Matrix 1.7.1

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Question

Show that the points $(0,0)$, $(2m,-4)$, and $(3,6)$ are collinear, and hence find m, using the rank method.

Solution

Let the given points be

$$A = (0,0), \quad B = (2m,-4), \quad C = (3,6).$$

Step 1: Form vectors

$$AB = B - A = \begin{bmatrix} 2m \\ -4 \end{bmatrix}, \quad AC = C - A = \begin{bmatrix} 3 \\ 6 \end{bmatrix}.$$

Step 2: Matrix form

Construct the matrix

$$M = \begin{bmatrix} 2m & 3 \\ -4 & 6 \end{bmatrix}.$$

For the points to be collinear, the two vectors AB and AC must be linearly dependent. This means

$$\text{rank}(M) = 1 \quad \Leftrightarrow \quad \det(M) = 0.$$

Step 3: Determinant condition

$$\det(M) = (2m)(6) - (-4)(3) = 12m + 12.$$

Setting this equal to zero:

$$12m + 12 = 0 \quad \Rightarrow \quad m = -1.$$

Final Answer

The given points are collinear when

$$\boxed{m = -1}$$

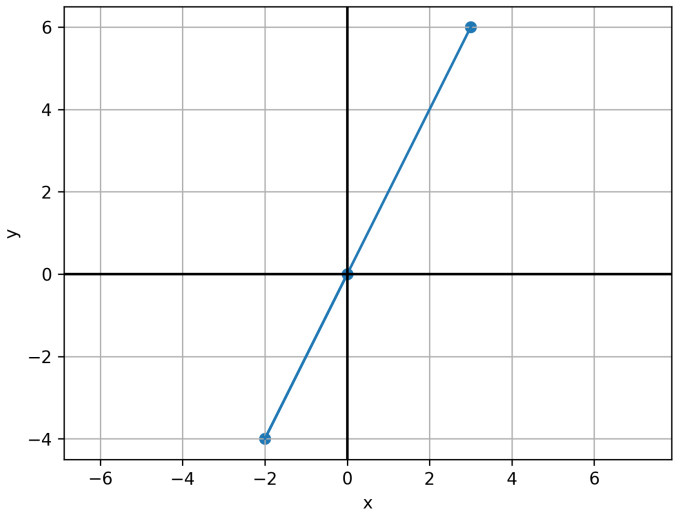

Figure 1: Graph