

frame=single, breaklines=true, columns=fullflexible

Matrix 2.10.4

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Question

Find the area of the triangle whose vertices are

$$A(1, -1, 2), \quad B(2, 0, -1), \quad C(3, -1, 2),$$

Vectors

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, \quad (2)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}. \quad (3)$$

Cross Product and Area

$$\mathbf{X} \times \mathbf{Y} = \begin{pmatrix} \begin{vmatrix} \mathbf{X}_{23} & \mathbf{Y}_{23} \\ \mathbf{X}_{31} & \mathbf{Y}_{31} \\ \mathbf{X}_{12} & \mathbf{Y}_{12} \end{vmatrix} \end{pmatrix} \quad (4)$$

$$(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -2 \end{pmatrix}. \quad (5)$$

Cross Product and Area

$$\mathbf{X} \times \mathbf{Y} = \begin{pmatrix} \begin{vmatrix} \mathbf{X}_{23} & \mathbf{Y}_{23} \\ \mathbf{X}_{31} & \mathbf{Y}_{31} \end{vmatrix} \\ \begin{vmatrix} \mathbf{X}_{31} & \mathbf{Y}_{31} \\ \mathbf{X}_{12} & \mathbf{Y}_{12} \end{vmatrix} \\ \begin{vmatrix} \mathbf{X}_{12} & \mathbf{Y}_{12} \\ \mathbf{X}_{23} & \mathbf{Y}_{23} \end{vmatrix} \end{pmatrix} \quad (4)$$

$$(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -2 \end{pmatrix}. \quad (5)$$

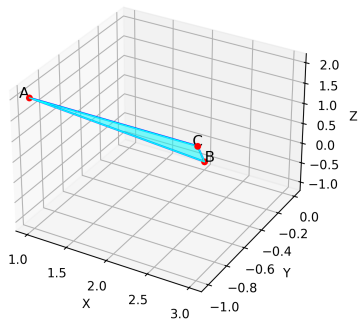
$$\text{Area} = \frac{1}{2} \left\| \begin{pmatrix} 0 \\ -6 \\ -2 \end{pmatrix} \right\| = \frac{1}{2} \sqrt{0^2 + (-6)^2 + (-2)^2} \quad (6)$$

$$= \frac{1}{2} \sqrt{36 + 4} = \frac{\sqrt{40}}{2} = \sqrt{10}. \quad (7)$$

$\text{Area} = \sqrt{10}$

Figure

Area : 3.1622776601683795



C Code

```
1
2 #include <stdio.h>
3
4 int main() {
5     int arr1[3] = {1, -1, 2};
6     int arr2[3] = {2, 0, -1};
7     int arr3[3] = {3, -1, 2};
8     FILE *fp = fopen("var.dat", "w");
9     if (fp == NULL) {
10         printf("Error opening file!\n");
11         return 1;
12     }
13     for (int i = 0; i < 3; i++) fprintf(fp, "%d%c", arr1
14         [i], i < 2 ? ' ' : '\n');
15     for (int i = 0; i < 3; i++) fprintf(fp, "%d%c", arr2
16         [i], i < 2 ? ' ' : '\n');
17     for (int i = 0; i < 3; i++) fprintf(fp, "%d%c", arr3
18         [i], i < 2 ? ' ' : '\n');
19     fclose(fp);
20     return 0;
21 }
```

Python Code (Part 1)

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from mpl_toolkits.mplot3d.art3d import Poly3DCollection
4
5 # Vertices
6 A = np.array([1, -1, 2])
7 B = np.array([2, 0, -1])
8 C = np.array([3, -1, 2])
9
10 AB = B - A
11 AC = C - A
12
13 # Area calculation
14 area = 0.5 * np.linalg.norm(np.cross(AB, AC))
15 print("Area of triangle ABC:", area)
16
17 # Create figure
18 fig = plt.figure()
19 ax = fig.add_subplot(111, projection='3d')
20
21 # Triangle vertices
22 triangle = np.array([A, B, C])
```


Python Code (Part 2)

```
1 # Plot triangle edges
2 ax.plot([A[0], B[0]], [A[1], B[1]], [A[2], B[2]], color='
  blue')
3 ax.plot([B[0], C[0]], [B[1], C[1]], [B[2], C[2]], color='
  blue')
4 ax.plot([C[0], A[0]], [C[1], A[1]], [C[2], A[2]], color='
  blue')
5
6 # Fill triangle surface
7 ax.add_collection3d(Poly3DCollection([triangle], color='cyan
  ', alpha=0.5))
8
9 # Scatter vertices
10 ax.scatter(A[0], A[1], A[2], color='red')
11 ax.text(A[0], A[1], A[2], "A", fontsize=12, ha='right')
12 ax.scatter(B[0], B[1], B[2], color='red')
13 ax.text(B[0], B[1], B[2], "B", fontsize=12, ha='left')
14 ax.scatter(C[0], C[1], C[2], color='red')
15 ax.text(C[0], C[1], C[2], "C", fontsize=12, ha='center')
16
17 # Labels
18 ax.set_xlabel("X")
19 ax.set_ylabel("Y")
20 ax.set_zlabel("Z")
21 ax.set_title("Area : " + str(area))
```