CHAPTER 41

RED-BLACK TREES

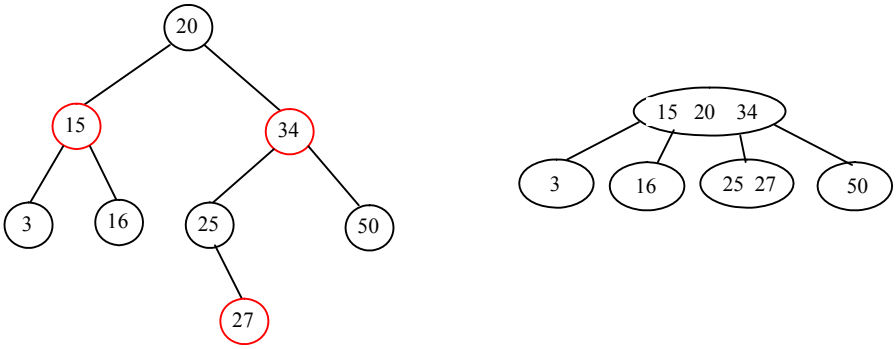
Objectives

- To know what a red-black tree is (§41.1).
- To convert a red-black tree to a 2-4 tree and vice versa (§41.2).
- To design the RBTree class that extends the BST class (§41.3).
- To insert an element in a red-black tree and resolve the double-red violation if necessary (§41.4).
- To delete an element from a red-black tree and resolve the double-black problem if necessary (§41.5).
- To implement and test the RBTree class (§§41.6–41.7).
- To compare the performance of AVL trees, 2-4 trees, and RBTree (§41.8).

41.1 Introduction

Key Point: A red-black tree is a balanced binary search tree derived from a *2-4 tree*. A red-black tree corresponds to a 2-4 tree.

Each node in a red-black tree has a *color attribute* red or black, as shown in Figure 41.1(a). A node is called *external* if its left or right subtree is empty. Note that a leaf node is external, but an external node is not necessarily a leaf node. For example, node 25 is external, but it is not a leaf. The *black depth* of a node is defined as the number of black nodes in a path from the node to the root. For example, the black depth of node 25 is 2 and that of node 27 is 2.



(a) A red-black tree (b) A 2-4 tree

Figure 41.1
A red-black tree can be represented using a 2-4 tree, and vice versa.

NOTE: The red nodes appear in blue in the text.

A red-black tree has the following properties:

1. The root is black.
2. Two adjacent nodes cannot be both red.
3. All external nodes have the same black depth.

The red-black tree in Figure 41.1(a) satisfies all three properties. A red-black tree can be converted to a 2-4 tree, and vice versa. Figure 41.1(b) shows an equivalent 2-4 tree for the red-black tree in Figure 41.1(a).

41.2 Conversion between Red-Black Trees and 2-4 Trees

Key Point: This section discusses the correspondence between a red-black tree and a 2-4 tree.

You can design insertion and deletion algorithms for red-black trees without having knowledge of 2-4 trees. However, the correspondence between red-black trees and 2-4 trees provides useful intuition about the structure of red-black trees and operations. For this reason, this section discusses the correspondence between these two types of trees.

To convert a red-black tree to a 2-4 tree, simply merge every red node with its parent to create a 3-node or a 4-node. For example, the red nodes 15 and 34 are merged to their parent to create a 4-node, and the red node 27 is merged to its parent to create a 3-node, as shown in Figure 41.1(b).

To convert a 2-4 tree to a red-black tree, perform the following transformations for each node u:

1. If u is a 2-node, color it black, as shown in Figure 41.2(a).

2. If u is a 3-node with element values e_0 and e_1, there are two ways to convert it. Either make e_0 the parent of e_1 or make e_1 the parent of e_0. In any case, color the parent black and the child red, as shown in Figure 41.2(b).

3. If u is a 4-node with element values e_0, e_1, and e_2, make e_1 the parent of e_0 and e_2. Color e_1 black and e_0 and e_2 red, as shown in Figure 41.2(c).

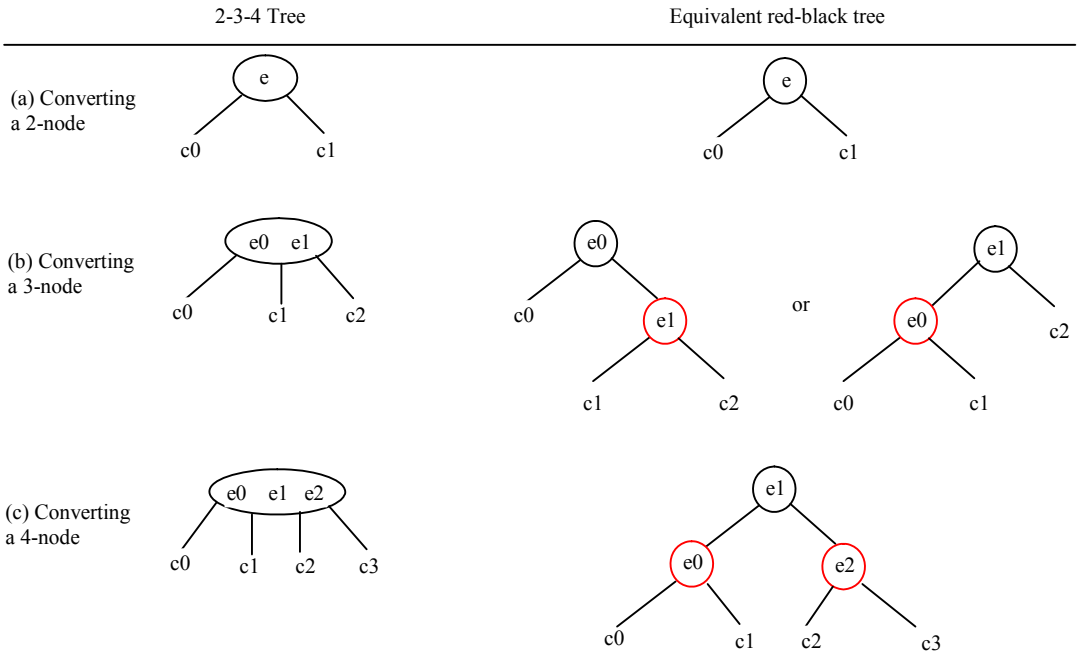


Figure 41.2
A node in a 2-4 tree can be transformed to nodes in a red-black tree.

Let us apply the transformation for the 2-4 tree in Figure 41.1(b). After transforming the 4-node, the tree is as shown in Figure 41.3(a). After transforming the 3-node, the tree is as shown in Figure 41.3(b). Note that the transformation for a 3-node is not unique. Therefore, the conversion from a 2-4 tree to a red-black tree is *not unique*. After transforming the 3-node, the tree could also be as shown in Figure 41.3(c).

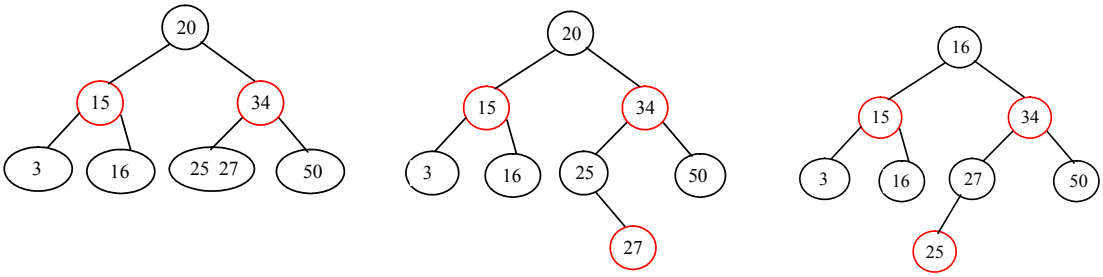


Figure 41.3
The conversion from a 2-4 tree to a red-black tree is not unique.

You can prove that the conversion results in a red-black tree that satisfies all three properties.

Property 1. The root is black.
Proof: If the root of a 2-4 tree is a 2-node, the root of the red-black tree is black. If the root of a 2-4 tree is a 3-node or 4-node, the transformation produces a black parent at the root.

Property 2. Two adjacent nodes cannot be both red.
Proof: Since the parent of a red node is always black, no two adjacent nodes can be both red.

Property 3. All external nodes have the same black depth.
Proof: When you covert a node in a 2-4 tree to red-black tree nodes, you get one black node and zero, one, or two red nodes as its children, depending on whether the original node is a 2-, 3-, or 4-node. Only a leaf 2-4 node may produce external red-black nodes. Since a 2-4 tree is perfectly balanced, the number of black nodes in any path from the root to an external node is the same.

Check point

41.1
What is a red-black tree? What is an external node? What is black-depth?

41.2
Describe the properties of a red-black tree.

41.3
How do you convert a red-black tree to a 2-4 tree? Is the conversion unique?

41.4
How do you convert a 2-4 tree to a red-black tree? Is the conversion unique?

41.3 Designing Classes for Red-Black Trees

Key Point: A red-black tree designs a class for a red-black tree.

A red-black tree is a binary search tree. So, you can define the RBTree class to extend the BST class, as shown in Figure 41.4. The BST and TreeNode classes are defined in §26.2.5.

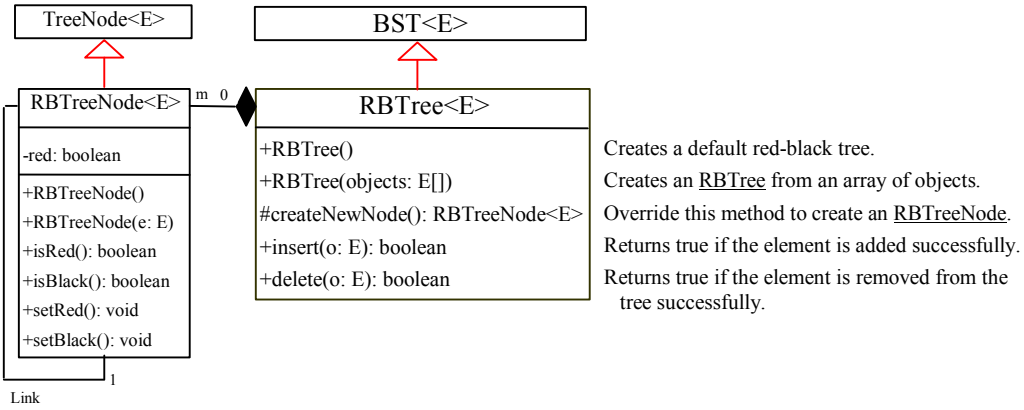


Figure 41.4
The RBTree class extends BST with new implementations for the insert and delete methods.

Each node in a red-black tree has a color property. Because the color is either red or black, it is efficient to use the boolean type to denote it. The RBTreeNode class can be defined to extend BST.TreeNode with the color property. For convenience, we also provide the methods for checking the color and setting a new color. Note that TreeNode is defined as a static inner class in BST. RBTreeNode will be defined as a static inner class in RBTree. Note that BSTNode contains the data fields element, left, and right, which are inherited in RBTreeNode. So, RBTreeNode contains four data fields, as pictured in Figure 41.5.

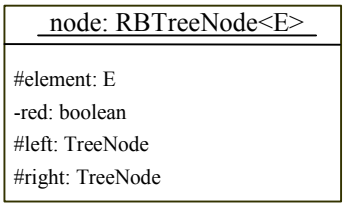


Figure 41.5
An RBTreeNode contains data fields element, red, left, and right.

In the BST class, the createNewNode() method creates a TreeNode object. This method is overridden in the RBTree class to create an RBTreeNode. Note that the return type of the createNewNode() method in the BST class is TreeNode, but the return type of the createNewNode() method in RBTree class is RBTreeNode. This is fine, since RBTreeNode is a subtype of TreeNode.

Searching an element in a red-black tree is the same as searching in a regular binary search tree. So, the search method defined in the BST class also works for RBTree.

The insert and delete methods are overridden to insert and delete an element and perform operations for coloring and restructuring if necessary to ensure that the three properties of the red-black tree are satisfied.

Pedagogical NOTE

Run from
www.cs.armstrong.edu/liang/animation/RBTreeAnimation.html
to see how a red-black tree works, as shown in Figure 41.6.

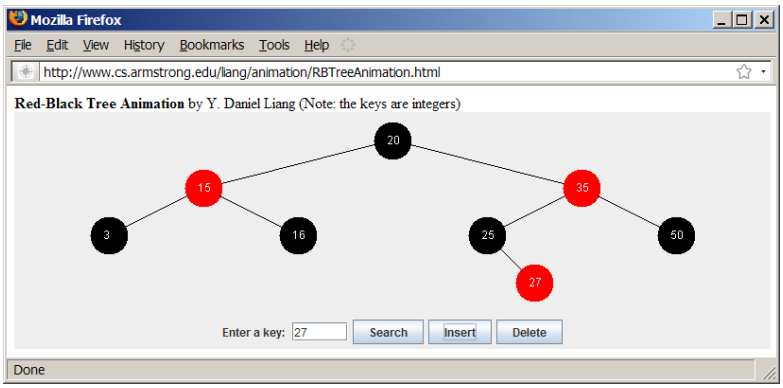


Figure 41.6
The animation tool enables you to insert, delete, and search elements in a red-black tree visually.

End NOTE

41.4 Overriding the insert Method

Key Point: This section discusses how to insert an element to red-black tree.

A new element is always inserted as a leaf node. If the new node is the root, color it black. Otherwise, color it red. If the parent of the new node is red, it violates Property 2 of the red-black tree. We call this a *double-red* violation.

Let u denote the new node inserted, v the parent of u, w the parent of v, and x the sibling of v. To fix the double-red violation, consider two cases:

Case 1: x is black or x is null. There are four possible configurations for u, v, w, and x, as shown in Figures 41.7(a), 41.8(a), 41.9(a), and 41.10(a). In this case, u, v, and w form a 4-node in the corresponding 2-4 tree, as shown in Figures 41.7(c), 41.8(c), 41.9(c), and 41.10(c), but are represented incorrectly in the red-black tree. To correct this error, restructure and recolor three nodes u, v, and w, as shown in Figures 41.7(b), 41.8(b), 41.9(b), and 41.10(b). Note that x, y_1, y_2, and y_3 may be null.

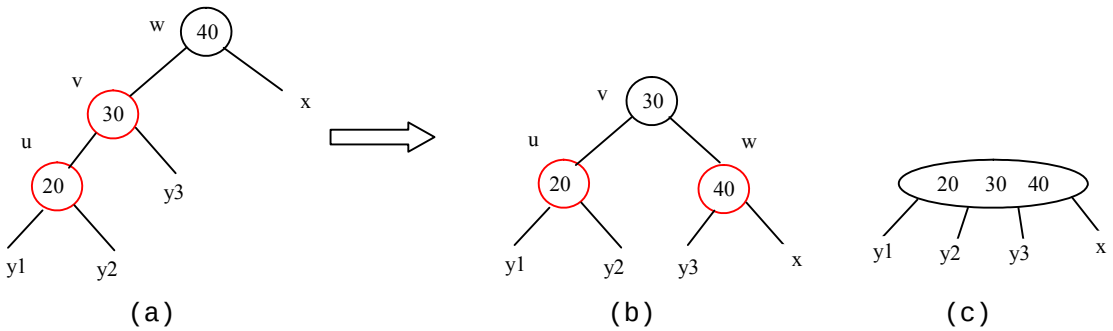


Figure 41.7

Case 1.1: u < v < w.

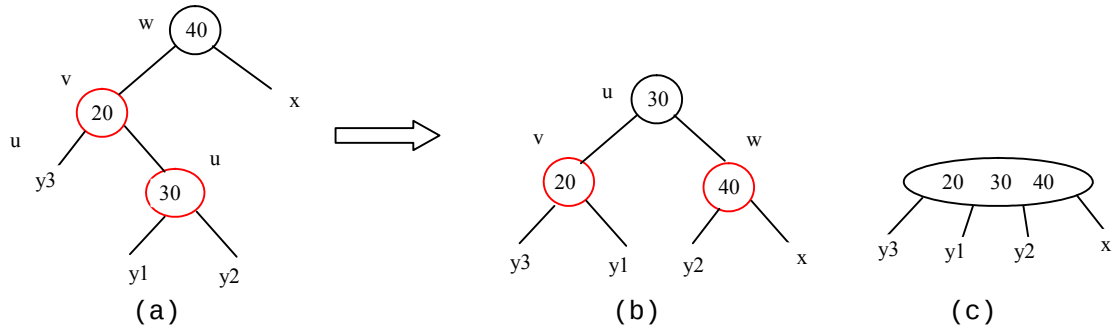


Figure 41.8
Case 1.2: v < u < w

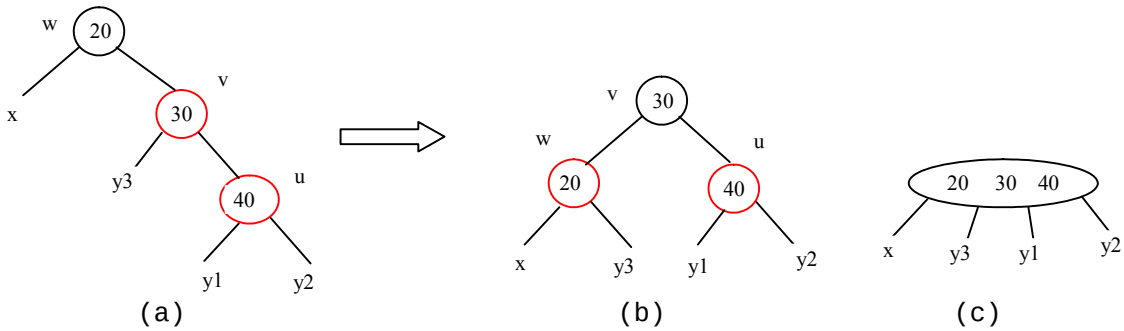


Figure 41.9
Case 1.3: w < v < u

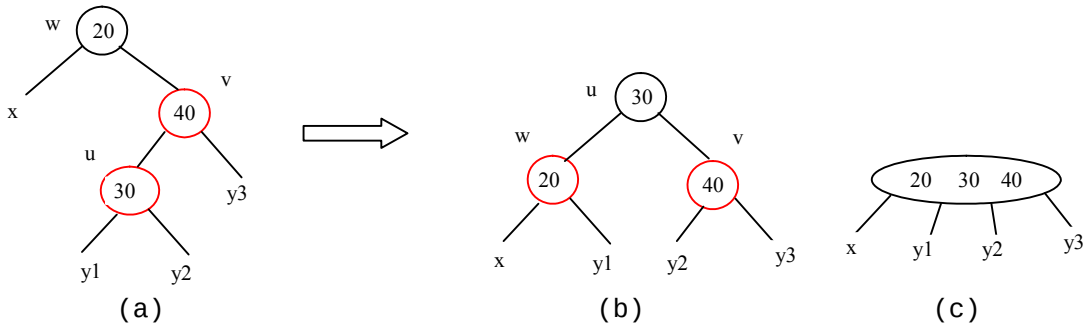


Figure 41.10
Case 1.4: w < u < v

Case 2: x is red. There are four possible configurations for u, v, w, and x, as shown in Figures 41.11(a), 41.11(b), 41.11(c), and 41.11(d). All of these configurations correspond to an overflow situation in the corresponding 4-node in a 2-4 tree, as shown in Figure 41.12(a). A splitting operation is performed to fix the overflow problem in a 2-4 tree, as shown in Figure 41.12(b). We perform an equivalent recoloring operation to fix the problem in a red-black tree. Color w and u red and color two children of w black. Assume u is a left child of v, as shown in Figure 41.11(a). After recoloring, the nodes are shown in Figure 41.12(c). Now w is red, if w's parent is black, the double-red violation is fixed. Otherwise, a new double-red violation occurs at node w. We need to continue the same process to eliminate the double-red violation at w, recursively.

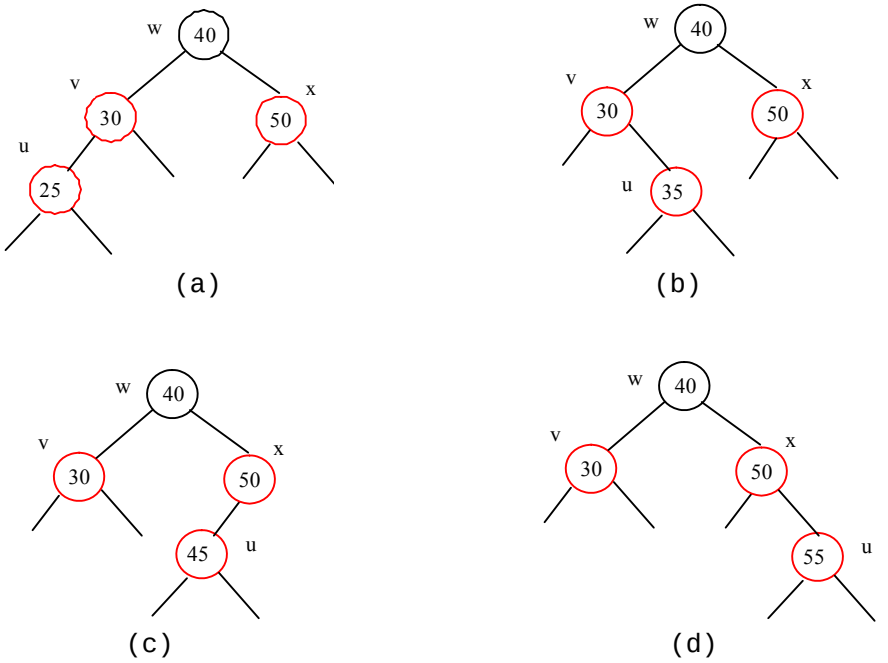


Figure 41.11
Case 2 has four possible configurations.

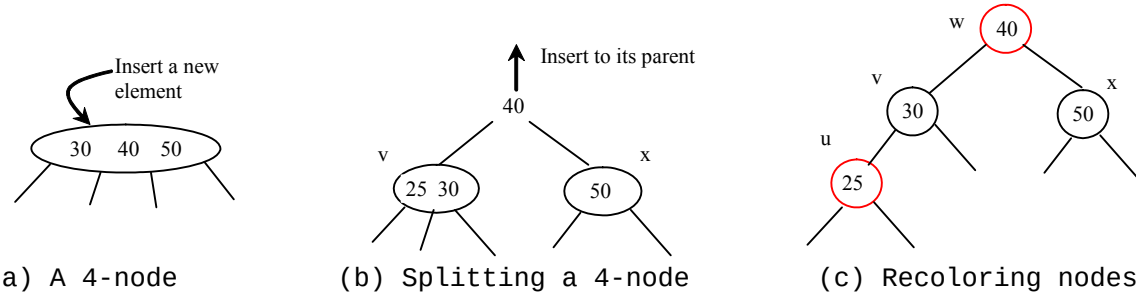


Figure 41.12
Splitting a 4-node corresponds to recoloring the nodes in the red-black tree.

A more detailed algorithm for inserting an element is described in Listing 41.1.

Listing 41.1 Inserting an Element to a Red-Black Tree

```
public boolean insert(E e) {
  boolean successful = super.insert(e);
  if (!successful)
    return false; // e is already in the tree
  else {
    ensureRBTree(e);
  }

  return true; // e is inserted
}
```

```
/** Ensure that the tree is a red-black tree */
private void ensureRBTree(E e) {
  Get the path that leads to element e from the root.
  int i = path.size() – 1; // Index to the current node in the path
  Get u, v from the path. u is the node that contains e and v
    is the parent of u.
  Color u red;

  if (u == root) // If e is inserted as the root, set root black
    u.setBlack();
  else if (v.isRed())
    fixDoubleRed(u, v, path, i); // Fix double-red violation at u
}

/** Fix double-red violation at node u */
private void fixDoubleRed(RBTreeNode<E> u, RBTreeNode<E> v,
    ArrayList<TreeNode<E>> path, int i) {
  Get w from the path. w is the grandparent of u.

  // Get v's sibling named x
  RBTreeNode<E> x = (w.left == v) ?
    (RBTreeNode<E>)(w.right) : (RBTreeNode<E>)(w.left);

  if (x == null || x.isBlack()) {
    // Case 1: v's sibling x is black
    if (w.left == v && v.left == u) {
      // Case 1.1: u < v < w, Restructure and recolor nodes
    }
    else if (w.left == v && v.right == u) {
      // Case 1.2: v < u < w, Restructure and recolor nodes
    }
    else if (w.right == v && v.right == u) {
      // Case 1.3: w < v < u, Restructure and recolor nodes
    }
    else {
      // Case 1.4: w < u < v, Restructure and recolor nodes
    }
  }
  else { // Case 2: v's sibling x is red
    Color w and u red
    Color two children of w black.

    if (w is root) {
      Set w black;
    }
    else if (the parent of w is red) {
      // Propagate along the path to fix new double-red violation
      u = w;
      v = parent of w;
      fixDoubleRed(u, v, path, i - 2); // i – 2 propagates upward
    }
  }
}
```

The insert(E e) method (lines 1–10) invokes the insert method in the BST class to create a new leaf node for the element (line 2). If the element

is already in the tree, return false (line 4). Otherwise, invoke ensureRBTree(e) (line 6) to ensure that the tree satisfies the color and black depth property of the red-black tree.

The ensureRBTree(E e) method (lines 13–24) obtains the path that leads to e from the root (line 14), as shown in Figure 41.13. This path plays an important role to implement the algorithm. From this path, you get nodes u and v (lines 16–17). If u is the root, color u black (lines 20–21). If v is red, a double-red violation occurs at node u. Invoke fixDoubleRed to fix the problem.

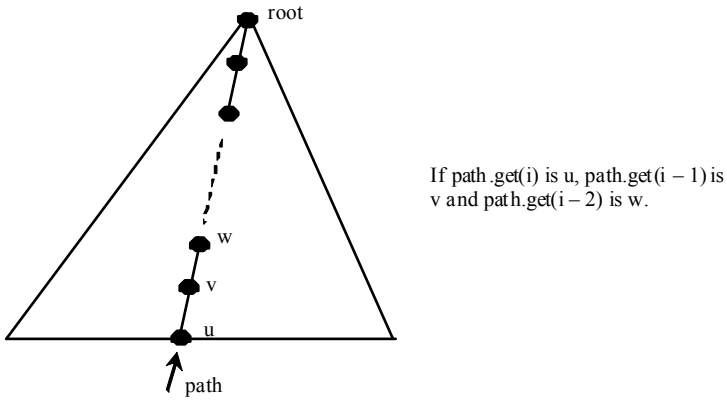
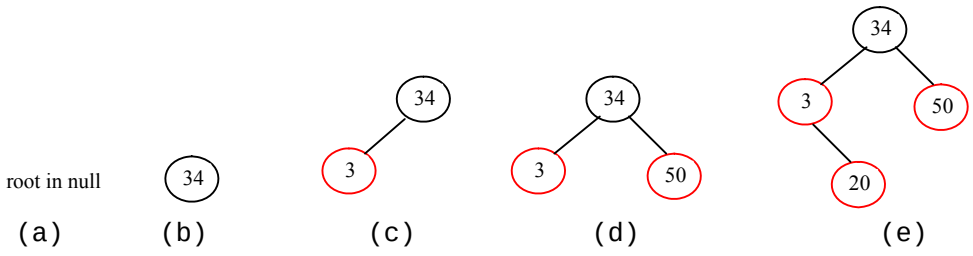


Figure 41.13
The path consists of the nodes from u to the root.

The fixDoubleRed method (lines 27–63) fixes the double-red violation. It first obtains w (the parent of v) from the path (line 29) and x (the sibling of v) (lines 32–33). If x is empty or a black node, restructure and recolor three nodes u, v, and w to eliminate the problem (lines 35–49). If x is a red node, recolor the nodes u, v, w and x (lines 51–52). If w is the root, color w black (lines 54–56). If the parent of w is red, the double-red violation reappears at w. Invoke fixDoubleRed with new u and v to fix the problem (line 61). Note that now i – 2 points to the new u in the path. This adjustment is necessary to locate the new nodes w and parent of w along the path.

Figure 41.14 shows the steps of inserting 34, 3, 50, 20, 15, 16, 25, and 27 into an empty red-black tree. When inserting 20 into the tree in (d), Case 2 applies to recolor 3 and 50 to black. When inserting 15 into the tree in (g), Case 1.4 applies to restructure and recolor nodes 15, 20, and 3. When inserting 16 into the tree in (i), Case 2 applies to recolor nodes 3 and 20 to black and nodes 15 and 16 to red. When inserting 27 into the tree in (l), Case 2 applies to recolor nodes 16 and 25 to black and nodes 20 and 27 to red. Now a new double-red problem occurs at node 20. Apply Case 1.2 to restructure and recolor nodes. The new tree is shown in (n).



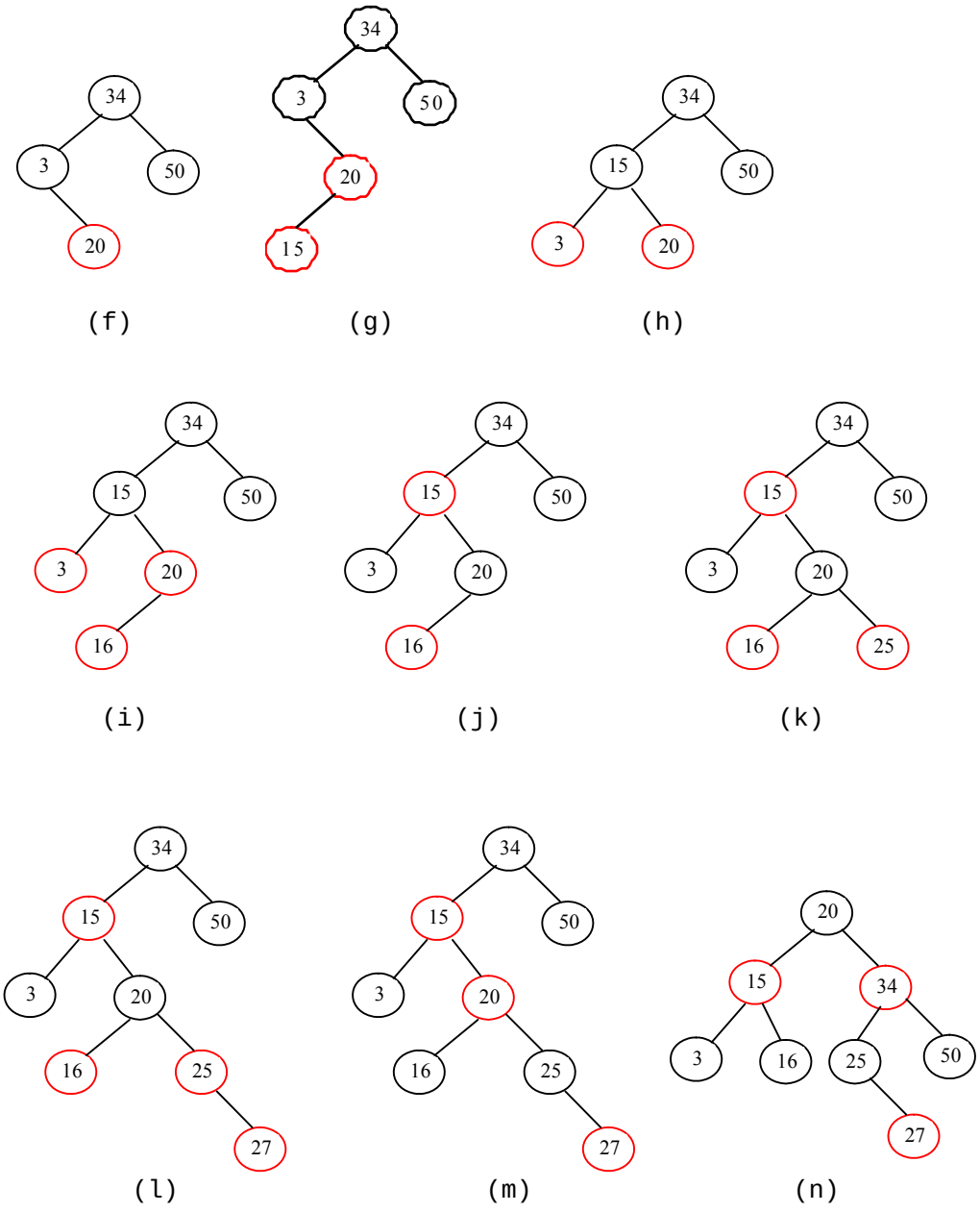


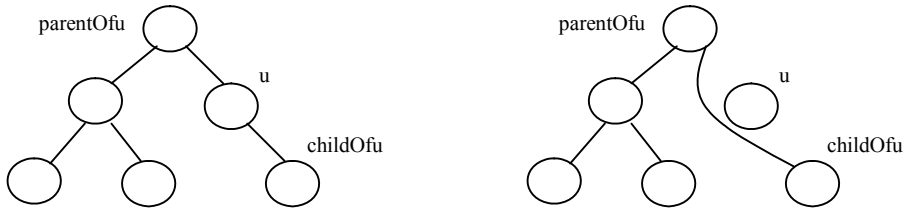
Figure 41.14
Inserting into a red-black tree: (a) initial empty tree; (b) inserting 34; (c) inserting 3; (d) inserting 50; (e) inserting 20 causes a double red; (f) after recoloring (Case 2); (g) inserting 15 causes a double red; (h) after restructuring and recoloring (Case 1.4); (i) inserting 16 causes a double red; (j) after recoloring (Case 2); (k) inserting 25; (l) inserting 27 causes a double red at 27; (m) a double red at 20 reappears after recoloring (Case 2); (n) after restructuring and recoloring (Case 1.2).

41.5 Overriding the delete Method

Key Point: This section discusses how to delete an element to red-black tree.

To delete an element from a red-black tree, first search the element in the tree to locate the node that contains the element. If the element is not in the tree, the method returns false. Let *u* be the node that contains the element. If *u* is an internal node with both left and right children, find the rightmost node in the left subtree of *u*. Replace the element in *u* with the element in the rightmost node. Now we will only consider deleting external nodes.

Let *u* be an external node to be deleted. Since *u* is an external node, it has at most one child, denoted by *childOfu*. *childOfu* may be null. Let *parentOfu* denote the parent of *u*, as shown in Figure 41.15(a). Delete *u* by connecting *childOfu* with *parentOfu*, as shown in Figure 41.15(b).

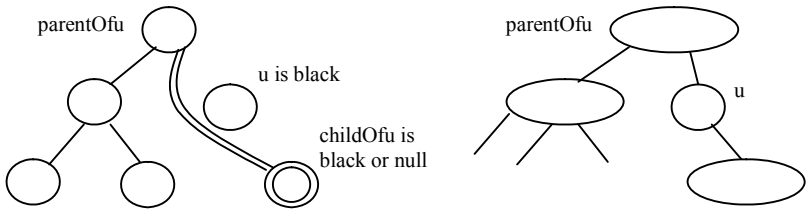


(a) Before deleting u (b) After deleting u

Figure 41.15
u is an external node and childOfu may be null.

Consider the following case:

- If *u* is red, we are done.
- If *u* is black and *childOfu* is red, color *childOfu* black to maintain the black height for *childOfu*.
- Otherwise, assign *childOfu* a fictitious *double black*, as shown in Figure 41.16(a). We call this a *double-black problem*, which indicates that the black-depth is short by 1, caused by deleting a black node u.



(a) **(b)**

Figure 41.16
(a) childOfu is denoted double black. (b) u corresponds to an empty node in a 2-4 tree.

A double black in a red-black tree corresponds to an empty node for u (i.e., underflow situation) in the corresponding 2-4 tree, as shown in Figure 41.16(b). To fix the double-black problem, we will perform equivalent transfer and fusion operations. Consider three cases:

Case 1: The sibling y of *childOfu* is black and has a red child. This case has four possible configurations, as shown in Figures 41.17(a), 41.18(a), 41.19(a), and 41.20(a). The dashed circle denotes that the node is either red or black. To eliminate the double-black problem, restructure and recolor the nodes, as shown in Figures 41.17(b), 41.18(b), 41.19(b), and 41.20(b).

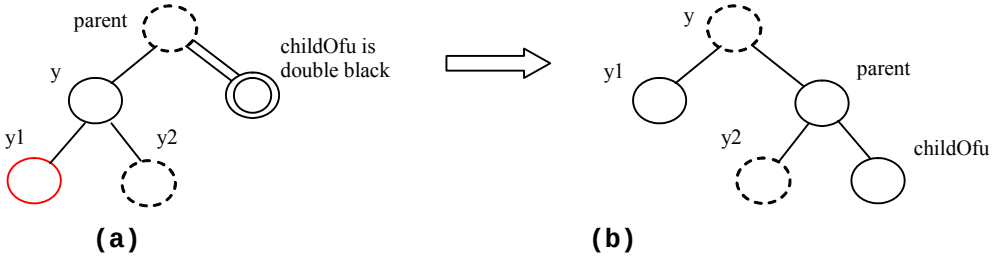


Figure 41.17
Case 1.1: The sibling y of childOfu is black and y1 is red.

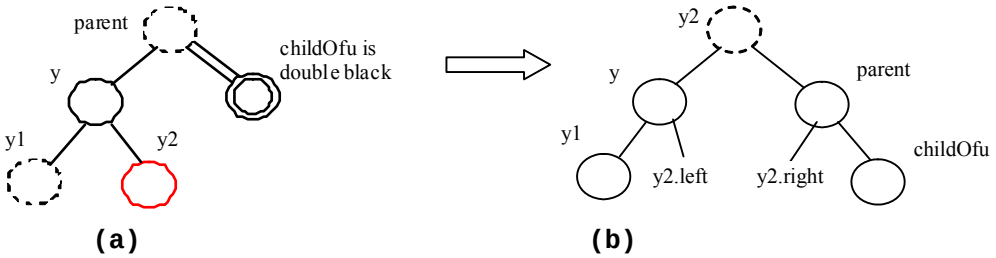


Figure 41.18
Case 1.2: The sibling y of childOfu is black and y2 is red.

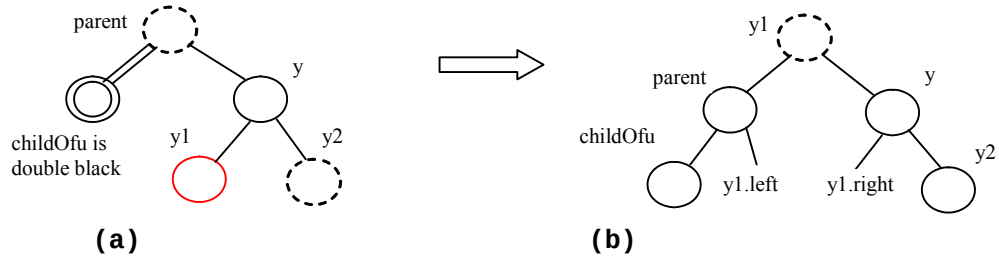


Figure 41.19
Case 1.3: The sibling y of childOfu is black and y1 is red.

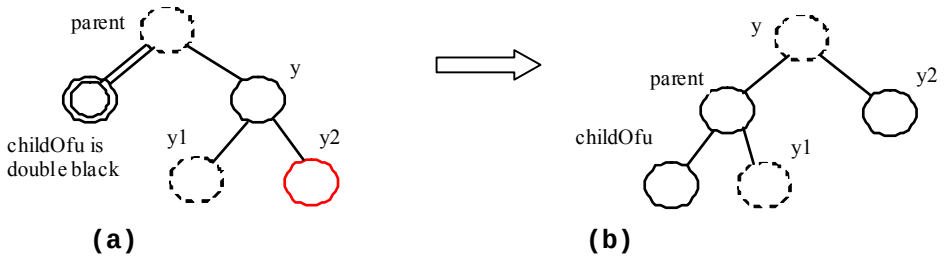


Figure 41.20

Case 1.4: the sibling y of childOfu is black and y2 is red.

Note

Case 1 corresponds to a *transfer* operation in the 2-4 tree. For example, the corresponding 2-4 tree for Figure 41.17(a) is shown in Figure 41.21(a), and it is transformed into 41.21(b) through a transfer operation.

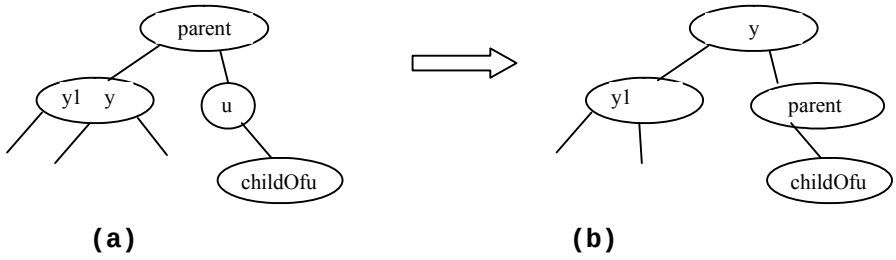


Figure 41.21

Case 1 corresponds to a transfer operation in the corresponding 2-4 tree.

Case 2: The sibling y of *childOfu* is black and its children are black or null. In this case, change y's color to red. If parent is red, change it to black, and we are done, as shown in Figure 41.22. If parent is black, we denote parent double black, as shown in Figure 41.23. The double-black problem *propagates* to the parent node.

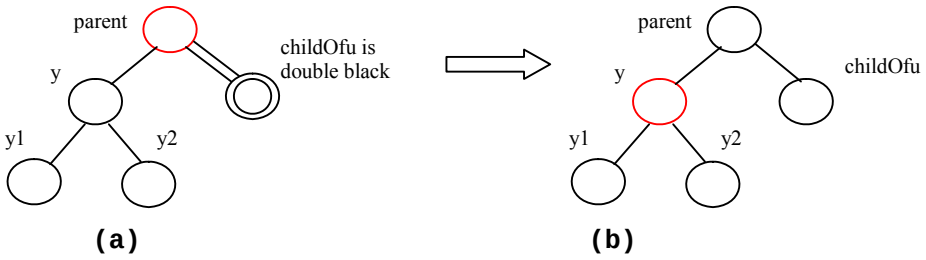


Figure 41.22

Case 2: Recoloring eliminates the double-black problem if parent is red.

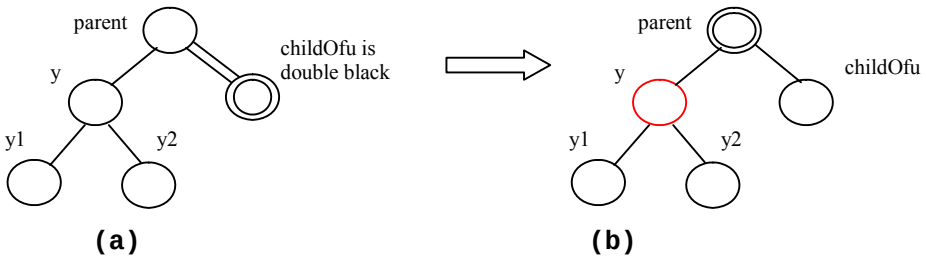


Figure 41.23

Case 2: Recoloring propagates the double-black problem if parent is black.

Note

Figures 41.22 and 41.22 show that childOfu is a right child of parent. If childOfu is a left child of parent, recoloring is performed identically.

Note

Case 2 corresponds to a *fusion* operation in the 2-4 tree. For example, the corresponding 2-4 tree for Figure 41.22(a) is shown in Figure 41.24(a), and it is transformed into 41.24(b) through a fusion operation.

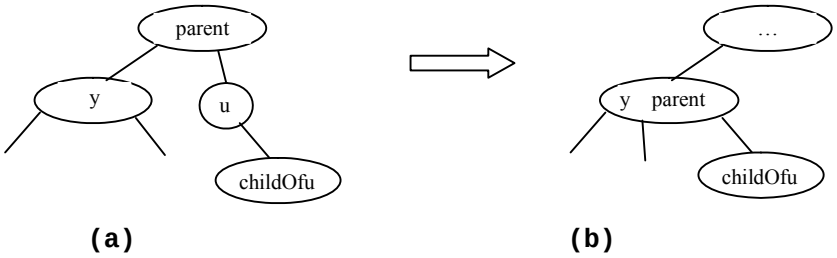


Figure 41.24

Case 2 corresponds to a fusion operation in the corresponding 2-4 tree.

Case 3: The sibling y of *childOfu* is red. In this case, perform an *adjustment* operation. If y is a left child of parent, let y1 and y2 be the left and right child of y, as shown in Figure 41.25. If y is a right child of parent, let y1 and y2 be the left and right child of y, as shown in Figure 41.26. In both cases, color y black and parent red. childOfu is still a fictitious double-black node. After the adjustment, the sibling of childOfu is now black, and either Case 1 or Case 2 applies. If Case 1 applies, a one-time restructuring and recoloring operation eliminates the double-black problem. If Case 2 applies, the double-black problem cannot reappear, since parent is now red. Therefore, one-time application of Case 1 or Case 2 will complete Case 3.

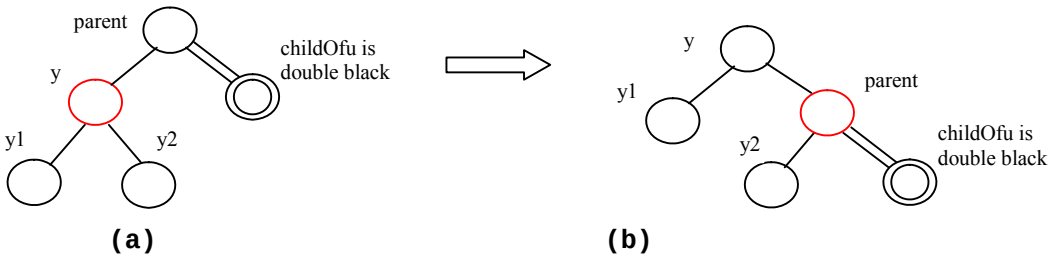


Figure 41.25

Case 3.1: y is a left red child of parent.

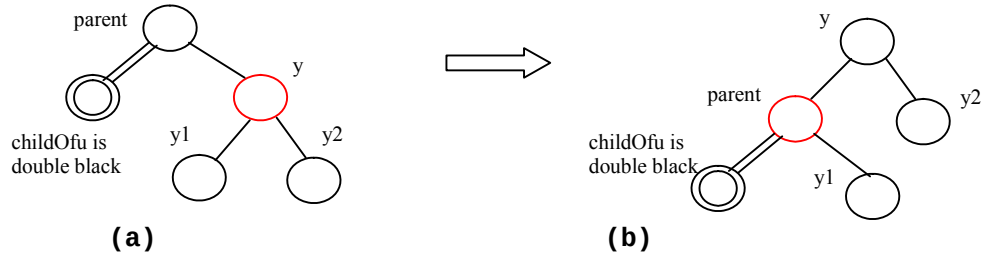


Figure 41.26

Case 3.2: y is a right red child of parent.

Note

Case 3 results from the fact that a 3-node may be transformed in two ways to a red-black tree, as shown in Figure 41.27.

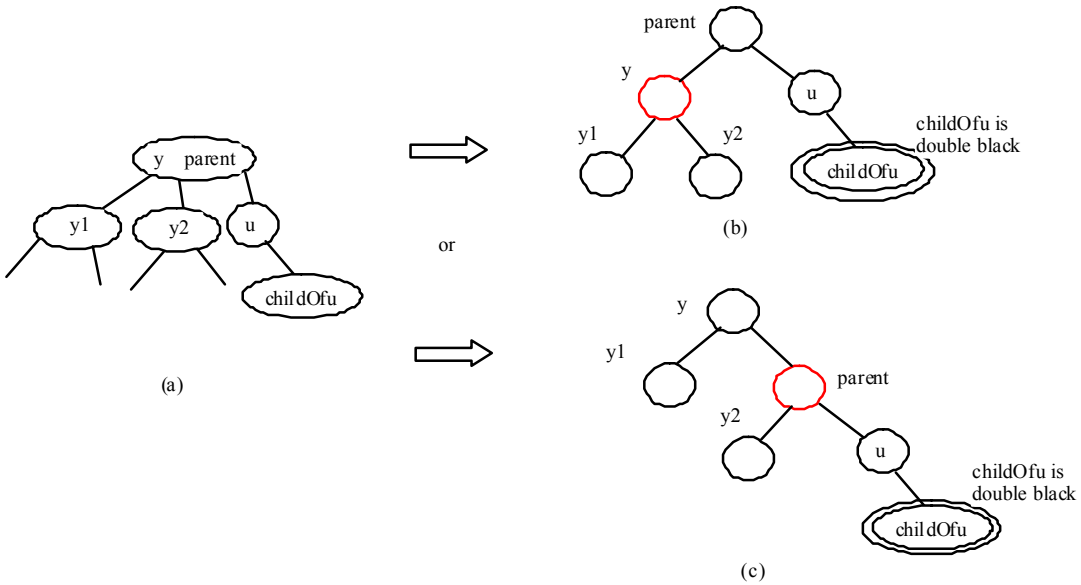


Figure 41.27

A 3-node may be transformed in two ways to red-black tree nodes.

Based on the foregoing discussion, Listing 41.2 presents a more detailed algorithm for deleting an element.

Listing 41.2 Deleting an Element from a Red-Black Tree

```
public boolean delete(E e) {
  Locate the node to be deleted
  if (the node is not found)
    return false;

  if (the node is an internal node) {
    Find the rightmost node in the subtree of the node;
    Replace the element in the node with the one in rightmost;
    The rightmost node is the node to be deleted now;
  }

  Obtain the path from the root to the node to be deleted;

  // Delete the last node in the path and propagate if needed
  deleteLastNodeInPath(path);

  size--; // After one element deleted
  return true; // Element deleted
}

/** Delete the last node from the path. */
public void deleteLastNodeInPath(ArrayList<TreeNode<E>> path) {
  Get the last node u in the path;
  Get parentOfu and grandparentOfu in the path;
  Get childOfu from u;
  Delete node u. Connect childOfu with parentOfu

  // Recolor the nodes and fix double black if needed
  if (childOfu == root || u.isRed())
    return; // Done if childOfu is root or if u is red
  else if (childOfu != null && childOfu.isRed())
    childOfu.setBlack(); // Set it black, done
  else // u is black, childOfu is null or black
    // Fix double black on parentOfu
    fixDoubleBlack(grandparentOfu, parentOfu, childOfu, path, i);
}

/** Fix the double black problem at node parent */
private void fixDoubleBlack(
    RBTreeNode<E> grandparent, RBTreeNode<E> parent,
    RBTreeNode<E> db, ArrayList<TreeNode<E>> path, int i) {
  Obtain y, y1, and y2

  if (y.isBlack() && y1 != null && y1.isRed()) {
    if (parent.right == db) {
      // Case 1.1: y is a left black sibling and y1 is red
      Restructure and recolor parent, y, and y1 to fix the problem;
    }
    else {
```

```
            // Case 1.3: y is a right black sibling and y1 is red
            Restructure and recolor parent, y1, and y to fix the problem;
          }
        }
        else if (y.isBlack() && y2 != null && y2.isRed()) {
          if (parent.right == db) {
            // Case 1.2: y is a left black sibling and y2 is red
            Restructure and recolor parent, y2, and y to fix the problem;
          }
          else {
            // Case 1.4: y is a right black sibling and y2 is red
            Restructure and recolor parent, y, and y2 to fix the problem;
          }
        }
        else if (y.isBlack()) {
          // Case 2: y is black and y's children are black or null
          Recolor y to red;

          if (parent.isRed())
            parent.setBlack(); // Done
          else if (parent != root) {
            // Propagate double black to the parent node
            // Fix new appearance of double black recursively
            db = parent;
            parent = grandparent;
            grandparent =
              (i >= 3) ? (RBTreeNode<E>)(path.get(i - 3)) : null;
            fixDoubleBlack(grandparent, parent, db, path, i - 1);
          }
        }
        else if (y.isRed()) {
          if (parent.right == db) {
            // Case 3.1: y is a left red child of parent
            parent.left = y2;
            y.right = parent;
          }
          else {
            // Case 3.2: y is a right red child of parent
            parent.right = y.left;
            y.left = parent;
          }

          parent.setRed(); // Color parent red
          y.setBlack(); // Color y black
          connectNewParent(grandparent, parent, y); // y is new parent
          fixDoubleBlack(y, parent, db, path, i - 1);
        }
      }
```

The delete(E e) method (lines 1–19) locates the node that contains e (line 2). If the node does not exist, return false (lines 3–4). If the node is an internal node, find the right most node in its left subtree and replace the element in the node with the element in the right most node (lines 6–9). Now the node to be deleted is an external node. Obtain the path from the root to the node (line 12). Invoke

deleteLastNodeInPath(path) to delete the last node in the path and ensure that the tree is still a red-black tree (line 15).

The deleteLastNodeInPath method (lines 22–36) obtains the last node u, parentOfu, grandparendOfu, and childOfu (lines 23–26). If childOfu is the root or u is red, the tree is fine (lines 29–30). If childOfu is red, color it black (lines 31–32). We are done. Otherwise, u is black and childOfu is null or black. Invoke fixDoubleBlack to eliminate the double-black problem (line 35).

The fixDoubleBlack method (lines 39–97) eliminates the double-black problem. Obtain y, y1, and y2 (line 42). y is the sibling of the double-black node. y1 and y2 are the left and right children of y. Consider three cases:

1. If y is black and one of its children is red, the double-black problem can be fixed by one-time restructuring and recoloring in Case 1 (lines 44–63).
2. If y is black and its children are null or black, change y to red. If parent of y is black, denote parent to be the new double-black node and invoke fixDoubleBlack recursively (line 77).
3. If y is red, adjust the nodes to make parent a child of y (lines 84, 89) and color parent red and y black (lines 92–93). Make y the new parent (line 94). Recursively invoke fixDoubleBlack on the same double-black node with a different color for parent (line 95).

Figure 41.28 shows the steps of deleting elements. To delete 50 from the tree in Figure 41.28(a), apply Case 1.2, as shown in Figure 41.28(b). After restructuring and recoloring, the new tree is as shown in Figure 41.28(c).

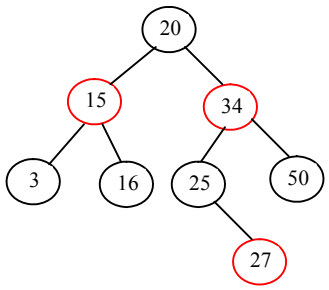
When deleting 20 in Figure 41.28(c), 20 is an internal node, and it is replaced by 16, as shown in Figure 41.28(d). Now Case 2 applies to deleting the rightmost node, as shown in Figure 41.28(e). Recolor the nodes results in a new tree, as shown in Figure 41.28(f).

When deleting 15, connect node 3 with node 20 and color node 3 black, as shown in Figure 41.28(g). We are done.

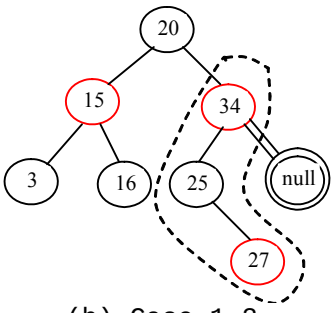
After deleting 25, the new tree is as shown in Figure 41.28(j). Now delete 16. Apply Case 2, as shown in Figure 41.28(k). The new tree is shown in Figure 41.28(l).

After deleting 34, the new tree is as shown in Figure 41.28(m).

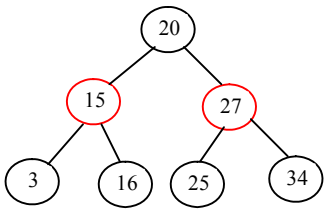
After deleting 27, the new tree is as shown in Figure 41.28(n).



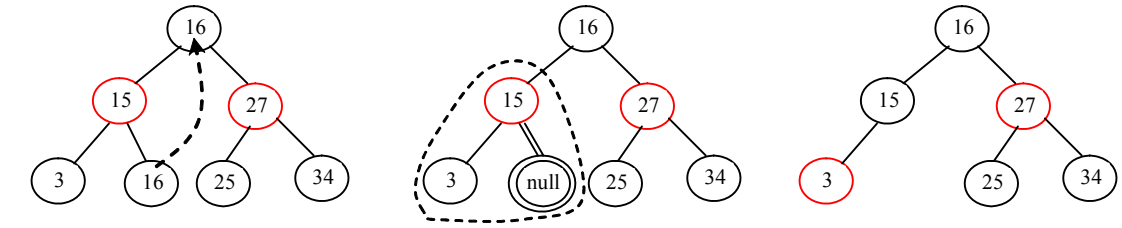
(a) Delete 50



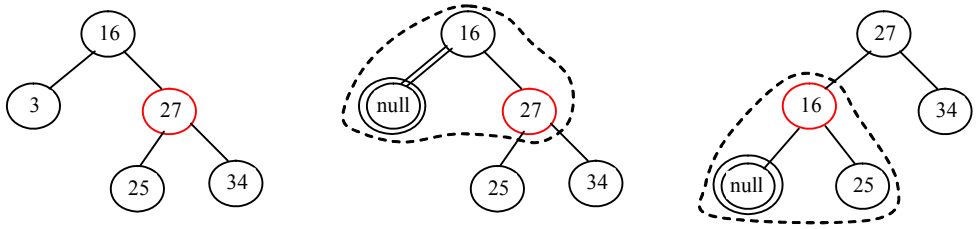
(b) Case 1.2



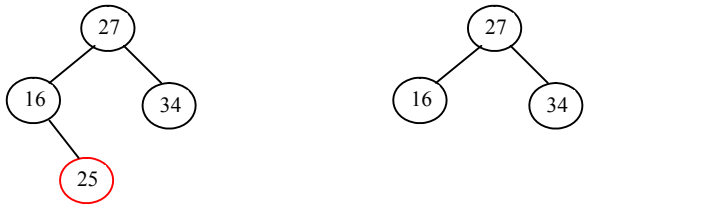
(c) Delete 20



(d) Copy 16 to replace 20 (e) Case 2 (f) Delete 15



(g) Delete 3 (h) Case 3 (i) Case 2



(i) Delete 25 (j) Delete 16 (k) Case 2

(l) Delete 34 (m) Delete 27 (n) Empty tree

Figure 41.28
Delete elements from a red-black tree.

Check point

41.5
What are the data fields in RBTreeNode?

41.6
How do you insert an element into a red-black tree and how do you fix the double-red violation?

41.7
How do you delete an element from a red-black tree and how do you fix the double-black problem?

41.8

Show the change of the tree when inserting 1, 2, 3, 4, 10, 9, 7, 5, 8, 6 into it, in this order.

41.9
For the tree built in the preceding question, show the change of the tree after deleting 1, 2, 3, 4, 10, 9, 7, 5, 8, 6 from it in this order.

41.6 Implementing RBTree Class

Key Point: This section implements the RBTree class.

Listing 41.3 gives a complete implementation for the RBTree class.

Listing 41.3 RBTree.java

```
import java.util.ArrayList;

public class RBTree<E extends Comparable<E>> extends BST<E> {
  /** Create a default RB tree */
  public RBTree() {
  }

  /** Create an RB tree from an array of elements */
  public RBTree(E[] elements) {
    super(elements);
  }

  /** Override createNewNode to create an RBTreeNode */
  protected RBTreeNode<E> createNewNode(E e) {
    return new RBTreeNode<E>(e);
  }

  /** Override the insert method to balance the tree if necessary */
  public boolean insert(E e) {
    boolean successful = super.insert(e);
    if (!successful)
      return false; // e is already in the tree
    else {
      ensureRBTree(e);
    }

    return true; // e is inserted
  }

  /** Ensure that the tree is a red-black tree */
  private void ensureRBTree(E e) {
    // Get the path that leads to element e from the root
    ArrayList<TreeNode<E>> path = path(e);

    int i = path.size() - 1; // Index to the current node in the path

    // u is the last node in the path. u contains element e
    RBTreeNode<E> u = (RBTreeNode<E>)(path.get(i));
```

```
    // v is the parent of of u, if exists
    RBTreeNode<E> v = (u == root) ? null :
      (RBTreeNode<E>)(path.get(i - 1));

    u.setRed(); // It is OK to set u red

    if (u == root) // If e is inserted as the root, set root black
      u.setBlack();
    else if (v.isRed())
      fixDoubleRed(u, v, path, i); // Fix double-red violation at u
  }

  /** Fix double-red violation at node u */
  private void fixDoubleRed(RBTreeNode<E> u, RBTreeNode<E> v,
      ArrayList<TreeNode<E>> path, int i) {
    // w is the grandparent of u
    RBTreeNode<E> w = (RBTreeNode<E>)(path.get(i - 2));
    RBTreeNode<E> parentOfw = (w == root) ? null :
      (RBTreeNode<E>)path.get(i - 3);

    // Get v's sibling named x
    RBTreeNode<E> x = (w.left == v) ?
      (RBTreeNode<E>)(w.right) : (RBTreeNode<E>)(w.left);

    if (x == null || x.isBlack()) {
      // Case 1: v's sibling x is black
      if (w.left == v && v.left == u) {
        // Case 1.1: u < v < w, Restructure and recolor nodes
        restructureRecolor(u, v, w, w, parentOfw);

        w.left = v.right; // v.right is y3 in Figure 41.6
        v.right = w;
      }
      else if (w.left == v && v.right == u) {
        // Case 1.2: v < u < w, Restructure and recolor nodes
        restructureRecolor(v, u, w, w, parentOfw);
        v.right = u.left;
        w.left = u.right;
        u.left = v;
        u.right = w;
      }
      else if (w.right == v && v.right == u) {
        // Case 1.3: w < v < u, Restructure and recolor nodes
        restructureRecolor(w, v, u, w, parentOfw);
        w.right = v.left;
        v.left = w;
      }
      else {
        // Case 1.4: w < u < v, Restructure and recolor nodes
        restructureRecolor(w, u, v, w, parentOfw);
        w.right = u.left;
        v.left = u.right;
        u.left = w;
        u.right = v;
      }
    }
```

```
    else { // Case 2: v's sibling x is red
      // Recolor nodes
      w.setRed();
      u.setRed();
      ((RBTreeNode<E>)(w.left)).setBlack();
      ((RBTreeNode<E>)(w.right)).setBlack();

      if (w == root) {
        w.setBlack();
      }
      else if (((RBTreeNode<E>)parentOfw).isRed()) {
        // Propagate along the path to fix new double-red violation
        u = w;
        v = (RBTreeNode<E>)parentOfw;
        fixDoubleRed(u, v, path, i - 2); // i - 2 propagates upward
      }
    }
  }

  /** Connect b with parentOfw and recolor a, b, c for a < b < c */
  private void restructureRecolor(RBTreeNode<E> a, RBTreeNode<E> b,
      RBTreeNode<E> c, RBTreeNode<E> w, RBTreeNode<E> parentOfw) {
    if (parentOfw == null)
      root = b;
    else if (parentOfw.left == w)
      parentOfw.left = b;
    else
      parentOfw.right = b;

    b.setBlack(); // b becomes the root in the subtree
    a.setRed(); // a becomes the left child of b
    c.setRed(); // c becomes the right child of b
  }

  /** Delete an element from the RBTree.
   * Return true if the element is deleted successfully
   * Return false if the element is not in the tree */
  public boolean delete(E e) {
    // Locate the node to be deleted
    TreeNode<E> current = root;
    while (current != null) {
      if (e.compareTo(current.element) < 0) {
        current = current.left;
      }
      else if (e.compareTo(current.element) > 0) {
        current = current.right;
      }
      else
        break; // Element is in the tree pointed by current
    }

    if (current == null)
      return false; // Element is not in the tree

    java.util.ArrayList<TreeNode<E>> path;
```

```
    // current node is an internal node
    if (current.left != null && current.right != null) {
      // Locate the rightmost node in the left subtree of current
      TreeNode<E> rightMost = current.left;
      while (rightMost.right != null) {
        rightMost = rightMost.right; // Keep going to the right
      }

      path = path(rightMost.element); // Get path before replacement

      // Replace the element in current by the element in rightMost
      current.element = rightMost.element;
    }
    else
      path = path(e); // Get path to current node

    // Delete the last node in the path and propagate if needed
    deleteLastNodeInPath(path);

    size--; // After one element deleted
    return true; // Element deleted
  }

  /** Delete the last node from the path. */
  public void deleteLastNodeInPath(ArrayList<TreeNode<E>> path) {
    int i = path.size() - 1; // Index to the node in the path
    // u is the last node in the path
    RBTreeNode<E> u = (RBTreeNode<E>)(path.get(i));
    RBTreeNode<E> parentOfu = (u == root) ? null :
      (RBTreeNode<E>)(path.get(i - 1));
    RBTreeNode<E> grandparentOfu = (parentOfu == null ||
      parentOfu == root) ? null :
      (RBTreeNode<E>)(path.get(i - 2));
    RBTreeNode<E> childOfu = (u.left == null) ?
      (RBTreeNode<E>)(u.right) : (RBTreeNode<E>)(u.left);

    // Delete node u. Connect childOfu with parentOfu
    connectNewParent(parentOfu, u, childOfu);

    // Recolor the nodes and fix double black if needed
    if (childOfu == root || u.isRed())
      return; // Done if childOfu is root or if u is red
    else if (childOfu != null && childOfu.isRed())
      childOfu.setBlack(); // Set it black, done
    else // u is black, childOfu is null or black
      // Fix double black on parentOfu
      fixDoubleBlack(grandparentOfu, parentOfu, childOfu, path, i);
  }

  /** Fix the double-black problem at node parent */
  private void fixDoubleBlack(
      RBTreeNode<E> grandparent, RBTreeNode<E> parent,
      RBTreeNode<E> db, ArrayList<TreeNode<E>> path, int i) {
    // Obtain y, y1, and y2
    RBTreeNode<E> y = (parent.right == db) ?
      (RBTreeNode<E>)(parent.left) : (RBTreeNode<E>)(parent.right);
```

```
    RBTreeNode<E> y1 = (RBTreeNode<E>)(y.left);
    RBTreeNode<E> y2 = (RBTreeNode<E>)(y.right);

    if (y.isBlack() && y1 != null && y1.isRed()) {
      if (parent.right == db) {
        // Case 1.1: y is a left black sibling and y1 is red
        connectNewParent(grandparent, parent, y);
        recolor(parent, y, y1); // Adjust colors

        // Adjust child links
        parent.left = y.right;
        y.right = parent;
      }
      else {
        // Case 1.3: y is a right black sibling and y1 is red
        connectNewParent(grandparent, parent, y1);
        recolor(parent, y1, y); // Adjust colors

        // Adjust child links
        parent.right = y1.left;
        y.left = y1.right;
        y1.left = parent;
        y1.right = y;
      }
    }
    else if (y.isBlack() && y2 != null && y2.isRed()) {
      if (parent.right == db) {
        // Case 1.2: y is a left black sibling and y2 is red
        connectNewParent(grandparent, parent, y2);
        recolor(parent, y2, y); // Adjust colors

        // Adjust child links
        y.right = y2.left;
        parent.left = y2.right;
        y2.left = y;
        y2.right = parent;
      }
      else {
        // Case 1.4: y is a right black sibling and y2 is red
        connectNewParent(grandparent, parent, y);
        recolor(parent, y, y2); // Adjust colors

        // Adjust child links
        y.left = parent;
        parent.right = y1;
      }
    }
    else if (y.isBlack()) {
      // Case 2: y is black and y's children are black or null
      y.setRed(); // Change y to red
      if (parent.isRed())
        parent.setBlack(); // Done
      else if (parent != root) {
        // Propagate double black to the parent node
        // Fix new appearance of double black recursively
        db = parent;
```

```
          parent = grandparent;
          grandparent =
            (i >= 3) ? (RBTreeNode<E>)(path.get(i - 3)) : null;
          fixDoubleBlack(grandparent, parent, db, path, i - 1);
        }
      }
      else { // y.isRed()
        if (parent.right == db) {
          // Case 3.1: y is a left red child of parent
          parent.left = y2;
          y.right = parent;
        }
        else {
          // Case 3.2: y is a right red child of parent
          parent.right = y.left;
          y.left = parent;
        }

        parent.setRed(); // Color parent red
        y.setBlack(); // Color y black
        connectNewParent(grandparent, parent, y); // y is new parent
        fixDoubleBlack(y, parent, db, path, i - 1);
      }
    }

    /** Recolor parent, newParent, and c. Case 1 removal */
    private void recolor(RBTreeNode<E> parent,
        RBTreeNode<E> newParent, RBTreeNode<E> c) {
      // Retain the parent's color for newParent
      if (parent.isRed())
        newParent.setRed();
      else
        newParent.setBlack();

      // c and parent become the children of newParent; set them black
      parent.setBlack();
      c.setBlack();
    }

    /** Connect newParent with grandParent */
    private void connectNewParent(RBTreeNode<E> grandparent,
        RBTreeNode<E> parent, RBTreeNode<E> newParent) {
      if (parent == root) {
        root = newParent;
        if (root != null)
          newParent.setBlack();
      }
      else if (grandparent.left == parent)
        grandparent.left = newParent;
      else
        grandparent.right = newParent;
    }

    /** Preorder traversal from a subtree */
    protected void preorder(TreeNode<E> root) {
      if (root == null) return;
```

```
    System.out.print(root.element +
      (((RBTreeNode<E>)root).isRed() ? " (red) " : " (black) "));
    preorder(root.left);
    preorder(root.right);
  }

  /** RBTreeNode is TreeNode plus color indicator */
  protected static class RBTreeNode<E extends Comparable<E>> extends
      BST.TreeNode<E> {
    private boolean red = true; // Indicate node color

    public RBTreeNode(E e) {
      super(e);
    }

    public boolean isRed() {
      return red;
    }

    public boolean isBlack() {
      return!red;
    }

    public void setBlack() {
      red = false;
    }

    public void setRed() {
      red = true;
    }

    int blackHeight;
  }
}
```

The RBTree class extends BST. Like the BST class, the RBTree class has a no-arg constructor that constructs an empty RBTree (lines 5–6) and a constructor that creates an initial RBTree from an array of elements (lines 9–11).

The createNewNode() method defined in the BST class creates a TreeNode. This method is overridden to return an RBTreeNode (lines 14–16). This method is invoked in the insert method in BST to create a node.

The insert method in RBTree is overridden in lines 19–28. The method first invokes the insert method in BST, then invokes ensureRBTree(e) (line 24) to ensure that tree is still a red-black tree after inserting a new element.

The ensureRBTree(E e) method first obtains the path of nodes that lead to element e from the root (line 33). It obtains u and v (the parent of u) from the path. If u is the root, color u black (lines 46–47). If v is red, invoke fixDoubleRed to fix the double red on both u and v (lines 48–49).

The fixDoubleRed(u, v, path, i) method fixes the double-red violation at node u. The method first obtains w (the grandparent of u from the path)

(line 56), parentOfw if exists (lines 57–58), and x (the sibling of v) (lines 61–62). If x is null or black, consider four subcases to fix the double-red violation (lines 66–95). If x is red, color w and u red and color w's two children black (lines 100–103). If w is the root, color w black (lines 103-105). Otherwise, propagate along the path to fix the new double-red violation (lines 108–110).

The delete(E e) method in RBTree is overridden in lines 133–173. The method locates the node that contains e (lines 135–145). If the node is null, no element is found (lines 147–148). The method considers two cases:

- If the node is internal, find the rightmost node in its left subtree (lines 155–158). Obtain a path from the root to the rightmost node (line 160), and replace the element in the node with the element in the rightmost node (line 163).

- If the node is external, obtain the path from the root to the node (line 166).

The last node in the path is the node to be deleted. Invoke deleteLastNodeInPath(path) to delete it and ensure the tree is a red-black after the node is deleted (line 169).

The deleteLastNodeInPath(path) method first obtains u, parentOfu, grandparendOfu, and childOfu (lines 179-186). u is the last node in the path. Connect childOfu as a child of parentOfu (line 189). This in effect deletes u from the tree. Consider three cases:

- If childOfu is the root or childOfu is red, we are done (lines 192-193).
- Otherwise, if childOfu is red, color it black (lines 194–195).
- Otherwise, invoke fixDoubleBlack to fix the double-black problem on childOfu (line 198).

The fixDoubleBlack method first obtains y, y1, and y2 (lines 206-209). y is the sibling of the first double-black node, and y1 and y2 are the left and right children of y. Consider three cases:

- If y is black and y1 or y2 is red, fix the double-black problem for Case 1 (lines 212–254).

- Otherwise, if y is black, fix the double-black problem for Case 2 by recoloring the nodes. If parent is black and not a root, propagate double black to parent and recursively invoke fixDoubleBlack (lines 263–267).

- Otherwise, y is red. In this case, adjust the nodes to make parent the child of y (lines 271–280). Invoke fixDoubleBlack with the adjusted nodes (line 285) to fix the double-black problem.

The preorder(TreeNode<E> root) method is overridden to display the node colors (lines 318-324).

41.7 Testing the RBTree Class

Key Point: This section gives a test program that uses the RBTree class.

Listing 41.4 gives a test program. The program creates an RBTree initialized with an array of integers 34, 3, and 50 (lines 4–5), inserts elements in lines 10–22, and deletes elements in lines 25–46.

Listing 41.4 TestRBTree.java

```
public class TestRBTree {
  public static void main(String[] args) {
    // Create an RB tree
    RBTree<Integer> tree =
      new RBTree<Integer>(new Integer[]{34, 3, 50});
    printTree(tree);

    tree.insert(20);
    printTree(tree);

    tree.insert(15);
    printTree(tree);

    tree.insert(16);
    printTree(tree);

    tree.insert(25);
    printTree(tree);

    tree.insert(27);
    printTree(tree);

    tree.delete(50);
    printTree(tree);

    tree.delete(20);
    printTree(tree);

    tree.delete(15);
    printTree(tree);

    tree.delete(3);
    printTree(tree);

    tree.delete(25);
    printTree(tree);

    tree.delete(16);
    printTree(tree);

    tree.delete(34);
    printTree(tree);

    tree.delete(27);
    printTree(tree);
  }

  public static void printTree(BST tree) {
    // Traverse tree
    System.out.print("\nInorder (sorted): ");
```

```
    tree.inorder();
    System.out.print("\nPostorder: ");
    tree.postorder();
    System.out.print("\nPreorder: ");
    tree.preorder();
    System.out.print("\nThe number of nodes is " + tree.getSize());
    System.out.println();
  }
}
```

Output

```
Inorder (sorted): 3 34 50
Postorder: 3 50 34
Preorder: 34 (black) 3 (red) 50 (red)
The number of nodes is 3

Inorder (sorted): 3 20 34 50
Postorder: 20 3 50 34
Preorder: 34 (black) 3 (black) 20 (red) 50 (black)
The number of nodes is 4

Inorder (sorted): 3 15 20 34 50
Postorder: 3 20 15 50 34
Preorder: 34 (black) 15 (black) 3 (red) 20 (red) 50 (black)
The number of nodes is 5

Inorder (sorted): 3 15 16 20 34 50
Postorder: 3 16 20 15 50 34
Preorder: 34 (black) 15 (red) 3 (black) 20 (black) 16 (red) 50 (black)
The number of nodes is 6

Inorder (sorted): 3 15 16 20 25 34 50
Postorder: 3 16 25 20 15 50 34
Preorder: 34 (black) 15 (red) 3 (black) 20 (black) 16 (red) 25 (red)
  50 (black)
The number of nodes is 7

Inorder (sorted): 3 15 16 20 25 27 34 50
Postorder: 3 16 15 27 25 50 34 20
Preorder: 20 (black) 15 (red) 3 (black) 16 (black) 34 (red) 25 (black)
  27 (red) 50 (black)
The number of nodes is 8

Inorder (sorted): 3 15 16 20 25 27 34
Postorder: 3 16 15 25 34 27 20
Preorder: 20 (black) 15 (red) 3 (black) 16 (black) 27 (red)
  25 (black) 34 (black)
The number of nodes is 7

Inorder (sorted): 3 15 16 25 27 34
Postorder: 3 15 25 34 27 16
Preorder: 16 (black) 15 (black) 3 (red) 27 (red) 25 (black) 34 (black)
The number of nodes is 6

Inorder (sorted): 3 16 25 27 34
Postorder: 3 25 34 27 16
```

```
Preorder: 16 (black) 3 (black) 27 (red) 25 (black) 34 (black)
The number of nodes is 5


Inorder (sorted): 16 25 27 34
Postorder: 25 16 34 27
Preorder: 27 (black) 16 (black) 25 (red) 34 (black)
The number of nodes is 4

Inorder (sorted): 16 27 34
Postorder: 16 34 27
Preorder: 27 (black) 16 (black) 34 (black)
The number of nodes is 3

Inorder (sorted): 27 34
Postorder: 34 27
Preorder: 27 (black) 34 (red)
The number of nodes is 2

Inorder (sorted): 27
Postorder: 27
Preorder: 27 (black)
The number of nodes is 1

Inorder (sorted):
Postorder:
Preorder:
The number of nodes is 0
```

Figure 41.14 shows how the tree evolves as elements are added to it, and Figure 41.28 shows how the tree evolves as elements are deleted from it.

41.8 Performance of the RBTree Class

Key Point: This search, insertion, and deletion operations take O(logn) time in a red-black tree.

The search, insertion, and deletion times in a red-black tree depend on the height of the tree. A red-black tree corresponds to a 2-4 tree. When you convert a node in a 2-4 tree to red-black tree nodes, you get one black node and zero, one, or two red nodes as its children, depending on whether the original node is a 2-node, 3-node, or 4-node. So, the height of a red-black tree is at most as twice that of its corresponding 2-4 tree. Since the height of a 2-4 tree is $\log n$, the height of a red-black tree is $2\log n$.

A red-black tree has the same time complexity as an AVL tree, as shown in Table 41.1. In general, a red-black is more efficient than an AVL tree, because a red-black tree requires only one time restructuring of the nodes for insert and delete operations.

A red-black tree has the same time complexity as a 2-4 tree, as shown in Table 41.1. In general, a red-black is more efficient than a 2-4 tree for two reasons:

1. A red-black tree requires only one-time restructuring of the nodes for insert and delete operations. However, a 2-4 tree may require many splits for an insert operation and fusion for a delete operation.
2. A red-black tree is a binary search tree. A binary tree can be implemented more space efficiently than a 2-4 tree, because a node in a 2-4 tree has at most three elements and four children. Space is wasted for 2-nodes and 3-nodes in a 2-4 tree.

Table 41.1
Time Complexities for Methods in RBTree, AVLTree, and Tree234

Mehtods	Red-Black Tree	AVL Tree	2-4 Tree
search(e: E)	$O(\log n)$	$O(\log n)$	$O(\log n)$
insert(e: E)	$O(\log n)$	$O(\log n)$	$O(\log n)$
delete(e: E)	$O(\log n)$	$O(\log n)$	$O(\log n)$
getSize()	$O(1)$	$O(1)$	$O(1)$
isEmpty()	$O(1)$	$O(1)$	$O(1)$

Listing 41.5 gives an empirical test of the performance of AVL trees, 2-4 trees, and red-black trees.

Listing 41.5 TreePerformanceTest.java

```
public class TreePerformanceTest {
  public static void main(String[] args) {
    final int TEST_SIZE = 500000; // Tree size used in the test

    // Create an AVL tree
    Tree<Integer> tree1 = new AVLTree<Integer>();
    System.out.println("AVL tree time: " +
      getTime(tree1, TEST_SIZE) + " milliseconds");

    // Create a 2-4 tree
    Tree<Integer> tree2 = new Tree24<Integer>();
    System.out.println("2-4 tree time: "
      + getTime(tree2, TEST_SIZE) + " milliseconds");

    // Create a red-black tree
    Tree<Integer> tree3 = new RBTree<Integer>();
    System.out.println("RB tree time: "
      + getTime(tree3, TEST_SIZE) + " milliseconds");
  }

  public static long getTime(Tree<Integer> tree, int testSize) {
    long startTime = System.currentTimeMillis(); // Start time

    // Create a list to store distinct integers
    java.util.List<Integer> list = new java.util.ArrayList<Integer>();
    for (int i = 0; i < testSize; i++)
```

```
      list.add(i);

    java.util.Collections.shuffle(list); // Shuffle the list

    // Insert elements in the list to the tree
    for (int i = 0; i < testSize; i++)
      tree.insert(list.get(i));

    java.util.Collections.shuffle(list); // Shuffle the list

    // Delete elements in the list from the tree
    for (int i = 0; i < testSize; i++)
      tree.delete(list.get(i));

    // Return elapse time
    return System.currentTimeMillis() - startTime;
  }
}
```

Output

```
AVL tree time: 7609 milliseconds
2-4 tree time: 8594 milliseconds
RB tree time: 5515 milliseconds
```

The getTestTime method creates a list of distinct integers from 0 to testSize – 1 (lines 25–27), shuffles the list (line 29), adds the elements from the list to a tree (lines 32–33), shuffles the list again (line 35), removes the elements from the tree (lines 38–39), and finally returns the execution time (line 42).

The program creates an AVL (line 6), a 2-4 tree (line 11), and a red-black tree (line 16). The program obtains the execution time for adding and removing 500000 elements in the three trees.

As you see, the red-black tree performs the best, followed by the AVL tree.

NOTE:
The java.util.TreeSet class in the Java API is implemented using a red-black tree. Each entry in the set is stored in the tree. Since the search, insert, and delete methods in a red-black tree take $O(\log n)$ time, the get, add, remove, and contains methods in java.util.TreeSet take $O(\log n)$ time.

NOTE:
The java.util.TreeMap class in the Java API is implemented using a red-black tree. Each entry in the map is stored in the tree. The order of the entries is determined by their keys. Since the search, insert, and delete methods in a red-black tree take $O(\log n)$ time, the get, put, remove, and containsKey methods in java.util.TreeMap take $O(\log n)$ time.

Key Terms

- black depth

- double-black violation
- double-red violation
- external node
- red-black tree

Chapter Summary

1. A red-black tree is a binary search tree, derived from a *2-4 tree*. A red-black tree corresponds to a 2-4 tree. You can convert a red-black tree to a 2-4 tree or vice versa.
2. In a red-black tree, each node is colored red or black. The root is always black. Two adjacent nodes cannot be both red. All external nodes have the same black depth.
3. Since a red-black tree is a binary search tree, the RBTree class extends the BST class.
4. Searching an element in a red-black tree is the same as in binary search tree, since a red-black tree is a binary search tree.
5. A new element is always inserted as a leaf node. If the new node is the root, color it black. Otherwise, color it red. If the parent of the new node is red, we have to fix the *double-red violation* by reassigning the color and/or restructuring the tree.
6. If a node to be deleted is internal, find the rightmost node in its left subtree. Replace the element in the node with the element in the rightmost node. Delete the rightmost node.
7. If the external node to be deleted is red, simply reconnect the parent node of the external node with the child node of the external node.
8. If the external node to be deleted is black, you need to consider several cases to ensure that black height for external nodes in the tree is maintained correctly.
9. The height of a red-black tree is O(logn). So, the time complexities for the search, insert, and delete methods are O(logn).

Quiz

Answer the quiz for this chapter online at www.cs.armstrong.edu/liang/intro10e/quiz.html.

Programming Exercises

41.1*
(*red-black tree to 2-4 tree*) Write a program that converts a red-black tree to a 2-4 tree.

41.2*
(*2-4 tree to red-black tree*) Write a program that converts a red-black tree to a 2-4 tree.

41.3***
(*red-black tree animation*) Write a GUI program that animates the red-black tree insert, delete, and search methods, as shown in Figure 41.6.

41.4**
(*Parent reference for RBTree*) Suppose that the TreeNode class defined in BST contains a reference to the node's parent, as shown in

Exercise 26.17. Implement the RBTree class to support this change. Write a test program that adds numbers 1, 2, ..., 100 to the tree and displays the paths for all leaf nodes.