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Matrix 2.6.24

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Question (2.6.24)

Find the area of the parallelogram whose adjacent sides are given by the vectors

$$\mathbf{a} = 3\hat{i} + \hat{j} + 4\hat{k}, \quad \mathbf{b} = \hat{i} - \hat{j} + \hat{k}.$$

Do not use determinant method; use $\|\mathbf{a}\|\|\mathbf{b}\| \sin\theta$.

Step 1: Representing Vectors

$$\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}. \tag{1}$$

Step 2: Magnitudes

$$\|\mathbf{a}\| = \sqrt{3^2 + 1^2 + 4^2} = \sqrt{26}, \tag{2}$$

$$\|\mathbf{b}\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}. \tag{3}$$

Step 3: Dot Product and $\cos\theta$

$$\mathbf{a}\cdot\mathbf{b} = 3(1) + 1(-1) + 4(1) = 6, \tag{4}$$

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} = \frac{6}{\sqrt{26}\sqrt{3}} = \frac{6}{\sqrt{78}}. \tag{5}$$

Step 4: Finding $\sin\theta$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{6}{\sqrt{78}}\right)^2}, \tag{6}$$

$$\sin\theta = \sqrt{\frac{7}{13}}. \tag{7}$$

Step 5: Area of Parallelogram

$$\text{Area} = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta, \tag{8}$$

$$= \sqrt{26}\sqrt{3}\sqrt{\tfrac{7}{13}} = \sqrt{42}. \tag{9}$$

Final Answer

$$\boxed{\text{Area} = \sqrt{42}} \tag{10}$$

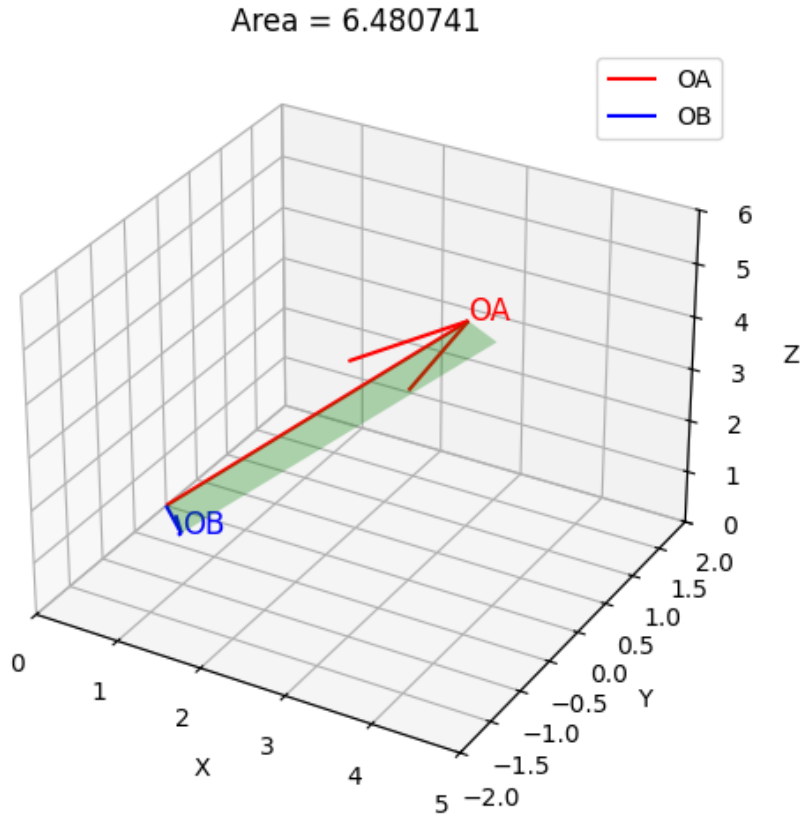

Figure: Parallelogram spanned by **a** and **b**.

C Code

```c
// C code to calculate area of parallelogram
#include <stdio.h>
#include "libs/matfun.h"
#include <math.h>

int main() {
    // create a b vectros
    double **a = createMat(3,1);
    a[0][0] = 3;
    a[1][0] = 1;
    a[2][0] = 4;

    double **b = createMat(3,1);
    b[0][0] = 1;
    b[1][0] = -1;
    b[2][0] = 1;
```

C Code

```
    double mag_a = sqrt(Matdot(a, a,3));
    double mag_b = sqrt(Matdot(b, b,3));

    double cos_theta = Matdot(a, b,3) / (mag_a * mag_b);
    double angle = acos(cos_theta);

    double area = mag_a * mag_b * sin(angle);

    FILE *fp = fopen("var.dat", "w");
    if (fp != NULL) {
        fprintf(fp, "%lf\n", area);
        fclose(fp);
    } else {
        printf("Error opening file for writing.\n");
    }
    return 0;
}
```

Python Code

```
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
from mpl_toolkits.mplot3d.art3d import Poly3DCollection
import matplotlib.pyplot as plt
with open('var.dat', 'r') as f:
        area = f.read().strip()

a = np.array([3, 1, 4])
b = np.array([1, -1, 1])
O = np.array([0, 0, 0])

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot vectors OA and OB
ax.quiver(*O, *a, color='r', label='OA')
ax.quiver(*O, *b, color='b', label='OB')
ax.text(a[0], a[1], a[2], 'OA', color='r', fontsize=12)
ax.text(b[0], b[1], b[2], 'OB', color='b', fontsize=12)

verts = [ [O, a, a+b, b] ]
ax.add_collection3d(Poly3DCollection(verts, alpha=0.3, facecolor='
    green'))
```

Python Code

```
# Set limits and labels
ax.set_xlim([0, max(a[0], b[0], a[0]+b[0])+1])
ax.set_ylim([min(0, a[1], b[1], a[1]+b[1])-1, max(a[1], b[1], a[1]+
    b[1])+1])
ax.set_zlim([0, max(a[2], b[2], a[2]+b[2])+1])
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.legend()

# Set the graph title to the area value
ax.set_title(f'Area = {area}')

plt.tight_layout()
plt.savefig('../figs/vectors_3d.png')
plt.close()
```