frame=single, breaklines=true, columns=fullflexible

4.13.84

ai25btech11015 – M Sai Rithik

Question

Find the value of k such that the line

$$\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$$

lies in the plane

$$2x - 4y + z = 7$$

Solution

- Parametric form of line:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ k \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

- Plane equation in matrix form:

$$\begin{bmatrix} 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 7$$

- Substitute line into plane:

$$\begin{bmatrix} 2 & -4 & 1 \end{bmatrix} \left(\begin{bmatrix} 4 \\ 2 \\ k \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right) = 7$$

- Multiply matrices:

$$\underbrace{\begin{bmatrix} 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}}_{=0} t + \underbrace{\begin{bmatrix} 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ k \end{bmatrix}}_{=k} = 7$$

- Solve:

$$k = 7$$

Answer

$$\boxed{k = 7}$$