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Matrix 2.6.25

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Question

Find the area of the triangle whose vertices are

$$A(1, -1, 2), \quad B(2, 0, -1), \quad C(3, -1, 2).$$

Solution

Step 1: Form vectors

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2-1 \\ 0-(-1) \\ -1-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, \tag{1}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3-1 \\ -1-(-1) \\ 2-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}. \tag{2}$$

Step 2: Cross product formula

The cross product of two vectors is defined as

$$\mathbf{X} \times \mathbf{Y} = \begin{pmatrix} \begin{vmatrix} \mathbf{X}_{23} & \mathbf{Y}_{23} \\ \mathbf{X}_{31} & \mathbf{Y}_{31} \\ \mathbf{X}_{12} & \mathbf{Y}_{12} \end{vmatrix} \end{pmatrix}. \tag{3}$$

Applying to $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$:

$$(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} \begin{vmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} -3 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{vmatrix} \\ \begin{vmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 2 \\ 0 \end{pmatrix} \end{vmatrix} \end{pmatrix}. \tag{4}$$

Step 3: Simplify

$$(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}(1)(0)-(-3)(0)\\(-3)(2)-(1)(0)\\(1)(0)-(1)(2)\end{pmatrix} \tag{1}$$
$$= \begin{pmatrix}0\\-6\\-2\end{pmatrix}. \tag{5}$$

Step 4: Area of triangle

$$\text{Area of } \triangle ABC = \tfrac{1}{2}\,\|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A})\| \tag{2}$$
$$= \tfrac{1}{2}\sqrt{0^2+(-6)^2+(-2)^2} \tag{3}$$
$$= \tfrac{1}{2}\sqrt{40} \tag{4}$$
$$= \sqrt{10}. \tag{6}$$

Final Answer

$$\boxed{\text{Area of } \triangle ABC = \sqrt{10}}$$

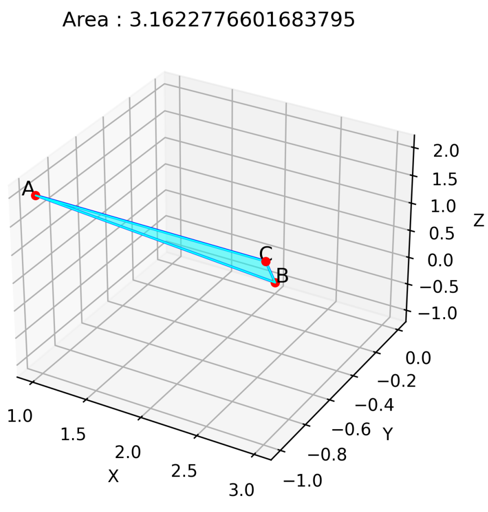

Figure 1: Triangle formed by points A, B, and C.