Lab 4 Report: EXERCISES

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I. Individual

References

Appendix A
Individual

A. Deliverable - Single-segment trajectory optimization

Consider the following minimum velocity ($r = 1$) single-segment trajectory optimization problem:

$$\min_{P(t)} \int_0^1 \left(P^{(1)}(t)\right)^2 dt, \tag{1}$$

$$s.t. \quad P(0) = 0, \tag{2}$$

$$P(1) = 1, \tag{3}$$

with $P(t) \in \mathbb{R}[t]$, i.e., $P(t)$ is a polynomial function in t with real coefficients:

$$P(t) = p_N t^N + p_{N-1} t^{N-1} + ... + p_1 t + p_0. \tag{4}$$

Note that because of constraint ((2)), we have $P(0) = p_0 = 0$, and we can parametrize $P(t)$ without a scalar part P_0.

1. Suppose we restrict $P(t) = p_1 t$ to be a polynomial of degree 1, what is the optimal solution of problem ((1))? What is the value of the cost function at the optimal solution?

2. Suppose now we allow $P(t)$ to have degree 2, i.e., $P(t) = p_2 t^2 + p_1 t$.

(a) Write $\min_{P(t)} \int_0^1 \left(P^{(1)}(t)\right)^2 dt$, the cost function of problem ((1)), as $\boldsymbol{p}^\top \boldsymbol{Q} \boldsymbol{p}$, where $\boldsymbol{p} = [p_1, p_2]^\top$ and $\boldsymbol{Q} \in \boldsymbol{S}^2$ is a symmetric 2×2 matrix.

(b) Write $P(1) = 1$, constraint ((3)), as $\boldsymbol{A}\boldsymbol{p} = \boldsymbol{b}$, where $\boldsymbol{A} \in \mathbb{R}^{1\times 2}$ and $\boldsymbol{b} \in \mathbb{R}$.

(c) Solve the Quadratic Program (QP):

$$\min_{\boldsymbol{p}} \boldsymbol{p}^\top \boldsymbol{Q} \boldsymbol{p} \quad s.t. \quad \boldsymbol{A}\boldsymbol{p} = \boldsymbol{b}. \tag{5}$$

You can solve it by hand, or you can solve it using numerical QP solvers (e.g., you can easily use the `quadprog` function in Matlab). What is the optimal solution you get for $P(t)$, and what is the value of the cost function at the optimal solution? Are you able to get a lower cost by allowing $P(t)$ to have degree 2?

3. Now suppose we allow $P(t) = p_3 t^3 + p_2 t^2 + p_1 t$:

(a) Let $\boldsymbol{p} = [p_1, p_2, p_3]^\top$, write down $\boldsymbol{Q} \in \boldsymbol{S}^3$, $\boldsymbol{A} \in \mathbb{R}^{1\times 3}$, $b \in \mathbb{R}$ for the QP (5).

(b) Solve the QP, what optimal solution do you get? Do this example agree with the result we learned from Euler-Lagrange equation in class?

4. Now suppose we are interested in adding one more constraint to problem ((1)):

$$\min_{p(t)} \int_0^1 \left(P^{(1)}(t)\right)^2 dt, s.t. P(0) = 0, P(1) = 1, P^{(1)}(1) = 1. \tag{6}$$

Using the QP method above, find the optimal solution and optimal cost of problem ((6)) in the case of:

(a) $P(t) = p_2 t^2 + p_1 t$, and

(b) $P(t) = p_3 t^3 + p_2 t^2 + p_1 t$.