Deterministic and Heuristic apprroach on finding the minimum of a function

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Abstract

This homework provides the introduction to genetic algorithms. It focuses on the aspect of modelling number candidates in a specific way, such as BitStrings. The operations are performed at bit level and will determine the specific evolution for all our number candidates. In this homework there will be two approaches: Hill Climbing and Simulated Annealing. The results will be compared and determine which approach works better on getting the minimum on specific functions in 2, 5 and 20 dimensions.

1 Objective

Determine which approach is better for getting the minimum for the next functions.

a. Booth

b. Eusom

c. Shubert

d. Rastrigin

1.1 Definitions

Booth Function

$$f(X) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2 \tag{1}$$

Eusom Function

$$f(x, y) = -cos(x_1)cos(x_2)exp(-(x - \pi)^2 - (y - \pi)^2) \tag{2}$$

Shubert Function

$$f(\mathbf{x}) = f(x_1, ..., x_n) = \prod_{i=1}^{n} \left(\sum_{j=1}^{5} cos((j+1)x_i + j) \right) \tag{3}$$

Rastrigin Function

$$f(x, y) = 10n + \sum_{i=1}^{n} (x_i^2 - 10cos(2\pi x_i)) \tag{4}$$

2 Setup

The language that tha algorith is written is *C++* to improve the speed.
Simple data structure were used: simple variables, matrices.

The experiments are done within 2, 5 and 20 dimensions. For each dimension, each function will be ran with a deterministic aproach to find the minimum. The non-deterministic aproach will be composed of 10000 runs of a random input and taken the minimum from all of these 30 runs.

Hill Climbing approach: Each run cosists of an evloutionary algorithm that is described like this:
Each candidate is represented in bitstring. His evaluation is done by transforming it into a real number and given as parameter into the specific function. A run represents an evolution of the candidate. The evolution is represented by having the candidate's bits changed, becoming a neighbour. By changing (negating) a single bit from the candidate we find ourselves with one of his n neighbours (if he has n bits).
The candidate evolves into the neighbour only if

$$eval(decoded(candidate)) > eval(decoded(neighbour)). \tag{5}$$

Simulated Annealing approach: In addition to the HC method, the simmulated annealing lets the algorithm make "involutions". When our evolving algorithm checks if expression (5), if the statements is false, it allows to involve into neighbour if

$$random[0, 1) < exp(-abs(eval(decoded(candidate)) - eval(decoded(neighbour))|)/T) \tag{6}$$

, where T is a temperature which on each run is decreased by a factor of 25%.

3 Sample Calculation

2 Dimensions - Hill Climbing				
Function \Value	F min	F Mean	F StdDev	F Avg
Booth	2	20.33818	30.17318	6.500013
Easom	-1	-0.4330835	0.5037164	-2.7e-09
Shubert	0.01151226	40.62198	22.8239	58.68443
Rastrigin	1e-10	3.134486	3.481765	1.994961

2 Dimensions - Simulated Annealing				
Function \Value	F min	F Mean	F StdDev	F Avg
Booth	4.07222	586.9657	544.5047	394.3619
Easom	-5e-10	-3.333333e-11	1.268541e-10	0
Shubert	0.3571665	77.25411	30.87428	76.6366
Rastrigin	4.365741	41.54321	22.93483	40.4976

5 Dimensions - Hill Climbing				
Function \Value	F min	F Mean	F StdDev	F Avg
Booth	2	17.51634	28.78196	2.281252
Easom	-1	-0.5996907	0.4980161	-0.9991743
Shubert	-6099756	-3725598	2728942	-5997212
Rastrigin	4e-10	10.49341	12.28039	5.589442

5 Dimensions - Simulated Annealing				
Function \Value	F min	F Mean	F StdDev	F Avg
Booth	13.7936	726.852	698.0556	465.2658
Easom	0	0	0	0
Shubert	-6055382	-290207.8	1639637	6.098384
Rastrigin	9.244849	86.36404	54.92566	66.9411

20 Dimensions - Hill Climbing				
Function \Value	F min	F Mean	F StdDev	F Avg
Booth	2	19.72947	30.50142	2.281252
Easom	-1	-0.3331948	0.4792641	-2.7e-09
Shubert	0	2.634213e+20	1.002482e+21	0
Rastrigin	1.5e-09	40.01471	40.76614	24.71551

20 Dimensions - Simulated Annealing				
Function \Value	F min	F Mean	F StdDev	F Avg
Booth	3.885553	616.2707	706.4712	342.4231
Easom	0	0	0	0
Shubert	163464.2	1.259035e+36	3.919845e+36	6.631709e+25
Rastrigin	21.36679	408.8735	219.8453	389.8166

4 Conclusion

In the case of Booth function in all dimensions we see that the HC algorithm is better. This is the cause of a smooth and ”predictable” curve nature of the function.

In the case of Easom function, the result is understandable because if we allow even the smallest involution, the chance of getting the minimum decreases drastically, cause of the nature of the function.

In the case of Shubert function, we see a lot of different values with a big difference between them. My attention is mostly attracted by the 20 dimension results which seem to be the cause of high number of local minima.

In case of Rastrigin function, the HC is better because the involution also determines a step back into a local minima and the function seems to respond better to constant evolution.