5.7.7

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Question

If $\mathbf{A}$ is a square matrix such that $\mathbf{A}^2 = \mathbf{A}$, then find the value of $(\mathbf{I} + \mathbf{A})^3 - 7\mathbf{A}$.

Solution

Solution

Let's expand the equation:

$$
\begin{aligned}
(\mathbf{I}+\mathbf{A})^3 - 7\mathbf{A} &= (\mathbf{I}+\mathbf{A})^2(\mathbf{I}+\mathbf{A}) - 7\mathbf{A} \\
&= (\mathbf{A}^2 + \mathbf{AI} + \mathbf{IA} + \mathbf{I}^2)(\mathbf{I}+\mathbf{A}) - 7\mathbf{A} \\
&= (\mathbf{A}+\mathbf{A}+\mathbf{A}+\mathbf{I})(\mathbf{I}+\mathbf{A}) - 7\mathbf{A} \\
&= (3\mathbf{A}+\mathbf{I})(\mathbf{I}+\mathbf{A}) - 7\mathbf{A} \\
&= 3\mathbf{AI} + 3\mathbf{A}^2 + \mathbf{I}^2 + \mathbf{IA} - 7\mathbf{A} \\
&= 3\mathbf{A} + 3\mathbf{A} + \mathbf{I} + \mathbf{A} - 7\mathbf{A} \\
&= \mathbf{I}
\end{aligned}
$$