Bayesian Causal Discovery with Preference-Guided Normalising Flows

Cesare Bergossi
Supervisor: Prof. Sonali Parbhoo

Department of Computing
Imperial College London

MSc Thesis Presentation
September 2025

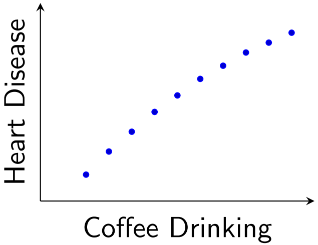
IMPERIAL

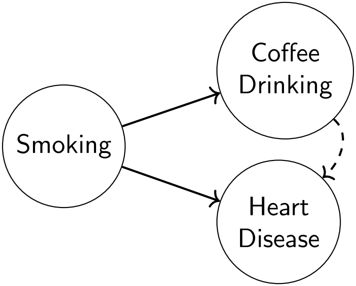
Outline

Why Causal Discovery?

Correlation $\neq$ Causation

- Correlations describe patterns in data.
- Causation explains what changes when we **intervene**.
- Medicine, policy, control: we must ask “what if we act?”





Challenges in Practice

Causal discovery is demanding in real-world settings.

Data Scarcity
costly or infeasible experiments

Noisy Observations
hidden confounders, errors

Costly Interventions
limited opportunities to test

Scalability Limits
classical methods struggle with large systems

More data is not always the answer – we need alternative signals.

Why Preferences?

Comparisons are easier than numerical outcomes

- Experts can judge "A is better than B"
- No need for precise or calibrated scores

Preferences still carry signal

- Encode knowledge about causal effects
- Expressed as rankings $x_i \succ x_j$

Preferential data provides indirect supervision when direct numeric labels are limited or ambiguous.

Research Question

Can preferences guide causal discovery?

Main ingredients

- Preferences: $x_i \succ x_j$ instead of numeric outcomes
- Bayesian view: beliefs over graphs, updated with data
- Active loop: use feedback to decide what to test next

The idea: use simple choices to recover the true causal structure.

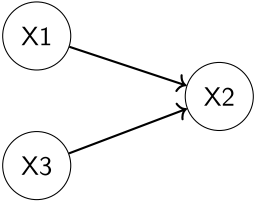
Outline

Structural Causal Models (SCMs)

Definition

$$X_i = f_i(\mathrm{Pa}(X_i)) + \epsilon_i \tag{1}$$

- Directed Acyclic Graph (DAG) – nodes and causal edges
- Parent variables influence each node
- Interventions described via *do*-calculus



SCMs formalise causal reasoning: they tell us what happens under interventions.

Bayesian Causal Discovery

Probabilistic Representation

- Graph G is treated as a random variable
- Keep a distribution over possible graphs, not a single guess
- Quantifying uncertainty is useful in data-limited or noisy settings

Bayesian Update Rule

$$P(G|D) \propto P(D|G) \cdot P(G) \tag{2}$$

- $P(G|D)$ posterior distribution $\rightarrow$ current beliefs about graph
- $P(D|G)$ likelihood of data given graph G
- $P(G)$ prior distribution over graphs

Bayesian Causal Discovery

Active Learning

- Observations alone leave ambiguities (Markov equivalence classes)
- *Acquisition functions* select interventions to solve these ambiguities
- Criterion: *information theory* – choose action expected to most reduce uncertainty about graph G

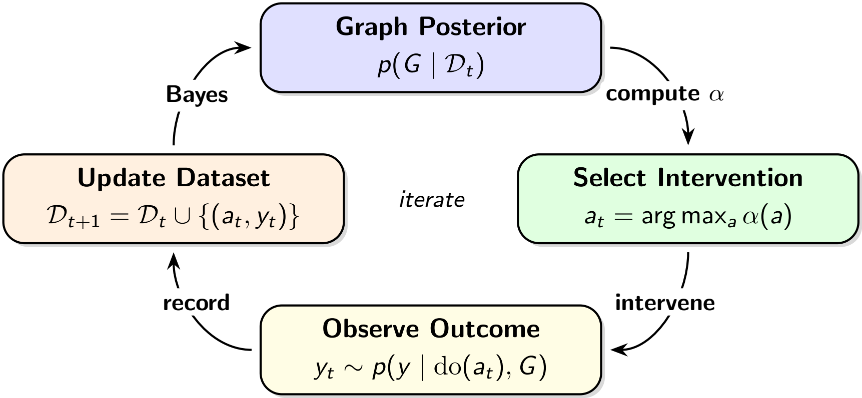
$$\alpha(x) = I(G; Y \mid \mathrm{do}(x), D) \tag{3}$$

for mutual information (entropy reduction)

$$I(A; B) = H(A) - H(A \mid B) \tag{4}$$

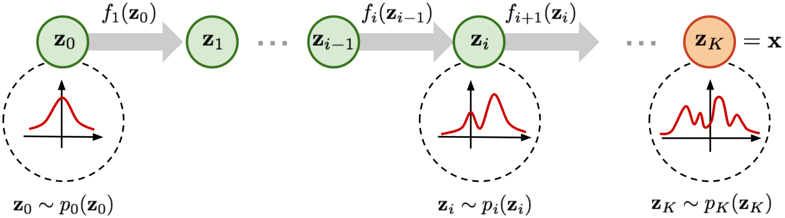
Causal Bayesian Optimisation (CBO)

Causal Bayesian Optimisation treats discovery as a sequential decision process



Normalising Flows (NFs)

Normalising Flows are flexible generative models that learn complex probability distributions

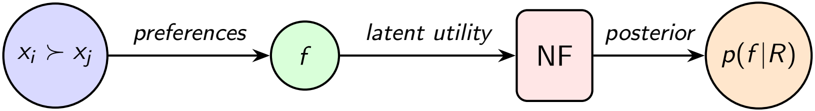


Properties

- Invertible transformations
- Exact density calculations
- Fully differentiable

Preferential Normalising Flows (PNFs)

Key Idea: Learn from *preferences* (comparisons, rankings) instead of explicit numeric outcomes.



Properties

- Model *latent utilities* behind observed choices
- Flexible: can capture multimodal preference distributions
- Bayesian: provide uncertainty-aware utility posteriors

Outline

From CBO to PCBO

Causal Bayesian Optimisation (CBO)

- *Numeric outcomes* $\rightarrow$ update beliefs about the graph
- *Posterior over graphs* $\rightarrow$ guides next intervention

Preferential Causal Bayesian Optimisation (PCBO)

- *Preferences* $(x_i \succ x_j)$ $\rightarrow$ learn latent utility function f (through PNFs)
- *Utility function* f $\rightarrow$ informs which causal graphs are plausible
- *Causal graph* $\rightarrow$ determines the most informative intervention

PCBO is a Bayesian feedback loop that combines preference-guided utility learning and Bayesian causal inference.

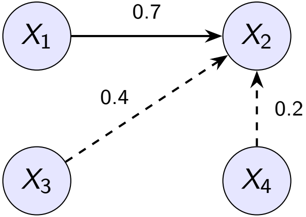
Parent Posteriors in PCBO

Local Edge Beliefs

- Instead of tracking full graph posterior $p(G|D)$ (infeasible for large G)
- Maintain *parent posterior* π_i for each node X_i (prob. distribution over possible parent sets S)
- Update each π_i locally (factorised approximation)

$$\pi_i(S) \propto \ell(S) \cdot p(S) \tag{5}$$

  for $\ell(S)$ marginal likelihood, $p(S)$ sparsity prior
- Scalable variant with MCMC approximation for > 10 nodes



Acquisition Functions: PIG and EEIG

Preference Information Gain (PIG)

$$\mathrm{PIG}(a,b) = H(p(a \succ b)) \tag{6}$$

- **Utility-focused:** uses entropy of the flow's win probability
- Picks comparisons where the preference model is maximally uncertain

Expected Edge Information Gain (EEIG)

$$\mathrm{EEIG}(a,b) = H(\pi) - \left[p(a \succ b)H(\pi^{(a)}) + (1 - p(a \succ b))H(\pi^{(b)})\right] \tag{7}$$

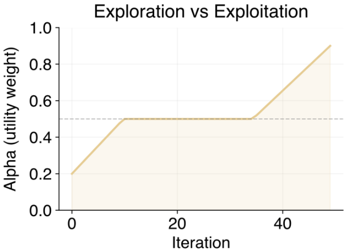
where π *= edge marginals,* $\pi^{(i)}$ *= edge marginals if candidate* i *won.*

- **Graph-focused:** expected entropy reduction in edge marginals
- Rewards interventions that most reduce structural uncertainty

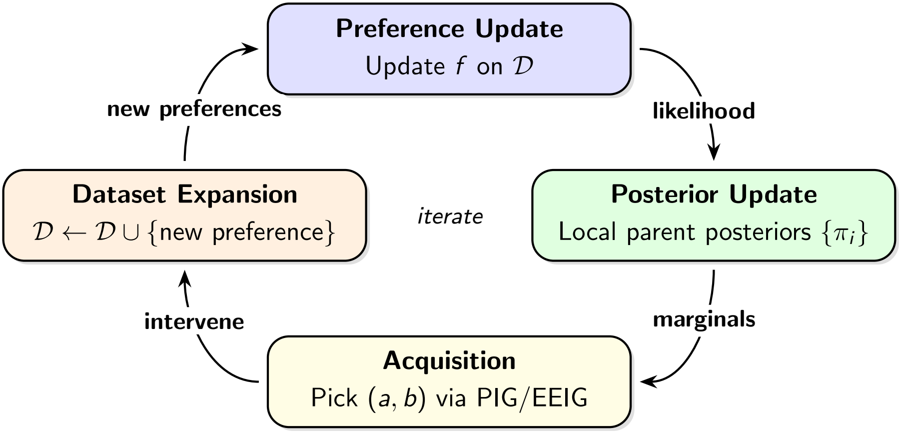
Balancing PIG and EEIG

- **PIG**: cheap, utility-focused exploration
- **EEIG**: expensive, but targets graph structure
- **Mixing** balances graph exploration (early iterations, structure) and exploitation (later iterations, preferences)

$$S_t(a, b) = \alpha_t \,\mathrm{PIG}(a, b) + (1 - \alpha_t)\,\mathrm{EEIG}(a, b) \tag{8}$$



PCBO Loop



Preference Update
Update f on $\mathcal{D}$
likelihood
Posterior Update
Local parent posteriors $\{\pi_i\}$
marginals
Acquisition
Pick (a, b) via PIG/EEIG
intervene
Dataset Expansion
$\mathcal{D} \leftarrow \mathcal{D} \cup \{\text{new preference}\}$
new preferences
iterate

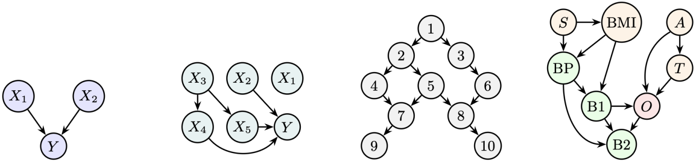
Outline

Main Questions

- Can PCBO recover causal graphs from *preferences alone*?
- How does it compare against baselines trained on full outcomes?
- What role do *thresholding* and acquisition functions play?

Datasets

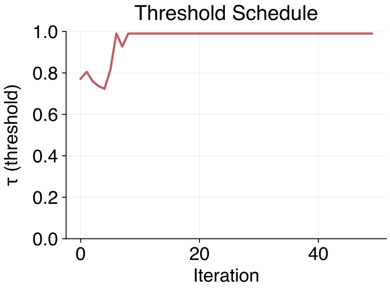
- Synthetic DAGs: 3-node, 6-node, Erdős–Rényi (15, 20 nodes)
- Medical case study with domain-specific variables

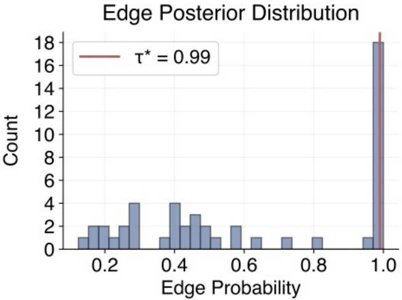


(a) 3-node toy **(b)** 6-node mixed toy **(c)** ER DAG (10 nodes) **(d)** Medical toy

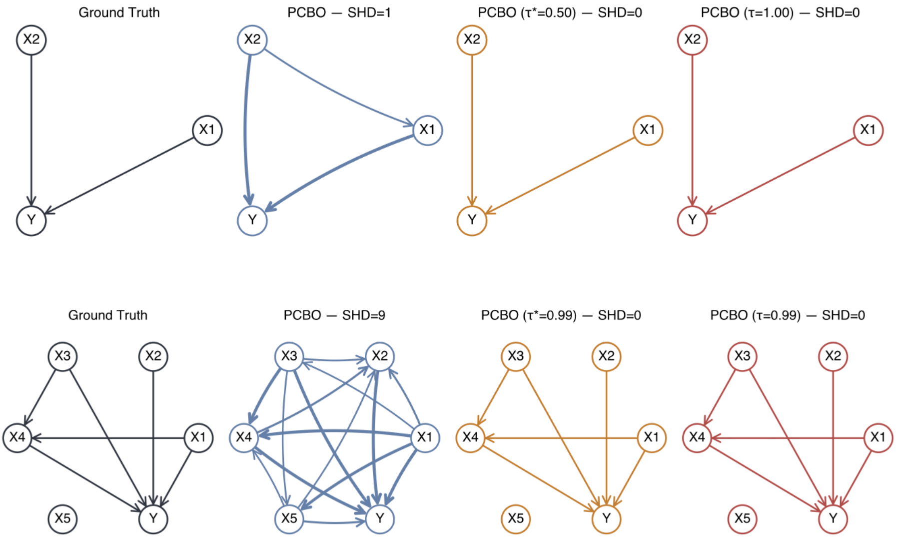
Threshold with Beta Mixture

Beta mixture model sets the threshold adaptively based on the edge marginals distribution.

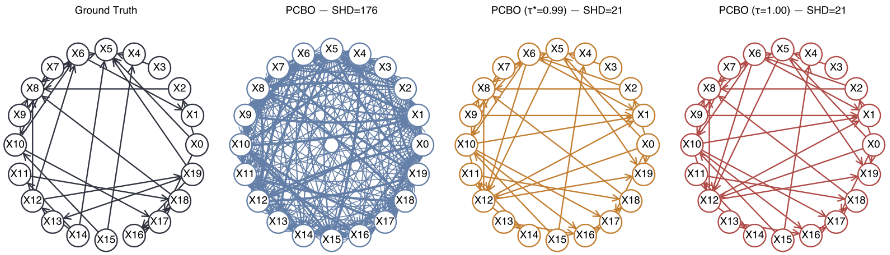




Small Graphs: 3, 6 Nodes

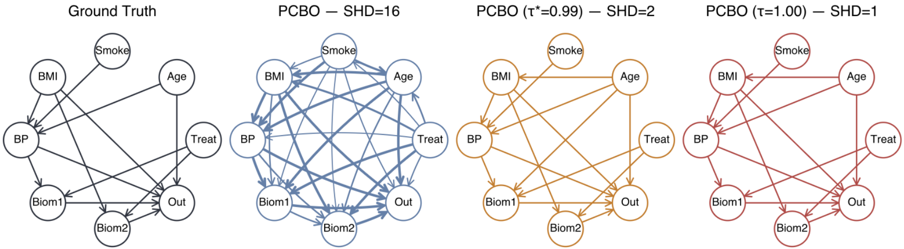


Larger Graphs: ER-20



Method	Precision	Recall	F1	SHD	#Edges
PCBO	0.111	0.750	0.193	176	190
PCBO ($\tau^* = 0.99$)	0.600	0.750	0.667	21	35
PCBO ($\tau = 1.00$)	0.600	0.750	0.667	21	35
Random	0.083	0.179	0.114	78	60
LASSO	0.190	1.000	0.320	119	147
PC	0.000	0.000	0.000	45	17
NOTEARS-Lite	0.857	0.857	0.857	8	28
Causal Sufficiency	0.000	0.000	0.000	28	0
Fully Connected	0.063	0.429	0.110	194	190

Medical Case Study



Method	Precision	Recall	F1	SHD	#Edges
PCBO ($\tau = 1.00$)	0.923	1.000	0.960	1	13
Random	0.600	0.250	0.353	11	5
LASSO	1.000	0.917	0.957	1	11
PC	0.000	0.000	0.000	14	2
NOTEARS-Lite	1.000	0.750	0.857	3	9
Causal Sufficiency	0.636	0.583	0.609	9	11
Fully Connected	0.429	1.000	0.600	16	28

Experimental Takeaways

- **Scalability:** accurate recovery from small up to 20-node graphs using only preferences
- **Thresholding:** automatic (Beta mixture) or manual (intuitive), essential to denoise posteriors and refine graph structures
- **Medical case study:** near-perfect recovery, outperforming baselines that use numerical outcomes
- **Flow variants:** structure stable across NF types; differences only in preference calibration

Outline

Summary of Contributions

- Proposed **PCBO**: a Bayesian framework for causal discovery from preferences
- Designed **acquisition strategies** (PIG, EEIG, hybrid scheduling) to jointly refine graph structure and preference learning
- Developed **scalable Bayesian inference** via local parent posteriors
- Empirically demonstrated PCBO's ability to **recover graph structure**, with refinements for stability and efficiency
- Integrated all components into a **coherent system**, ensuring each part works smoothly with the others

Future Directions & Impact

Limitations

- Assumes reliable preference data and causal sufficiency
- Relies on clear separation between strong and weak edges for thresholding
- EEIG becomes costly on larger graphs

Future Directions

- Apply to real-world domains (medicine, biology, decision-making)
- Improve scalability with faster inference strategies
- Extend to dynamic (temporal) causal graphs
- Make the framework interactive with expert-in-the-loop preferences

Thank you for your attention!

Questions?