An observer on the ground measures the positions of P and Q at different times t_P and t_Q respectively in the ground frame, and finds the difference between them to be $\frac{9L}{10}$. What is the value of $t_Q - t_P$?



a) $\frac{L}{3c}$
b) $\frac{L}{5c}$
c) $\frac{L}{6c}$
d) $\frac{L}{3c}$

2) A symmetric top has principal moments of inertia $I_1 = I_2 = \frac{2\alpha}{3}$, $I_3 = 2\alpha$ about a set of principal axes 1, 2, 3 respectively, passing through its center of mass, where α is a positive constant. There is no force acting on the body, and the angular speed of the body about the 3-axis is $\omega_3 = \frac{1}{8}$ rad/s. With what angular frequency in rad/s does the angular velocity vector ω_1 precess about the 3-axis?

a) 2
b) 3
c) 5
d) 7

3) A particle of mass m is free to move on a frictionless horizontal two-dimensional (r, θ) plane and is acted upon by a force $F = -\frac{k}{2r^3}\,\hat{r}$ with k being a positive constant. If p_r and p_θ are the generalised momenta corresponding to r and θ respectively, then what is the value of $\frac{dp_r}{dt}$?

a) $\frac{p_\theta^2 - 2mk}{2mr^3}$
b) $\frac{2p_\theta^2 - mk}{mr^3}$
c) $\frac{p_\theta^2 - 2mk}{mr^3}$
d) $\frac{2p_\theta^2 - mk}{2mr^3}$

4) Consider two real functions

$$U(x, y) = xy\left(x^2 - y^2\right),$$

$$V(x, y) = ax^4 + by^4 + cx^2y^2 + k,$$

where k is a real constant and a, b, c are real coefficients. If $U(x, y) + iV(x, y)$ is analytic, then what is the value of $a \times b \times c$?

a) $\frac{1}{8}$
b) $\frac{3}{28}$
c) $\frac{5}{36}$
d) $\frac{3}{32}$

5) Young's double slit experiment is performed using a beam of C_{60} (fullerene) molecules, each molecule being made up of 60 carbon atoms. When the slit separation is 50 nm, fringes are formed on a screen kept at a distance of 1 m from the slits. Now, the experiment is repeated with C_{70} molecules with a slit separation of 92.5 nm. The kinetic energies of both the beams are the same. The position of the 4^{th} bright fringe for C_{60} will correspond to the n^{th} bright fringe for C_{70}. What is the value of n (rounded off to the nearest integer)?

a) 5
b) 6
c) 7
d) 8

6) A neutron beam with a wave vector $\mathbf{k}$ and an energy 20.4 meV diffracts from a crystal with an outgoing wave vector $\mathbf{k}'$. One of the diffraction peaks is observed for the reciprocal lattice vector $\mathbf{G}$ of magnitude 3.14 Å^{-1}. What is the diffraction angle in degrees (rounded off to the nearest integer) that $\mathbf{k}$ makes with the plane? (Use mass of neutron $= 1.67 \times 10^{-27}$ kg)

a) 15
b) 30
c) 45
d) 60

7) In the first Brillouin zone of a rectangular lattice (lattice constants $a = 6\,\text{Å}$ and $b = 4\,\text{Å}$), three incoming phonons with the same wave vector $\langle 1.2\,\text{Å}^{-1}, 0.6\,\text{Å}^{-1} \rangle$ interact to give one phonon. Which one of the following is the CORRECT wave vector of the resulting phonon?

a) $\langle 2.56\,\text{Å}^{-1}, 0.23\,\text{Å}^{-1} \rangle$
b) $\langle 3.60\,\text{Å}^{-1}, 1.80\,\text{Å}^{-1} \rangle$
c) $\langle 0.48\,\text{Å}^{-1}, 0.23\,\text{Å}^{-1} \rangle$
d) $\langle 3.60\,\text{Å}^{-1}, -0.80\,\text{Å}^{-1} \rangle$

8) For a covalently bonded solid consisting of ions of mass m, the binding potential can be assumed to be given by

$$U(r) = -\epsilon \left(\frac{r}{r_0} \right) e^{-\frac{r}{r_0}},$$

where ϵ and r_0 are positive constants. What is the Einstein frequency of the solid in Hz?

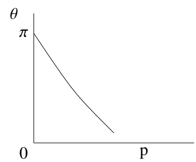
a) $\frac{1}{2\pi} \sqrt{\frac{\epsilon e}{m r_0^2}}$

b) $\frac{1}{2\pi}\sqrt{\frac{\epsilon}{mer_0^2}}$

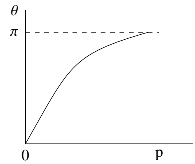
c) $\frac{1}{2\pi}\sqrt{\frac{\epsilon}{2mer_0^2}}$

d) $\frac{1}{2\pi}\sqrt{\frac{\epsilon e}{2mr_0^2}}$

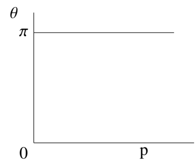
9) In a hadronic interaction, π^0s are produced with different momenta, and they immediately decay into two photons with an opening angle θ between them. Assuming that all these decays occur in one plane, which one of the following figures depicts the behaviour of θ as a function of the π^0 momentum p?



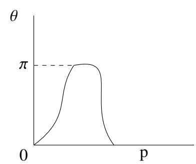
a)



b)



c)



d)

10) A particle has wavefunction

$$\psi(x, y, z) = Nze^{-\alpha(x^2+y^2+z^2)},$$

where N is a normalization constant and α is a positive constant. In this state, which one of the following options represents the eigenvalues of L^2 and L_z respectively? Some values of $Y_{\ell m}$ are:

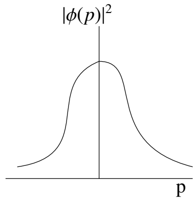
$$Y_0^0 = \sqrt{\frac{1}{4\pi}}, Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta, Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\phi}.$$

a) 0 and 0
b) $\hbar^2$ and $-\hbar$
c) $2\hbar^2$ and 0
d) $\hbar^2$ and $\hbar$

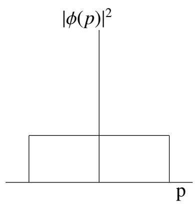
11) The wavefunction of a particle in one dimension is given by

$$\psi(x) = \begin{cases} M, & -a < x < a \\ 0, & \text{otherwise} \end{cases}$$

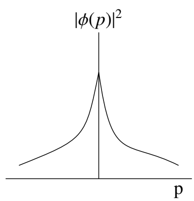
Here M and a are positive constants. If $\phi(p)$ is the corresponding momentum space wavefunction, which one of the following plots best represents $|\phi(p)|^2$?



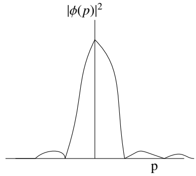
a)



b)



c)



d)

12) Consider a particle in a two-dimensional infinite square well potential of side L, with $0 \leq x \leq L$ and $0 \leq y \leq L$. The wavefunction of the particle is zero only along the line $y = \frac{L}{2}$, apart from the boundaries of the well. If the energy of the particle in this state is E, what is the energy of the ground state?
   a) $\frac{1}{4}E$
   b) $\frac{2}{5}E$
   c) $\frac{3}{8}E$
   d) $\frac{1}{2}E$
13) Consider two non-identical spin-$\frac{1}{2}$ particles labelled 1 and 2 in the spin product state $|\frac{1}{2}, \frac{1}{2}\rangle|\frac{1}{2}, -\frac{1}{2}\rangle$. The Hamiltonian of the system is

$$H = \frac{4\lambda}{\hbar^2}\mathbf{S}_1 \cdot \mathbf{S}_2,$$

where $\mathbf{S}_1$ and $\mathbf{S}_2$ are the spin operators of particles 1 and 2, respectively, and λ is a constant with appropriate dimensions. What is the expectation value of H in the above state?
   a) $-\lambda$
   b) -2λ
   c) λ
   d) 2λ