BSc Thesis Project
(Mathematics)

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2D Mesh Generation based on Conformal Mappings

Field. Finite Element Methods, Mesh Generation, Numerical Complex Analysis

Topic. The finite element method (FEM) relies on meshes of a bounded computational domain for the approximate solution of boundary value problems for partial differential equations, see [1, Sections 2.4 & 2.5.12]. We focus on (curvilinear) triangular meshes of simply connected 2D bounded computational domains $\Omega \subset \mathbb{R}^2$. These domains are given through a parameterization of their boundary

$$\partial\Omega := \boldsymbol{\gamma}([0, 2\pi]) , \quad \boldsymbol{\gamma} \in \left(C^0([0, 2\pi])\right)^2 \ 2\pi - \text{periodic} .$$

A natural way to prescribe $\boldsymbol{\gamma}$ is through a Fourier series representation.

Let us assume that we already have a curvilinear triangular mesh $\widehat{\mathcal{M}}$ of the unit disk D, whose triangles are all "nicely shape regular" in the sense that the ratio of their diameter and the radius of the largest inscribed circle is small for all "triangles". According to the Riemann mapping theorem there is a bijective conformal mapping $\boldsymbol{\Phi} : D \to \Omega$. We can use the image of $\widehat{\mathcal{M}}$ under Φ as a mesh on Ω.

The goal of the project is to implement and evaluate numerical approximations of these conforming mappings. It is desirable that the algorithm provides $\boldsymbol{\Phi}$ is a form that allows efficient point evaluations of both $\boldsymbol{\Phi}$ and its Jacobian $\mathsf{D}\boldsymbol{\Phi}$.

Tasks.

1. Research literature about the numerical computation of conformal mappings starting from [3] and [2, Chapter 4].

2. Select a suitable algorithm for the approximate computation of $\boldsymbol{\Phi}$.

3. Implement the algorith in Pyhton and validate your code.

4. Study the accuracy of the approximation of $\boldsymbol{\Phi}$ depending on discretization parameters.

5. Study the quality of the meshes generated by the algorithm depending on for a number of "typical" domains Ω with different degrees of smoothness of $\partial\Omega$.

6. Set up a code repository complete with installation instructions and a short documentation.

References

[1] R. Hiptmair. *Numerical Methods for Partial Differential Equations*. Lecture Notes, SAM, ETH Zürich. 2023. URL: `https://www.sam.math.ethz.ch/~grsam/NUMPDEFL/NUMPDE.pdf` (cit. on p. 1).

[2] F. Wechsung. "Shape optimisation and robust solvers for incompressible flow". PhD thesis. University of Oxford. Trinity College, 2019. URL: `https://ora.ox.ac.uk/objects/uuid:b8ffd339-32e8-43cd-a304-cc6ab9777442` (cit. on p. 1).

[3] Rudolf Wegmann. "Methods for numerical conformal mapping". In: *Handbook of complex analysis: geometric function theory. Vol. 2*. Elsevier Sci. B. V., Amsterdam, 2005, pp. 351–477. DOI: `10.1016/S1874-5709(05)80013-7`. URL: `https://doi.org/10.1016/S1874-5709(05)80013-7` (cit. on p. 1).

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