1.2.23 – Matgeo Assignment

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Question

Represent graphically a displacement of $40\,\text{km}$, 30° west of south.

Coordinate Convention

We choose the coordinate axes as:

$$\text{East} \equiv +x, \quad \text{West} \equiv -x, \quad \text{North} \equiv +y, \quad \text{South} \equiv -y.$$

The unit column for South is

$$\mathbf{s} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

Rotation Matrix

For rotation by angle θ,

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

Since "30° west of south" means a clockwise rotation of 30°, we apply

$$\mathbf{u} = R(-30^\circ)\mathbf{s}.$$

Direction Column

$$\mathbf{u} = \begin{bmatrix} \cos 30^\circ & \sin 30^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix}.$$

Displacement Column

With magnitude 40 km:

$$\mathbf{d} = 40\mathbf{u} = 40\begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} -20 \\ -20\sqrt{3} \end{bmatrix} \text{ km.}$$

Endpoint:

$$(x, y) = (-20,\ -20\sqrt{3}) \text{ km.}$$

Graphical Representation

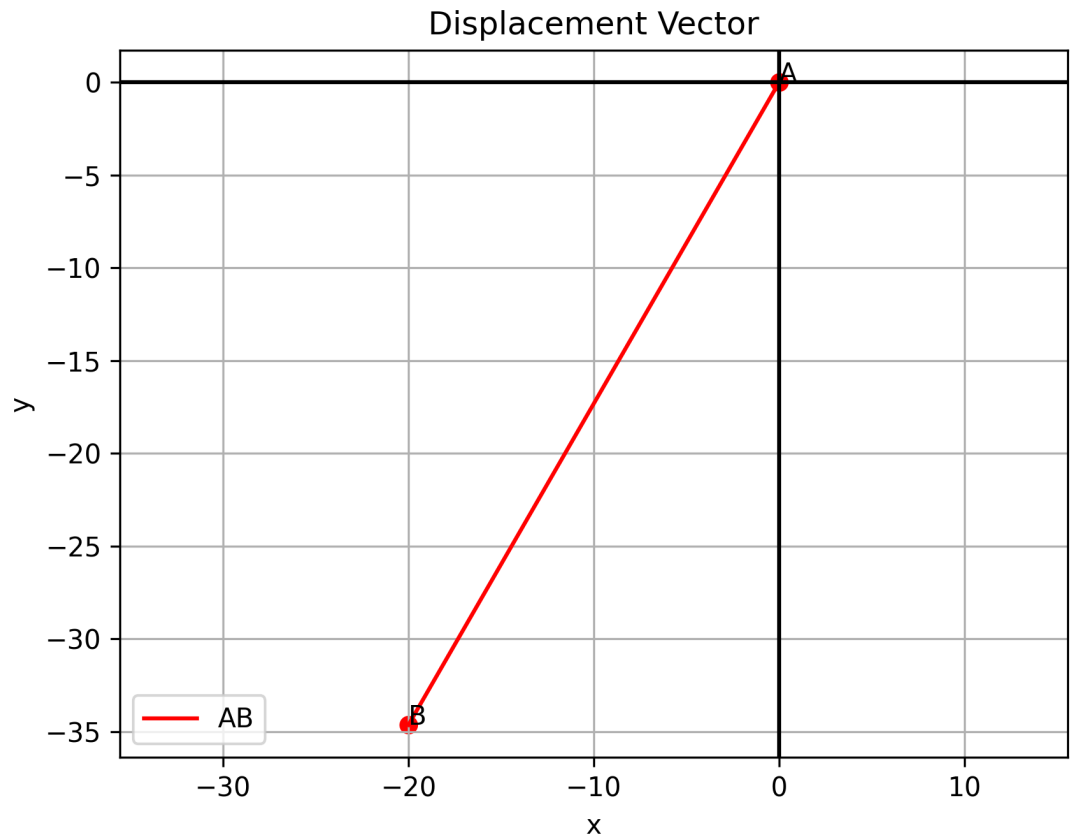

Figure: Displacement vector: 40 km, 30° west of south