AI25BTECH11034 - SUJAL CHAUHAN
5.7.7

Question:

If $\mathbf{A}$ is square matrix such that$\mathbf{A}^2 = \mathbf{A}$ then find value of $(\mathbf{I} + \mathbf{A})^3 - 7\mathbf{A}$.

Solution Let's expand the equation:

$$= (\mathbf{I} + \mathbf{A})^2(\mathbf{I} + \mathbf{A}) - 7\mathbf{A} \tag{1}$$

$$= (\mathbf{A}^2 + \mathbf{AI} + \mathbf{IA} + \mathbf{I}^2)(\mathbf{I} + \mathbf{A}) - 7\mathbf{A} \tag{2}$$

$$= (\mathbf{A} + \mathbf{A} + \mathbf{A} + \mathbf{I})(\mathbf{I} + \mathbf{A}) - 7\mathbf{A} \tag{3}$$

$$= (3\mathbf{A} + \mathbf{I})(\mathbf{I} + \mathbf{A}) - 7\mathbf{A} \tag{4}$$

$$= 3\mathbf{AI} + 3\mathbf{A}^2 + \mathbf{I}^2 + \mathbf{IA} - 7\mathbf{A} \tag{5}$$

$$= 3\mathbf{A} + 3\mathbf{A} + \mathbf{I} + \mathbf{A} - 7\mathbf{A} \tag{6}$$

$$= \mathbf{I} \tag{7}$$