frame=single, breaklines=true, columns=fullflexible

Matrix 2.6.24

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Question (2.6.24)

Find the area of the parallelogram whose adjacent sides are given by the vectors

$$\mathbf{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k} \quad \text{and} \quad \mathbf{b} = \hat{\imath} - \hat{\jmath} + \hat{k}.$$

Solution

Consider vectors a and b

$$\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}. \tag{1}$$

Magnitude of $\mathbf{a}$:

$$\|\mathbf{a}\| = \sqrt{3^2 + 1^2 + 4^2} = \sqrt{9 + 1 + 16} = \sqrt{26}. \tag{2}$$

Magnitude of $\mathbf{b}$:

$$\|\mathbf{b}\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{1 + 1 + 1} = \sqrt{3}. \tag{3}$$

Dot product $\mathbf{a} \cdot \mathbf{b}$ (to get $\cos\theta$):

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 3 \cdot 1 + 1 \cdot (-1) + 4 \cdot 1 = 3 - 1 + 4 = 6. \tag{4}$$

Cosine of the angle:

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \, \|\mathbf{b}\|} = \frac{6}{\sqrt{26}\,\sqrt{3}} = \frac{6}{\sqrt{78}}. \tag{5}$$

Sine of the angle (use $\sin\theta = \sqrt{1 - \cos^2\theta}$):

$$\sin\theta = \sqrt{1 - \left(\frac{6}{\sqrt{78}}\right)^2} = \sqrt{1 - \frac{36}{78}} = \sqrt{\frac{78 - 36}{78}} = \sqrt{\frac{42}{78}} = \sqrt{\frac{7}{13}}. \tag{6}$$

Area of parallelogram $= \|\mathbf{a}\| \, \|\mathbf{b}\| \sin\theta$:

$$\text{Area} = \sqrt{26}\,\sqrt{3}\,\sqrt{\frac{7}{13}} = \sqrt{\frac{26 \cdot 3 \cdot 7}{13}} = \sqrt{\frac{26}{13} \cdot 21} = \sqrt{2 \cdot 21} = \sqrt{42}. \tag{7}$$

Final Answer

$$\boxed{\text{Area} = 6.4807} \tag{8}$$

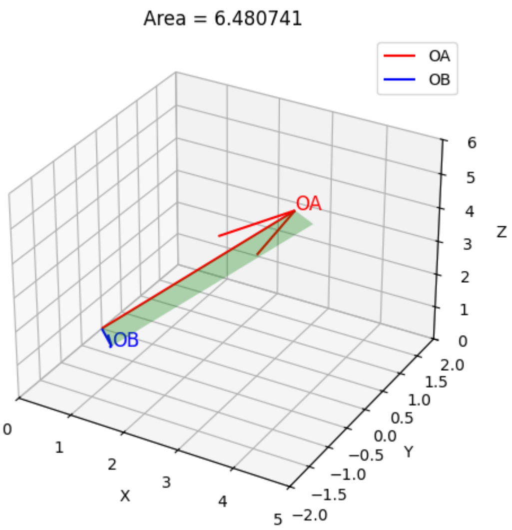

Figure 1: Parallelogram spanned by **a** and **b**