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Matrix 4.8.11

ai25btech11015 – M Sai Rithik

Question

Find the vector equation of the plane determined by the points

$$A(3, -1, 2), \quad B(5, 2, 4), \quad C(-1, -1, 6),$$

and hence find the distance of this plane from the origin.

Step 1: Position vectors

$$\mathbf{A} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix}. \tag{1}$$

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$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix}. \tag{2}$$

Step 2: Normal via cross product

$$(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}) = \begin{pmatrix} \left|\begin{pmatrix}3\\2\end{pmatrix} \quad \begin{pmatrix}0\\4\end{pmatrix}\right| \\ \left|\begin{pmatrix}2\\2\end{pmatrix} \quad \begin{pmatrix}-4\\4\end{pmatrix}\right| \\ \left|\begin{pmatrix}2\\3\end{pmatrix} \quad \begin{pmatrix}-4\\0\end{pmatrix}\right| \end{pmatrix}. \tag{3}$$

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$$= \begin{pmatrix} 12 \\ 16 \\ 12 \end{pmatrix}. \tag{4}$$

Step 2: Normal via cross product

$$(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}) = \begin{pmatrix} \begin{vmatrix} \begin{pmatrix}3\\2\end{pmatrix} & \begin{pmatrix}0\\4\end{pmatrix} \end{vmatrix} \\ \begin{vmatrix} \begin{pmatrix}2\\2\end{pmatrix} & \begin{pmatrix}-4\\4\end{pmatrix} \end{vmatrix} \\ \begin{vmatrix} \begin{pmatrix}2\\3\end{pmatrix} & \begin{pmatrix}-4\\0\end{pmatrix} \end{vmatrix} \end{pmatrix}. \tag{3}$$

$$= \begin{pmatrix}12\\16\\12\end{pmatrix}. \tag{4}$$

Simplify by dividing through 4:

$$\mathbf{N} = \begin{pmatrix}3\\-4\\3\end{pmatrix}. \tag{5}$$

Step 3: Finding place Eqn

Using $\mathbf{A}$:

$$d = \mathbf{N}^{\top}\mathbf{A} = \begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}. \tag{6}$$

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Equation of plane:

$$\mathbf{N}^\top x = 19. \tag{8}$$

Step 4: Normalize to $N^\top x = 1$

$$\widetilde{\mathbf{N}} = \frac{1}{19}\mathbf{N} = \begin{pmatrix} \frac{3}{19} \\ -\frac{4}{19} \\ \frac{3}{19} \end{pmatrix}. \tag{9}$$

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$$\boxed{\widetilde{\mathbf{N}}^\top x = 1} \tag{10}$$

Step 5: Distance from origin

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$$D = \frac{|1|}{\|\widetilde{\mathbf{N}}\|}. \tag{11}$$

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$$D = \frac{19}{\sqrt{34}} \approx 3.260. \tag{14}$$

Final Answer

$$\boxed{\widetilde{\mathbf{N}}^\top x = 1, \quad \widetilde{\mathbf{N}} = \begin{pmatrix} \frac{3}{19} \\ -\frac{4}{19} \\ \frac{3}{19} \end{pmatrix}} \tag{15}$$

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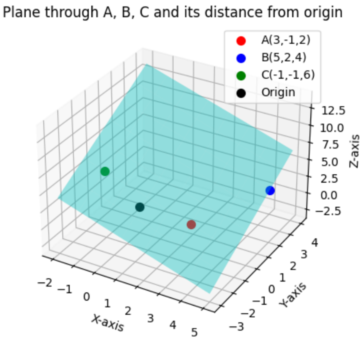
Plane through A, B, C and its distance from origin
A(3,-1,2)
B(5,2,4)
C(-1,-1,6)
Origin
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
Z-axis
−2
−1
0
1
2
3
4
5
X-axis
−3
−2
−1
0
1
2
3
4
Y-axis

Figure: