

## 1.7.1 – Matgeo Assignment

ai25btech11015 – M Sai Rithik

## Question

Show that the points  $(0, 0)$ ,  $(2m, -4)$ , and  $(3, 6)$  are collinear, and hence find  $m$ , using the rank method.

## Step 1: Form vectors

$$A = (0, 0), \quad B = (2m, -4), \quad C = (3, 6)$$

$$AB = \begin{bmatrix} 2m \\ -4 \end{bmatrix}, \quad AC = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

## Step 2: Matrix form

Form the matrix with  $AB$  and  $AC$  as columns:

$$M = \begin{bmatrix} 2m & 3 \\ -4 & 6 \end{bmatrix}$$

For collinearity,  $\text{rank}(M) = 1$ , i.e.  $\det(M) = 0$ .

### Step 3: Using RREF

$$M = \begin{bmatrix} 2m & 3 \\ -4 & 6 \end{bmatrix}.$$

We use RREF  $M$  and look for when its rank drops below 2.

$$\begin{aligned} \begin{bmatrix} 2m & 3 \\ -4 & 6 \end{bmatrix} &\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} -4 & 6 \\ 2m & 3 \end{bmatrix} \xrightarrow{R_1 \leftarrow -\frac{1}{4}R_1} \begin{bmatrix} 1 & -\frac{3}{2} \\ 2m & 3 \end{bmatrix} \\ &\xrightarrow{R_2 \leftarrow R_2 - 2m R_1} \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 3(m+1) \end{bmatrix}. \end{aligned}$$

If  $m \neq -1$ , the second row has a pivot (divide by  $3(m+1)$ ), so the RREF is  $I_2$  and  $\text{rank}(M) = 2$ . For the rank to drop (and hence the RREF to have a zero row), we need

$$3(m+1) = 0 \Rightarrow m = -1.$$

When  $m = -1$ ,

$$\begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 0 \end{bmatrix}$$

## Conclusion

The given points are collinear when

$$m = -1$$

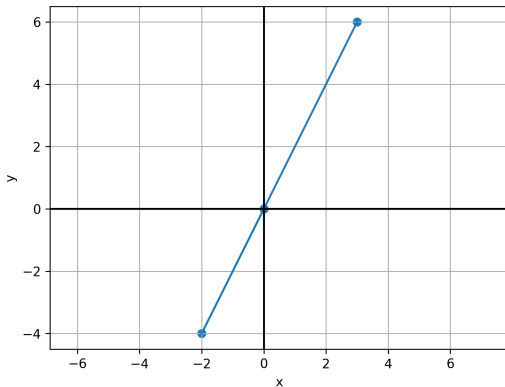


Figure: Graph