1.2.23

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Question

Represent graphically a displacement of 40 km, 30° west of south.

Solution (Matrix Method)

Coordinate convention. Let the x-axis point *East* and the y-axis point *North*. Thus:

$$\text{East} \equiv +x, \qquad \text{West} \equiv -x, \qquad \text{North} \equiv +y, \qquad \text{South} \equiv -y.$$

Rotation matrix. For a counterclockwise rotation by an angle θ, use

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

Direction setup. The unit direction for *South* is the column matrix

$$\mathbf{s} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

"30° west of south" means rotate the south direction *towards west* by 30°. On the standard ($+x$ from East, counterclockwise positive) angle circle, this is a *clockwise* rotation of 30°, i.e. by $-30°$.

Hence the required unit direction column is

$$\mathbf{u} = R(-30°)\,\mathbf{s} = \begin{bmatrix} \cos 30° & \sin 30° \\ -\sin 30° & \cos 30° \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\sin 30° \\ -\cos 30° \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix}.$$

Displacement column. With magnitude 40 km, the displacement (as a 2×1 matrix) is

$$\mathbf{d} = 40\,\mathbf{u} = 40 \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} -20 \\ -20\sqrt{3} \end{bmatrix} \text{ (km)}.$$

So the endpoint relative to the origin is

$$(x, y) = \left(-20,\, -20\sqrt{3}\right) \text{ km},$$

which lies in the third quadrant (west and south components), consistent with the description.

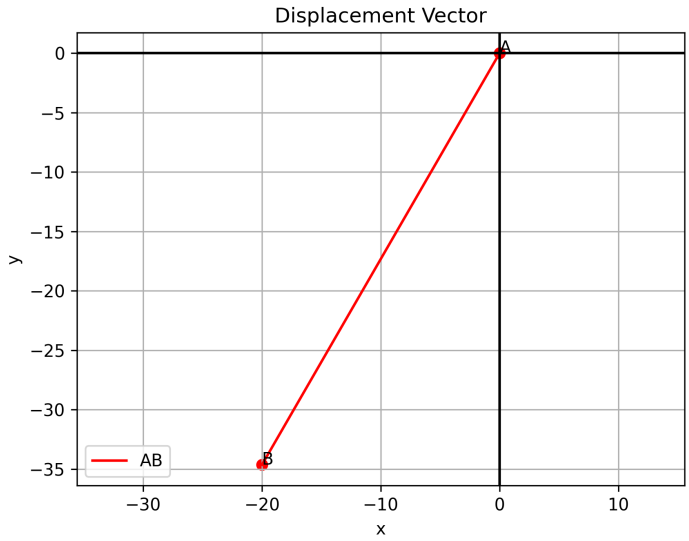

Figure 1: Displacement vector: 40 km, 30° west of south