Bonus Question

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Q : Prove the Condition for Two Points to Lie on the Same or Opposite Side of a Line

Let the line be

$$L: \ l_1x + l_2y + c = 0$$

and let two points be

$$P_1(x_1, y_1), \quad P_2(x_2, y_2).$$

Using Section Formula

If a point $P(x, y)$ divides P_1P_2 internally in the ratio $m : 1$, then

$$x = \frac{mx_2 + x_1}{m+1}, \qquad y = \frac{my_2 + y_1}{m+1}.$$

If m is ¿ 0 then points lie on opposite side If m is ¡ o then points lie on same side

Substituting in Line Equation

Substituting (x, y) into the line equation:

$$l_1\left(\frac{mx_2 + x_1}{m+1}\right) + l_2\left(\frac{my_2 + y_1}{m+1}\right) + c = 0$$

Multiplying through by $(m + 1)$:

$$m(l_1x_2 + l_2y_2 + c) + (l_1x_1 + l_2y_1 + c) = 0$$

$$mL_2 + L_1 = 0$$

where

$$L_1 = l_1x_1 + l_2y_1 + c, \qquad L_2 = l_1x_2 + l_2y_2 + c.$$

Thus,

$$m = -\frac{L_1}{L_2}.$$

- If $m > 0 \quad \Rightarrow \quad L_1L_2 < 0$, the point lies **between** P_1 and $P_2 \implies$ points are on **opposite sides** of the line. - If $m < 0 \quad \Rightarrow \quad L_1L_2 > 0$, the section ratio is negative $\implies$ points are on the **same side** of the line.

Hence

$$L_1 \cdot L_2 > 0 \quad \Rightarrow \quad \text{Same side of the line.}$$

$$L_1 \cdot L_2 < 0 \quad \Rightarrow \quad \text{Opposite sides of the line.}$$