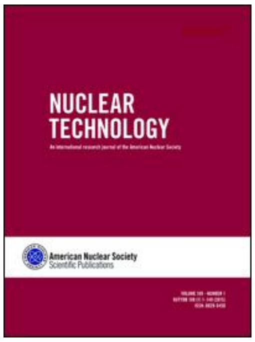

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DERIVATION OF GEOMETRY FACTORS FOR INTERNAL GAMMA DOSE CALCULATIONS FOR A CYLINDRICAL RADIOACTIVE WASTE PACKAGE

RADIATION PROTECTION

KEYWORDS: *geometry factor, internal gamma dose, cylindrical waste package*

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A general methodology was developed to estimate geometry factors for internal gamma dose rate calculations within a cylindrical radioactive waste container. In particular, an average geometry factor is needed to calculate the average energy deposition rate within the container for determination of the internal gas generation rate. Such a calculation is required in order to assess the potential for radioactive waste packages to radiolytically generate combustible gases.

This work therefore provides a method for estimating the point and average geometry factors for internal dose assessment for a cylindrical geometry. This analysis is compared to other results where it is shown that the classical work of Hine and Brownell do not correspond to the average geometry factors for a cylindrical body but rather to values at the center of its top or bottom end. The current treatment was further developed into a prototype computer code (PC-CAGE) that calculates the geometry factors numerically for a cylindrical body of any size and material, accounting both for gamma absorption and buildup effects.

I. INTRODUCTION

Radioactive waste packages containing water and/or organic substances have the potential to radiolytically generate hydrogen and other combustible gases such as methane and carbon monoxide. The rate of radiolytic gas generation in a radioactive waste package is directly related to the rate of energy deposition within the material in the package. It is estimated from the product of the energy deposition rate and the radiolytic gas yield (i.e., *G* value).

While the energy or dose deposited from alpha- and beta-emitting radionuclides in a radioactive waste package is easy to estimate because it is completely deposited locally, estimation of the dose from gamma emitters is not straightforward because of the penetrating nature of gamma radiation. Typically, a point kernel approach is utilized to formulate an expression for the gamma dose rate at an internal dose point within a package.[1] Taking an integral over the volume of the package provides an estimate of the dose rate at the selected point from all volume elements in the package. The integral of the geometry-dependent terms in the dose rate expression is called a point geometry factor. An average geometry factor can be estimated by volume averaging the point geometry factor (the average dose rate is generally of greater interest than the dose rate at a specific point within the package).

Unlike the case of a spherical body,[1–3] analytical derivations for the geometry factor of a cylindrical body have not been reported. However, Hine and Brownell[4] have published tabular values for the geometry factor of a cylindrical body, but the data are limited to a cylinder with a height and radius of only 100 and 35 cm, respectively.[3] Swyler et al.[5] extrapolated these values to estimate the cumulative dose to Epicor-II ion exchange media waste in a cylindrical container with a radius of 60 cm. However, Focht et al.[6] suggested that the tabular data presented in the classical analysis of Hine and Brownell[4] apply to dose calculations on the surface of a cylinder at the end of its axis and do not represent the average geometry factor for the body as claimed. Although Focht

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et al.[6] published corrected values in tabular form, they did not provide specific details of their methodology, and hence, calculations for a larger cylinder cannot generally be made.

In this work, geometry factors are derived for a cylindrical body, accounting for both attenuation and buildup effects. Because of the difficulty in obtaining analytical solutions for a cylindrical geometry, a numerical treatment was developed and implemented as a prototype Visual C++ computer code, PC-CAGE (Cylinder Average G Estimate). This numerical treatment is applicable to cylinders of any specified dimensions and material.

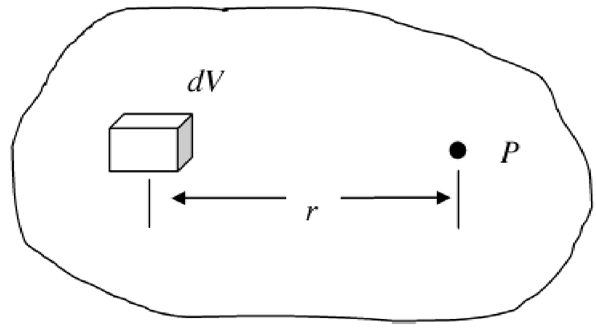

Fig. 1. Diagram for calculating dose at point P from gamma rays emitted from the volume element dV in a mass containing a uniformly distributed radionuclide.

II. MODEL DEVELOPMENT

Consider the case of a radionuclide that emits a gamma photon with energy E (MeV) at the rate of S_V $[\gamma/(\mathrm{s\ m}^3)]$ inside a source volume V_S. The dose rate, $\dot{D}$ [MeV/(s kg)], received by a target with volume V_T is given by the following expression[1]:

$$\dot{D} = \frac{\int_{V_S}\int_{V_T} \frac{S_V\,dV_S}{4\pi r^2}\, e^{-\mu r} E \mu_a B(\mu r)\, dV_T}{\rho V_T} \quad , \tag{1}$$

where

μ = total linear attenuation coefficient (m^{-1}) for gamma photons of energy E

μ_a = linear energy absorption coefficient (m^{-1})

$B(\mu r)$ = buildup factor

ρ = density of the target material ($\mathrm{kg/m}^3$).

Defining a geometry factor g as[1]

$$g = \frac{1}{V_T}\int_{V_T} dV_T \int_{V_S} \frac{e^{-\mu r}}{r^2} B(\mu r)\, dV_S \quad , \tag{2}$$

Eq. (1) can be written as

$$\dot{D} = \frac{S_V}{4\pi} E \left(\frac{\mu_a}{\rho}\right) g \quad . \tag{3}$$

Note that g has dimensions of length. When the source is also the target (i.e., $V_S = V_T = V$) as depicted in Fig. 1, Eq. (2) can be simplified to[3]

$$g = \int_V \frac{B(\mu r) e^{-\mu r}}{r^2}\, dV \quad , \tag{4a}$$

where B can be reasonably approximated as an exposure buildup factor for an isotropic point source.[7,8] The effect of buildup may be approximated (i.e., for water and/or tissue) by replacing the linear attenuation coefficient μ by a linear energy absorption coefficient μ_a resulting in the simplified expression for g (Refs. 3, 4, and 6):

$$g = \int_V \frac{e^{-\mu_a r}}{r^2}\, dV \quad . \tag{4b}$$

When gamma rays interact with matter, electrons or positrons are set in motion, which dissipate their energy as they are brought to rest. The total linear attenuation coefficient in Eq. (4a) depends on the total photon interaction cross section, which takes into account all elementary absorption and scattering processes including photoelectric absorption, Compton collision, and pair production. The buildup factor in Eq. (4a) further considers the rescatter of secondary photons, produced in the absorber by Compton scattering, back to the target point. On the other hand, the linear energy absorption coefficient in Eq. (4b) discounts the amount of energy that escapes from the volume in the form of the Compton-scattered photons. At energies greater than 10 MeV, the concept of the linear absorption coefficient becomes somewhat limited because the range of the secondary electrons is comparable with the gamma-ray mean free path; however, below 10 MeV, the travel of the electron is less significant so that the problem can be treated using the approximation of a linear energy absorption coefficient in Eq. (4b) (Refs. 3 and 9).

Mass attenuation coefficients μ/ρ and mass absorption coefficients μ_a/ρ as required for the estimation of the linear coefficients in Eqs. (4a) and (4b) are tabulated, for example, in Refs. 7, 8, and 9.

II.A. Geometry Factor for a Cylindrical Body

The solution of Eq. (4) is examined in this section for three specific cases:

1. geometry factor at the center of a cylinder (g_c) (Sec. II.A.1)
2. geometry factor at any point in a cylinder (g_P) (Sec. II.A.2)
3. average geometry factor for the cylinder ($\bar{g}$) (Sec. II.A.3).

II.A.1. Central Geometry Factor g_c

The geometry factor at the center of a cylinder (g_c) of radius R and height H can be evaluated from Fig. 2. For the cylindrical coordinate system shown in Fig. 2 and taking into account the azimuthal symmetry of the problem in which $dV_{shell} = 2\pi r dr dz$, Eq. (4b) can be written as

$$g_c = 2\int_0^{H/2}\int_0^R \left(\frac{e^{-\mu_a \xi}}{\xi^2}\right) 2\pi r dr dz \ . \tag{5a}$$

Using the Pythagorean theorem $\xi^2 = z^2 + r^2$, one can apply a change of variable $r \to \xi$ where $dr = (\partial r/\partial \xi) d\xi$. Hence, the transformation $rdr = \xi d\xi$ applies, and Eq. (5a) becomes

$$g_c = 4\pi \int_0^{H/2} \left[\int_z^{\sqrt{z^2+R^2}} \frac{e^{-\mu_a \xi} d\xi}{\xi}\right] dz$$

$$= 4\pi \int_0^{H/2} [E_1(\mu_a z) - E_1(\mu_a \sqrt{z^2+R^2})] dz \ . \tag{5b}$$

Here, the exponential integral function is generally defined by[7,10]

$$E_n(x) = x^{n-1}\int_x^{\infty} \frac{e^{-u}}{u^n} du \ , \tag{6}$$

whose values are tabulated in most mathematical handbooks. Because of its complexity, Eq. (5) must be solved numerically. It can be readily evaluated using Maple, a commercially available software package.[11] Maple uses a Clenshaw-Curtis quadrature as the default routine with an error tolerance of eps $= 0.5 \times 10^{(1\text{-Digits})}$ (a default value of ten was used in the current analysis for Digits). Sample results are shown in Fig. 3 for a cylinder with a height and radius of 100 and 35 cm, respectively, containing material with $\mu_a = 0.028$ cm^{-1} (see also discussion in Sec. IV).

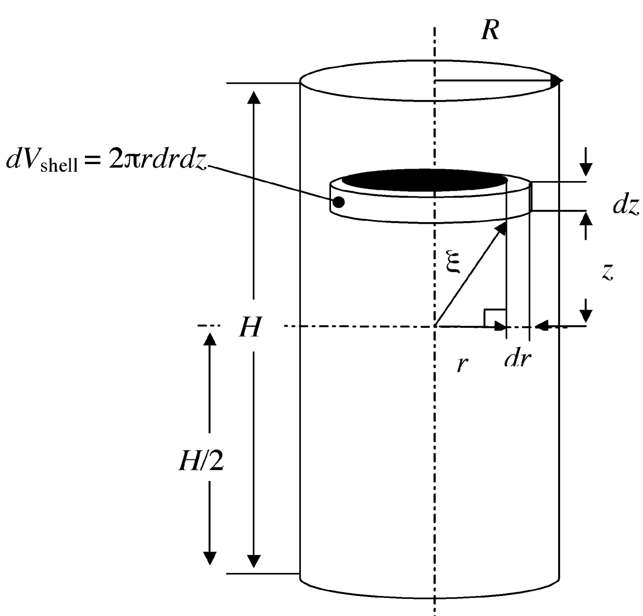

Fig. 2. Geometry for evaluating Eq. (5a).

II.A.2. Geometry Factor for a General Point in a Cylinder g_P

Consider a point P in the cylinder (see Fig. 4) located a distance D_c from the base of the cylinder and a distance R_c from the central axis of the cylinder. In this more general case, the geometry factor g_P can be evaluated from Eq. (4a) where buildup effects are considered explicitly. It follows that

$$g_P = \int_0^H \int_0^{2\pi} \int_0^R B(\mu\xi) \left(\frac{e^{-\mu\xi}}{\xi^2}\right) r dr d\theta dz \ . \tag{7}$$

The parameter ξ in Eq. (7) can be evaluated as a function of r, θ, and z for the geometry of Fig. 4 using the Pythagorean theorem $\xi^2 = \ell^2 + (z - D_c)^2$ (side view) and the cosine law $\ell^2 = R_c^2 + r^2 - 2R_c r\cos(\pi - \theta)$ (top view), yielding $\xi = \sqrt{R_c^2 + r^2 + 2R_c r\cos\theta + (z - D_c)^2}$. For the buildup factor, a simple Taylor form consisting of the sum of two exponential terms, namely,

$$B(\mu\xi) = Ae^{-\alpha\mu\xi} + (1 - A)e^{-\beta\mu\xi} \ , \tag{8}$$

was employed. This form for the buildup factor was selected because it can be readily combined with the attenuation exponential in Eq. (7). The buildup coefficients A, α, and β for an isotropic point source are tabulated in Ref. 7 for various materials as a function of the gamma-ray energy E. Unfortunately, Eq. (7) is now too complicated to be solved using the Maple package and must be evaluated by other numerical means (Sec. III).

```
> mu:=0.028;
                    mu := .028
> H:=100;
                    H := 100
> R:=35;
                    R := 35
> g:=evalf(Int(Ei(1,mu*z)-Ei(1,mu*sqrt(z^2+R^2)),z=0..H/2));
                  g := 24.90320096
> g_c:=evalf(g*4*Pi);
                  g_c := 312.9428528
```

Fig. 3. Sample Maple program for numerical integration of Eq. (5b).

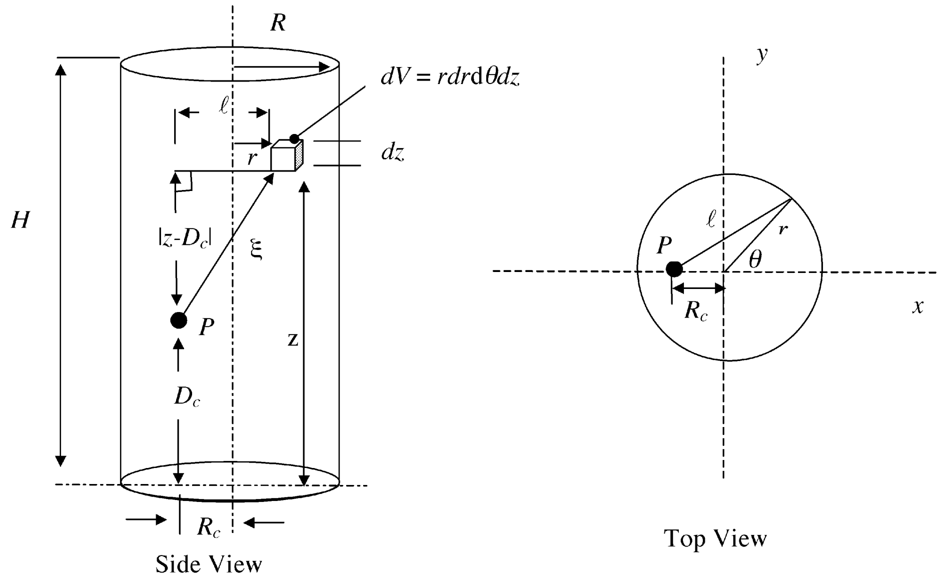

Fig. 4. Geometry for dose calculation at any point P inside a cylinder.

II.A.3. Average Geometry Factor $\bar{g}$

The average geometry factor $\bar{g}$ can be derived using the general relation[3]:

$$\bar{g} = \frac{1}{V}\int g_P \, dV \ . \tag{9}$$

For a cylinder of radius R and height H, Eq. (7) yields a value of g_P at the point (R_c, D_c). By replacing R_c by r and D_c by z, g_P values at other points in the cylinder can similarly be estimated. The quantity $\bar{g}$ can then be evaluated by numerical integration over the complete volume of the cylinder via Eq. (9) (see Sec. III).

III. NUMERICAL IMPLEMENTATION

In order to obtain the required accuracy in a reasonable computational time, a Gauss quadrature method was selected for estimating the geometry factor g_P. A transformation $x = b(t+1)/2$ can be used to provide the required integration limits for this method[12]:

$$\int_0^b f(x)dx = \frac{b}{2}\int_{-1}^{1} f\left(\frac{b(t+1)}{2}\right)dt \ . \tag{10}$$

Thus, using Eqs. (8) and (10), Eq. (7) becomes

$$g_P = \frac{\pi R^2 H}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}(x+1)\left[\frac{Ae^{-\mu\xi(\alpha+1)} + (1-A)e^{-\mu\xi(\beta+1)}}{\xi^2}\right]dxdydz \ , \tag{11}$$

where

$$\xi = \sqrt{R_c^2 + R^2(x+1)^2/4 + R_c R(x+1)\cos\{\pi(y+1)\} + (H(z+1)/2 - D_c)^2} \ .$$

Employing a Gauss quadrature procedure, Eq. (11) can be reduced to a triple summation:

$$g_P = \frac{\pi R^2 H}{8}\sum_{i=1}^{96}\sum_{j=1}^{96}\sum_{k=1}^{96} a_i a_j a_k f(x_i, y_j, z_k) \ , \tag{12}$$

where

$$f(x,y,z) = (x+1)\left[\frac{A\exp\{-\mu\xi(x,y,z)[\alpha+1]\} + (1-A)\exp\{-\mu\xi(x,y,z)[\beta+1]\}}{\xi(x,y,z)^2}\right] \ .$$

Here, the a_i's (as well as the a_j's and a_k's) and x_i's (as well as the y_j's and z_k's) correspond to the weight factors and zeros for a Legendre polynomial, $P_m(x)$, of order m. A polynomial of order $m = 96$ was chosen to obtain reasonable accuracy (relative error was less than ~2%), without encompassing too much computational effort. Accordingly, for each point in the cylinder, $96 \times 96 \times 96$ computations were required for evaluation of the g_P integral. The required weight factors and zeros are tabulated in Ref. 13.

Because g_c is a special case of Eq. (7) or (11) with $D_c = H/2$ and $R_c = 0$, an improved estimate for g_c (compared to that obtained using Maple) can be obtained with the Gauss quadrature method by exploiting the symmetry of Fig. 2 where only a double integral is required; i.e.,

$$g_c = 4\pi \int_0^{H/2} \int_0^R B(\mu\xi)\left(\frac{e^{-\mu\xi}}{\xi^2}\right) rdrdz$$

$$= \frac{\pi R^2 H}{4} \int_{-1}^{1}\int_{-1}^{1} (x+1)\left\{\frac{Ae^{-\mu\sqrt{\frac{R^2(x+1)^2}{4}+\frac{H^2(y+1)^2}{16}}(\alpha+1)} + (1-A)e^{-\mu\sqrt{\frac{R^2(x+1)^2}{4}+\frac{H^2(y+1)^2}{16}}(\beta+1)}}{\frac{R^2(x+1)^2}{4}+\frac{H^2(y+1)^2}{16}}\right\} dxdy$$

$$\approx \frac{\pi R^2 H}{4} \sum_{i=1}^{96}\sum_{j=1}^{96} a_i a_j f(x_i, y_j) \ . \tag{13}$$

Finally, one can take advantage of the symmetry of g_p about the midplane and azimuthal direction to evaluate $\bar{g}$ in Eq. (9) as

$$\bar{g} = \frac{1}{\pi R^2 H}\left\{2\int_0^{H/2}\int_0^R g_P(r,z)\cdot 2\pi rdrdz\right\} = \frac{4}{R^2 H}\left\{\int_0^{H/2}\int_0^R g_P(r,z)\cdot rdrdz\right\} \ , \tag{14}$$

where the function $g_P(r, z)$ is evaluated via Eq. (12). For the numerical integration of Eq. (14), an equally spaced trapezoidal rule can be employed:

$$\bar{g} = \left(\frac{4}{R^2 H}\right)\left(\frac{hk}{4}\right)\sum_{i=0}^{n}\sum_{j=0}^{n} w_{ij}\, g_p(r_i, z_j)\cdot r_i \ , \tag{15}$$

where $h = R/n$ and $k = H/(2n)$ are the radial and axial step sizes, respectively, for the required mesh. The number of intervals n is chosen to provide the required accuracy. Here, w_{ij} are weighting factors as tabulated in Ref. 14. A value of $n = 7$ yields an accuracy of ~7%, but this accuracy can be improved to ~3% by choosing $n = 20$ although at the expense of increased computational effort. The computational time will depend on the given processor speed and computer memory. In particular, on a Pentium III computer with 128 megabytes of random access memory and a 1-GHz processor, a $\bar{g}$ estimate for $n = 20$ requires ~14 min and only 2 min for $n = 7$.

Equations (12), (13), and (15) have been developed into a personal computer (PC)-based computer code entitled PC-CAGE (Cylinder Average G Estimate). This code is written under a Visual C++ (Version 6.0) platform with a graphical user interface (see Fig. 5) so that it can operate under a Windows or NT environment.

IV. COMPARISON OF ESTIMATED GEOMETRY FACTORS WITH LITERATURE-BASED RESULTS

The PC-CAGE program was validated by comparing calculated g_c results with those based on Maple (see Table I). Calculations were based on Eq. (13) where B was set equal to one to facilitate calculations with Maple and μ was chosen equal to 0.028 cm^{-1}. The results obtained should thus be equivalent to the case where buildup is not explicitly considered but μ is replaced by μ_a with the same numerical value. Hence, the case considered was analogous to that for which Hine and Brownell[4] developed their tabular data. As shown in Table I, results based on PC-CAGE deviate less than 0.1% from values generated using Maple.

Calculations were next done using PC-CAGE to address the suggestion by Focht et al.[6] that the estimates in the classical work of Hine and Brownell[4] do not represent $\bar{g}$ values as claimed but rather a geometry factor on the surface of a cylinder at the end of its axis. For this purpose, g_p was evaluated at the position $R_c = 0$ and $D_c = 0$ (see Fig. 4) using Eq. (12). Indeed, as shown in Table II, the values for $g_p(0,0)$ agree within 0.7% of the values proposed by Hine and Brownell.[4] This substantiates the claim made by Focht et al. and calls into question the methodology used by Swyler et al.[5] to estimate $\bar{g}$ for a larger cylinder based on the extrapolation of the Hine and Brownell results; the latter are given for the maximum height and radius of 100 cm and 35 cm, respectively.

Table III presents a comparison of PC-CAGE results for $\bar{g}$ with those estimated by Focht et al.[6] The latter calculation is possibly based on the analysis developed by Bush[2] using geometrical arguments, although the specific details have never been published. The two sets of values in Table III are in good agreement and typically

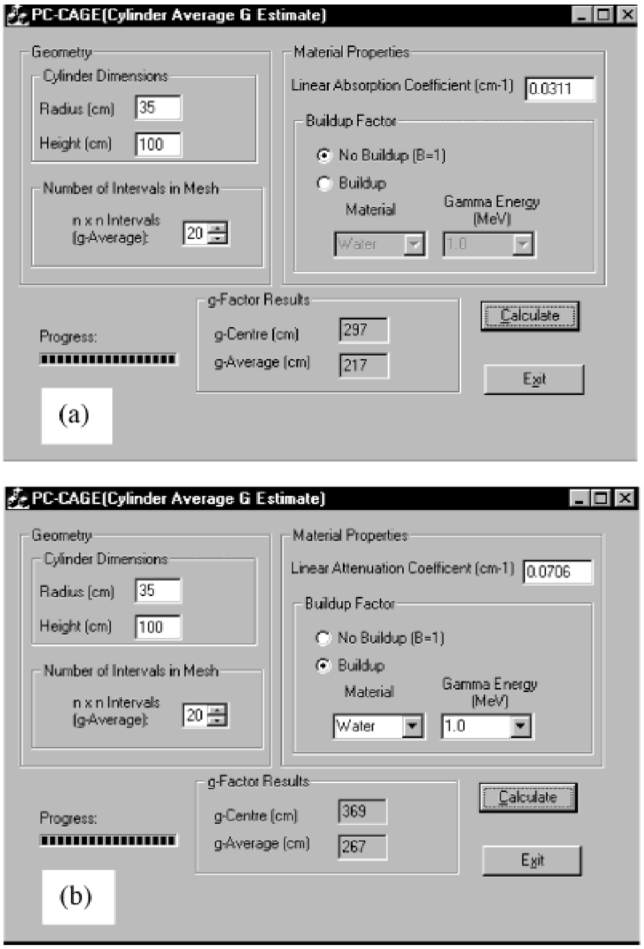

Fig. 5. Sample dialogue for PC-CAGE: (a) buildup considered implicitly and (b) buildup considered explicitly for the calculation in Table IV.

deviate by less than ~5%. The slight discrepancy can be attributed to the approximations involved in the numerical integrations of Eqs. (11) and (14) and to those made by Focht et al. in their analysis.

According to the PC-CAGE results in Tables I and III, the ratio $\bar{g}/g_c$ for a relatively square cylinder equals approximately 0.72 and increases with an increased elongation ratio.[a] These observations are consistent with Bush's results presented in Fig. 6. The latter represents a plot of the average-to-central dose rate ratio (this ratio is equivalent to the ratio between the average and central geometry factors) as a function of the elongation ratio.[2] According to Fig. 6, the average-to-central dose rate ratio is approximately equal to 0.74 for a cylinder with an elongation ratio of 1 to 3, and approaches unity in the case of infinitely thin cylinders or thin disks.

[a]The elongation ratio is defined as the greater ratio of the length-to-diameter or diameter-to-length for the cylinder.

TABLE I

Central Geometry Factors g_c (cm) Calculated Using PC-CAGE and Maple (with $\mu_a = 0.028\ \text{cm}^{-1}$)*

Height H (cm)	Radius R (cm)					
	10	15	20	25	30	35
10	96.2	113	124	131	136	140
	96.1	113	124	131	136	140
20	122	151	171	185	195	202
	122	151	171	184	195	202
30	133	170	196	215	229	240
	133	169	196	215	229	239
40	138	180	210	233	251	264
	138	179	210	233	250	264
60	143	189	225	253	275	292
	143	189	225	253	275	292
80	145	193	231	262	286	306
	144	193	231	262	286	306
100	145	195	234	266	292	313
	145	195	234	266	292	313

*The first number is obtained using PC-CAGE and the second value using Maple.

TABLE II

Point Geometry Factor (cm) on the Surface of a Cylinder at the End of the Axis: Comparison of Calculations Using PC-CAGE with Values Proposed by Hine and Brownell (with $\mu_a = 0.028\ \text{cm}^{-1}$)*

Height H (cm)	Radius R (cm)					
	10	15	20	25	30	35
10	60.9	75.6	85.4	92.3	97.3	101
	61.3	76.1	86.5	93.4	98.4	103
20	69.0	89.8	105	117	125	132
	68.9	89.8	105	117	126	133
30	71.4	94.6	113	126	137	146
	71.3	94.6	112	126	137	146
40	72.3	96.5	116	131	143	153
	72.4	96.5	116	131	143	153
60	72.9	97.9	118	134	148	159
	73.0	97.8	118	134	148	159
80	73.1	98.3	119	135	149	161
	73.3	98.4	119	135	150	161
100	73.2	98.5	119	136	150	161
	73.3	98.5	119	136	150	162

*The first number is the value estimated using PC-CAGE, and the second is that given by Hine and Brownell.

TABLE III

Average Geometry Factors $\bar{g}$ (cm): Comparison of Calculations Using PC-CAGE with Values Proposed by Focht et al. (with μ_a = 0.028 cm^{-1})*

Height H (cm)	Radius R (cm)					
	10	15	20	25	30	35
10	69.0 70.2	81.6 83.2	89.8 94.0	95.4 103	99.3 109	102 113
20	87.5 89.6	108 111	122 127	132 139	140 147	145 152
30	96.6 98.8	121 124	139 144	153 159	163 172	171 179
40	102 103	130 133	150 156	166 175	178 187	187 197
60	109 109	140 143	164 171	182 193	196 206	208 216
80	113 112	146 148	171 176	191 198	207 214	220 226
100	116 114	150 150	177 179	198 201	215 218	228 230

*The first number is the PC-CAGE value, and the second value is that proposed by Focht et al.

The mutual consistency of g_c values based on the present PC-CAGE analysis (Table I), $\bar{g}$ values based on PC-CAGE, or equivalently the results of Focht et al.[6] (Table III), and the $\bar{g}/g_c$ values based on Bush's calculations (Fig. 6) can be further illustrated by the example of a cylinder with $R = 35$ cm and $H = 100$ cm. For this case, Table I indicates a value of $g_c = 313$ cm, and Fig. 6 yields an average-to-central ratio of $\sim$0.74. Hence, $\bar{g} = 0.74 \times 313 = 232$ cm, which is in good agreement with the value of 230 cm in Table III.

Bush also provided the means (see the smaller graph of Fig. 6) for estimating the average geometry factor for a cylindrical or spheroid body by subtracting a percentage from the average geometry factor value for a sphere of equal volume.[2] An average value of $\bar{g}$ for a sphere can be evaluated from the corresponding central geometry factor for a sphere[2,3]

$$g_c = \int_{\phi=0}^{\phi=2\pi} d\phi \int_{\theta=0}^{\theta=\pi} \sin\theta\, d\theta \int_{r=0}^{r=R} \left(\frac{e^{-\mu_a r}}{r^2}\right) r^2 dr$$

$$= \frac{4\pi}{\mu_a}(1 - e^{-\mu_a R}) \tag{16}$$

and the ratio of the average-to-central geometry factor for a sphere given by Bush[2]:

$$\frac{\text{Average geometrical factor}}{\text{Central geometrical factor}} = K = \frac{1 - \dfrac{3}{4\mu_a R} - \dfrac{3e^{-2\mu_a R}}{4\mu_a^2 R^2} + \dfrac{3(1 - e^{-2\mu_a R})}{8\mu_a^3 R^3}}{1 - e^{-\mu_a R}} . \tag{17}$$

Using the same example of a cylinder with $R_{cyl} = 35$ cm and $H_{cyl} = 100$ cm, the equivalent sphere would have a radius of $R_{sphere} = [(\frac{3}{4}) R_{cyl}^2 H_{cyl}]^{1/3} = 45$ cm. Hence, from Eq. (17), with $\mu_a = 0.028$ cm^{-1}and $R_{sphere} = 45$ cm, the average-to-central geometry factor ratio for a sphere, $K = 0.753$. Using Eq. (16), this equivalent sphere has a central geometry factor of $g_c = 322$, leading to a value for $\bar{g}$ of $0.753 \times 322 = 242$ cm. The small graph in Fig. 6 indicates that $\sim$3.6% must be subtracted from this average value to correct for the effect of elongation, yielding a final $\bar{g} = 242 - 8.7 = 233$ cm for the cylinder. This value, again, is in good agreement with the value of 230 cm as estimated by Focht et al. and using PC-CAGE (see Table III). Therefore, the various approaches yield consistent values of $\bar{g}$. However, as previously explained, the values differ from those quoted in the original Hine and Brownell work.[4]

V. EFFECT OF BUILDUP ON GEOMETRY FACTOR ESTIMATES

Sample calculations were performed using PC-CAGE for three cylinder sizes (denoted as cylinders A, B, and C in Table IV) each assumed to be filled with water and for a 1-MeV photon. Dimensions for cylinder B correspond to the maximum radius and height limits for which data have been published in the literature.[4] Dimensions for cylinder C were supplied by Ontario Power Generation. For each case, three sets of geometry factors were computed:

1. *No buildup*: Calculations are based on Eq. (4a) where B was set equal to 1 and μ equaled 0.0706 cm^{-1} (Ref. 7).

2. *Approximate buildup correction considered*: Calculations are based on Eq. (4b) with the corresponding value of μ_a equal to 0.0311 cm^{-1} (Ref. 7).

3. *Explicit buildup correction considered*: Calculations are based on Eq. (4a) with μ equal to 0.0706 cm^{-1} and a Taylor's form (Ref. 7) for the buildup factor.

For each case, the geometry factor at the center of the cylinder g_c and the average geometry factor for the entire cylinder $\bar{g}$ were calculated. The sample calculations are shown in Table IV and indicate the following results:

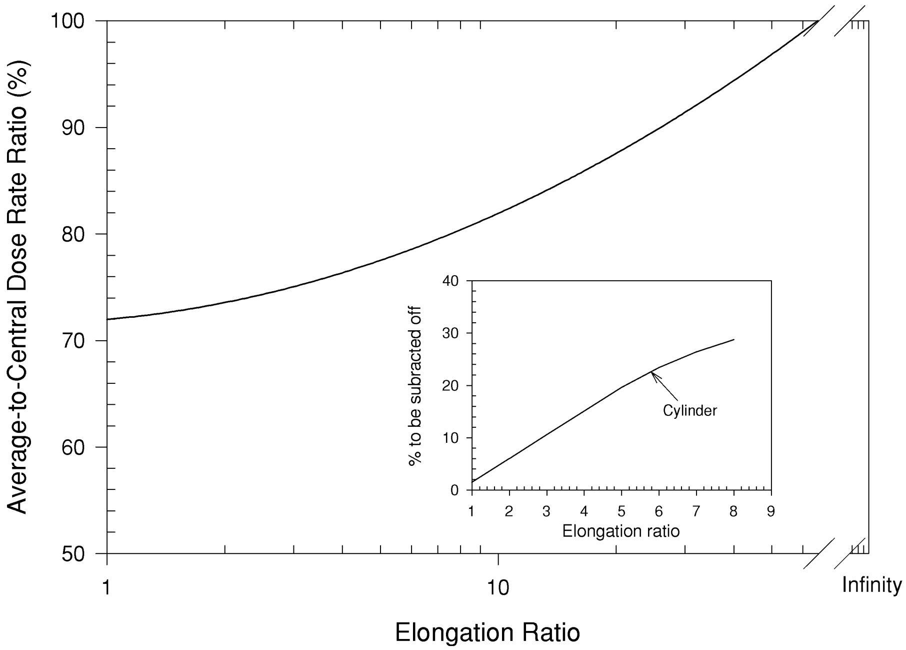

Fig. 6. Average-to-central dose rate ratio (or $\bar{g}/g_c$) and correction factor to obtain $\bar{g}$ for a cylinder from $\bar{g}$ for a sphere (taken from Ref. 2).

1. Neglecting buildup can result in a greater than 100% error in the case of large cylinders (see Table IV column 7).

2. Compared to the explicit treatment for the buildup factor, the approximate treatment (where B was set equal to unity and μ_a replaced μ) results in $\bar{g}$ being underestimated by less than 25% (see Table IV column 8). Thus, the approximate treatment may suffice in many cases of practical interest. This treatment is computationally less intensive.

3. The ratio $\bar{g}/g_c$ appears to be approximately constant, indicating the feasibility of calculating $\bar{g}$ from g_c, which is computationally less demanding.

TABLE IV

Effect of Buildup on Estimated Geometry Factors for a Cylinder

			Geometry Factor (cm)			Difference (%)	
			No Buildup Considered	Buildup Considered			
	Radius (cm)	Height (cm)	(I)	Approximate (II)	Explicit (III)	(III-I)/I $\times$ 100	(III-II)/II $\times$ 100
Cylinder A	10	10	$g_c = 80.5$	$g_c = 94.9$	$g_c = 111$	38	17
			$\bar{g} = 57.2$	$\bar{g} = 68.0$	$\bar{g} = 79.6$	39	17
			$\bar{g}/g_c = 0.71$	$\bar{g}/g_c = 0.72$	$\bar{g}/g_c = 0.72$	---	---
Cylinder B	35	100	$g_c = 169$	$g_c = 297$	$g_c = 369$	119	25
			$\bar{g} = 130$	$\bar{g} = 217$	$\bar{g} = 267$	105	23
			$\bar{g}/g_c = 0.77$	$\bar{g}/g_c = 0.73$	$\bar{g}/g_c = 0.72$	---	---
Cylinder C	81.5	170	$g_c = 178$	$g_c = 381$	$g_c = 435$	144	14
			$\bar{g} = 143$	$\bar{g} = 286$	$\bar{g} = 337$	136	18
			$\bar{g}/g_c = 0.80$	$\bar{g}/g_c = 0.75$	$\bar{g}/g_c = 0.78$	---	---

VI. CONCLUSIONS

1. A mathematical framework was developed to estimate the central and average geometry factors required for estimating dose to uniformly distributed radioactive material contained within cylinders. The resultant integral equations have been solved numerically and developed into a prototype computer code, PC-CAGE (Cylinder Average G Estimate), which supports a graphical user interface written under a Visual C++ platform.

2. Compared with previous treatments, the present treatment is more versatile because it is applicable to a cylinder of any dimension and accounts for both attenuation and buildup effects. Results, based on PC-CAGE, for the average geometry factor of a cylinder, are in reasonable agreement with the results from the pioneering work of Bush[2] and the values estimated by Focht et al.[6] As suspected by Focht et al., and confirmed in this study, the original data of Hine and Brownell,[4] which are often reproduced and quoted in the literature, correspond to dose estimates on the surface of a cylinder at the end of the axis rather than being values for the average geometry factor for the cylinders. This calls into question the extrapolation of Hine and Brownell's tabular data by Swyler et al.[5] to predict the average geometry factor for a larger cylinder.

3. Neglecting buildup can result in a greater than 100% error in the case of large cylinders. Compared to the explicit treatment for the buildup factor, the approximate treatment (where B is set equal to unity and μ_a replaces μ) results in $\bar{g}$ being underestimated by less than 25% (i.e., for a cylinder containing water and/or tissue material). Thus, the approximate treatment may suffice in many cases of practical interest. Additionally, the ratio $\bar{g}/g_c$ may be approximately constant indicating the feasibility of calculating $\bar{g}$ from g_c further reducing the computational effort.

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