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4.13.84

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Question

Find the value of k such that the line

$$\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$$

lies in the plane

$$2x - 4y + z = 7$$

Step 1: Parametric Form of Line

Let the line parameter be t:

$$\begin{aligned} x &= 4 + t, \\ y &= 2 + t, \\ z &= k + 2t. \end{aligned}$$

Matrix form:

$$\mathbf{r}(t) = \begin{pmatrix} 4 \\ 2 \\ k \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

Step 2: Plane in Matrix Form

Plane equation:

$$2x - 4y + z = 7$$

Normal vector:

$$\mathbf{n} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}.$$

Substitute line into plane:

$$\mathbf{n}^T \mathbf{r}(t) = \mathbf{n}^T \begin{pmatrix} 4 \\ 2 \\ k \end{pmatrix} + \mathbf{n}^T \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} t = 7$$

Step 3: Multiply Matrices

$$\underbrace{\begin{pmatrix}2 & -4 & 1\end{pmatrix}\begin{pmatrix}1\\1\\2\end{pmatrix}}_{0}t+\underbrace{\begin{pmatrix}2 & -4 & 1\end{pmatrix}\begin{pmatrix}4\\2\\k\end{pmatrix}}_{k}=7$$

$$0\cdot t+k=7$$

Step 4: Solve for k

$$k = 7$$

$$\boxed{k = 7}$$