AI25BTECH11034 - SUJAL CHAUHAN

4.4.30

Question:

Determine the product $\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$

and use this to solve the equations:

$$\begin{cases} x - y + z = 4 \\ x - 2y - 2z = 9 \\ 2x + y + 3z = 1 \end{cases}$$

Solution

Given equation can be write in form $\mathbf{AX} = \mathbf{b}$

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix} \tag{1}$$

Let's name our two matrices:

$$\mathbf{P} = \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix} \tag{2}$$

We can observe that $\mathbf{Q} = \mathbf{A}$

Now, Let's determine the product $\mathbf{PQ}$ of the given two matrices:

$$\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} \tag{3}$$

$$\mathbf{PQ} = 8\mathbf{I} \tag{4}$$

Which is multiple of Identity matrix we can use that fact and multiply $\mathbf{P}$ both side of our equation.

$$\mathbf{PAX} = \mathbf{Pb} \tag{5}$$

$$\mathbf{X} = \frac{\mathbf{Pb}}{8} \tag{6}$$

$$\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix} \tag{7}$$

$$\begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \tag{9}$$

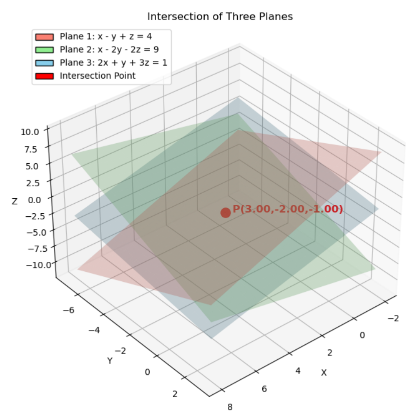

Figure 1: Intersection of three planes