Fin Geometry Calculations

Root Chord Length: $c_r = 9.0\,\text{in}$

Tip Chord Length: $c_t = 3.0\,\text{in}$

Fin Thickness: $FT = 0.37\,\text{in}$

Fin semi-span root to tip: $b = 4.75\,\text{in}$

Shear modulus of fin material (fiberglass): $G = 400{,}000.0\,\text{psi}$

Fin surface area:
$$\begin{aligned} S &= \frac{1}{2}\left(c_r + c_t\right)\cdot b \qquad \text{(eqn. 1)}\\ &= \frac{1}{2}\left(9.0 + 3.0\right)\cdot 4.0\\ &= 24.0\,\text{in}^2 \end{aligned}$$

Fin Aspect Ratio:
$$\begin{aligned} AR &= \frac{b^2}{S} \qquad \text{(eqn. 2)}\\ &= \frac{(4.0)^2}{24.0\,\text{in}^2}\\ &= 0.667 \end{aligned}$$

Fin taper ratio:
$$\begin{aligned} \lambda &= \frac{c_t}{c_r} \qquad \text{(eqn. 3)}\\ &= \frac{3.0}{9.0}\\ &= 0.33 \end{aligned}$$

Atmospheric Conditions Calculations

Sea level atmospheric pressure: $P_0 = 14.69\,\text{psi}$

Altitude at launchpad: $h_g = 4595.0\,\text{ft}$

Altitude of max velocity relative to launchpad: $h_{relVmax} = 1947.94\,\text{ft}$

Altitude of max velocity:
$$\begin{aligned} h_{Vmax} &= h_g + h_{relVmax}\\ &= 4595.0 + 1947.94 \qquad \text{(eqn.4)}\\ &= 6542.94\,\text{ft} \end{aligned}$$

Temperature at h_Vmax:
$$\begin{aligned} T &= 59 - 0.00356\cdot h_{Vmax} \qquad \text{(eqn. 5)}\\ &= 59 - 0.00356\cdot 6542.94\\ &= 35.71\,^{\circ}\text{F} \end{aligned}$$

Pressure at h_Vmax:
$$\begin{aligned} P &= P_0\cdot\left(\frac{T + 459.7}{518.6}\right)^{5.256} \qquad \text{(eqn.6)}\\ &= 14.69\cdot\left(\frac{35.71 + 459.7}{518.6}\right)^{5.256}\\ &= 11.55\,\text{psi} \end{aligned}$$

Speed of sound at h_Vmax:
$$a = \sqrt{1.4\cdot 1716.59\cdot (T + 459.7)} \qquad \text{(eqn. 7)}$$

$$= \sqrt{1.4 \cdot 1716.59 \cdot (35.71 + 459.7)}$$
$$= 1091.47\,\text{ft/s}$$

Fin Velocity Calculations

Max velocity of rocket: $V_{max} = 979.86\,\text{ft/s}$

Fin flutter velocity:
$$v_{fl} = \sqrt{\frac{G}{\left(\frac{39.3\cdot(AR)^3}{\left(\frac{FT}{c_r}\right)^3\cdot(AR+2)}\right)\left(\frac{\lambda+1}{2}\right)\left(\frac{P}{P_0}\right)}} \cdot a \qquad \text{(eqn. 8)}$$
$$= \sqrt{\frac{400000.0}{\left(\frac{39.3\cdot(0.667)^3}{\left(\frac{0.37}{9.0}\right)^3\cdot(0.667+2)}\right)\left(\frac{0.33+1}{2}\right)\left(\frac{11.55}{14.69}\right)}} \cdot 1091.47$$
$$= 3880.72\,\text{ft/s}$$

Fin flutter safety margin:
$$SM = \frac{V_{max}}{v_{fl}} \qquad \text{(eqn. 9)}$$
$$= \frac{979.86}{3880.72}$$
$$= 3.96$$

Parachute Descent Velocity Equation Proof

Drag force equation:
$$F_d = \frac{1}{2} \cdot \rho \cdot V^2 \cdot c_d \cdot A \qquad \text{(eqn. 10)}$$

Parachute area:
$$A = \pi \cdot \left(\frac{D^2}{4}\right)$$

Drag force equation with D:
$$F_d = \frac{1}{2} \cdot \rho \cdot V^2 \cdot c_d \cdot \pi \cdot \left(\frac{D^2}{4}\right)$$
$$\Longrightarrow F_d = \frac{1}{8} \cdot \rho \cdot V^2 \cdot c_d \cdot \pi \cdot D^2$$

Weight force equation: $F_w = m \cdot g$

Force balance of drag and weight: $F_w = F_d$
$$\Longrightarrow m \cdot g = \frac{1}{8} \cdot \rho \cdot V^2 \cdot c_d \cdot \pi \cdot D^2 \qquad \text{(eqn. 11)}$$

Re-arrange force balance equation for solution:
$$V_{chute} = \sqrt{\frac{8 \cdot m \cdot g}{\rho \cdot c_d \cdot \pi \cdot D^2}} \qquad \text{(eqn. 12)}$$

Descent Velocity Calculations

Parachute Descent Velocity Equation:
$$V_{chute} = \sqrt{\frac{8 \cdot m \cdot g}{\rho \cdot C_d \cdot \pi \cdot D^2}} \qquad \text{(eqn. 12)}$$

Drogue Chute Diameter: $D_d = 2.5\,\text{ft}$

Drogue Chute Drag Coefficient: $Cd_d = 1.55$

Air density under drogue chute descent: $\rho_d = 0.06\,\text{lbs/ft}^3$

Main Chute Diameter: $D_m = 12.0\,\text{ft}$

Main Chute Drag Coefficient: $Cd_m = 2.2$

Air density under main chute descent: $\rho_m = 0.07\,\text{lbs/ft}^3$

Rocket mass after motor burnout: $m = 63.3\,\text{lbs}$

Gravity: $g = 32.17\,\text{ft/s}^2$

Drogue chute descent velocity:
$$\begin{aligned} V_{drogue} &= \sqrt{\frac{8 \cdot 63.3 \cdot 32.2}{\pi \cdot 0.06 \cdot 1.55 \cdot 2.5^2}} \\ &= 91.6\,\text{ft/s} \end{aligned}$$

Main chute descent velocity:
$$\begin{aligned} V_{main} &= \sqrt{\frac{8 \cdot 63.3 \cdot 32.2}{\pi \cdot 0.07 \cdot 2.2 \cdot 12.0^2}} \\ &= 15.87\,\text{ft/s} \end{aligned}$$

Ejection Charge Size Calculations

Airframe Diameter: $D_a = 3.9\,\text{in}$

Bulkhead area:
$$\begin{aligned} A_{bh} &= \frac{D_a \cdot pi}{4} \\ &= 11.95\,\text{in}^2 \end{aligned} \qquad \text{(eqn. 13)}$$

Force applied to bulkheads by P_e:
$$\begin{aligned} F_e &= P_e \cdot A_{bh} \\ &= 179.19\,\text{lbs} \end{aligned}$$

Airframe Section Volume Equation:
$$Vol_s = \frac{\pi \cdot D^2 \cdot L}{4} \qquad \text{(eqn. 14)}$$

Volume of Drogue Chute Bay:
$$\begin{aligned} Vol_d &= \frac{\pi \cdot 3.9^2 \cdot 10.0}{4} \\ &= 119.46\,\text{in}^3 \end{aligned}$$

Volume of Main Chute Bay:
$$\begin{aligned} Vol_m &= \frac{\pi \cdot 3.9^2 \cdot 21.0}{4} \\ &= 250.86\,\text{in}^3 \end{aligned}$$

Combustion gas constant of black powder: $R = 265.92 \dfrac{in \cdot lbf}{lbm \cdot {}^\circ\text{R}}$

Combustion gas temperature of black powder: $T_c = 3307\,^\circ\text{R}$

Black powder charge mass equation: $$m_{bp} = \frac{454g}{1lbf} \cdot \frac{P_e \cdot Vol_s}{265.92 \frac{in \cdot lbf}{lbm \cdot {}^\circ\mathrm{R}} \cdot 3307 \cdot {}^\circ\mathrm{R}} \quad \text{(eqn. 15)}$$

Black powder charge mass for drogue chute bay: $$m_{bp,d} = \frac{454g}{1lbf} \cdot \frac{15.0 \cdot 119.46}{265.92 \cdot 3307 \cdot} = 0.93\,\mathrm{g}$$

Black powder charge mass for main chute bay: $$m_{bp,m} = \frac{454g}{1lbf} \cdot \frac{15.0 \cdot 250.86}{265.92 \cdot 3307} = 1.94\,\mathrm{g}$$

Ejection Charge Size Calculations Reversed

Airframe Diameter: $D_a = 3.9\,\mathrm{in}$

Bulkhead area: $$A_{bh} = \frac{D_a \cdot \pi}{4} = 11.95\,\mathrm{in}^2 \quad \text{(eqn. 13)}$$

Length of Drogue Chute Bay: $L_{d,bay} = 10.0\,\mathrm{in}$

Length of Main Chute Bay: $L_{m,bay} = 21.0\,\mathrm{in}$

Airframe Section Volume Equation: $$Vol_s = \frac{\pi \cdot D^2 \cdot L}{4} \quad \text{(eqn. 14)}$$

Volume of Drogue Chute Bay: $$Vol_d = \frac{\pi \cdot 3.9^2 \cdot 10.0}{4} = 119.46\,\mathrm{in}^3$$

Volume of Main Chute Bay: $$Vol_m = \frac{\pi \cdot 3.9^2 \cdot 21.0}{4} = 250.86\,\mathrm{in}^3$$

Combustion gas constant of black powder: $$R = 265.92 \frac{in \cdot lbf}{lbm \cdot {}^\circ\mathrm{R}}$$

Combustion gas temperature of black powder: $T_c = 3307\,{}^\circ\mathrm{R}$

Ejection pressure equation: $$P_e = \left(m_{bp} \cdot \frac{1\,\mathrm{lbf}}{454\,\mathrm{g}}\right) \cdot \frac{R \cdot T_c}{Vol_S} \quad \text{(eqn. 15)}$$

Force applied to bulkheads equaton: $$F_e = P_e \cdot A_{bh} \quad \text{(eqn. 16)}$$

Black powder charge mass for drogue chute bay: $m_{bp,d} = 2.5\,\mathrm{g}$

Ejection pressure for drogue chute: $$P_{e,d} = \left(2.5\,\mathrm{g} \cdot \frac{1lbf}{454g}\right) \cdot \frac{265.92\frac{in \cdot lbf}{lbm \cdot {}^\circ\mathrm{R}} \cdot 3307^\circ\mathrm{R}}{119.46}$$
$$= 40.54\,\mathrm{psi}$$

Force applied to drogue chute bay bulkheads: $$\begin{aligned} F_{e,d} &= P_e \cdot A_{bh} \\ &= 40.54 \cdot 11.95 \\ &= 484.25\,\mathrm{lbs} \end{aligned}$$

Black powder charge mass for main chute bay: $$m_{bp,m} = 5\,\mathrm{g}$$

Ejection pressure for main chute: $$P_{e,m} = \left(5\,\mathrm{g} \cdot \frac{1lbf}{454g}\right) \cdot \frac{265.92\frac{in \cdot lbf}{lbm \cdot {}^\circ\mathrm{R}} \cdot 3307^\circ\mathrm{R}}{250.86}$$
$$= 38.61\,\mathrm{psi}$$

Force applied to main chute bay bulkheads: $$\begin{aligned} F_{e,d} &= P_e \cdot A_{bh} \\ &= 38.61 \cdot 11.95 \\ &= 461.19\,\mathrm{lbs} \end{aligned}$$

Main Chute Opening Force Calculation

Descent velocity before main chute opening: $$V_i = 82.37\,\mathrm{ft/s}$$

Descent velocity after main chute opening: $$V_f = 15.69\,\mathrm{ft/s}$$

Main chute opening time: $$t_{infl} = 0.51\,\mathrm{s}$$

Main Chute Opening Force Equation: $$F_{max} = \left(\frac{2 \cdot m \cdot v_i}{g \cdot t_{infl}}\right)\left(1 - \frac{v_f}{v_i}\right) + 2 \cdot m \qquad \text{(eqn. 16)}$$
$$= \left(\frac{2 \cdot 63.3 \cdot 82.37}{32.17 \cdot 0.51}\right)\left(1 - \frac{15.69}{82.37}\right) + 2 \cdot 63.3$$
$$= 183.34\,\mathrm{lbf}$$