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4.12.20

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Question

One vertex of the equilateral triangle with centroid at the origin and one side as

$$x + y - 2 = 0$$

is

(a)
$$(-1,-1)$$
 (b) $(2,2)$ (c) $(-2,-2)$ (d) $(2,-2)$.

Step 1: Condition for centroid

Let the vertices be

$$\mathbf{A},\;\mathbf{B},\;\mathbf{C}.$$

Since centroid is at origin,

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}. \tag{1}$$

Step 2: Side equation

Suppose side BC lies on the line

$$x + y - 2 = 0, (2)$$

i.e.,

$$(1 \ 1) \mathbf{x} = 2.$$

Normal vector:

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Step 3: Alternative choice of side

If AB lies on line (??), then C lies on perpendicular from origin:

$$\mathbf{x} = t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
.

Check options: only (2,-2) satisfies this condition.

Final Answer

The required vertex is

$$(2, -2)$$

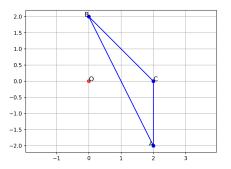


Figure: Equilateral triangle with centroid at origin and one side on x + y - 2 = 0.