AIP | Review of Scientific Instruments

Initialization of plasma density profiles from reflectometry

P. Varela, M. E. Manso, A. Silva, J. Fernandes, and F. Silva

Citation: Review of Scientific Instruments **66**, 4937 (1995); doi: 10.1063/1.1146178
View online: http://dx.doi.org/10.1063/1.1146178
View Table of Contents: http://scitation.aip.org/content/aip/journal/rsi/66/10?ver=pdfcov
Published by the AIP Publishing

Initialization of plasma density profiles from reflectometry

P. Varela, M. E. Manso, A. Silva, J. Fernandes, and F. Silva
Associação EURATOM/IST—Centro de Fusão Nuclear/Instituto Superior Técnico, 1096 Lisboa Codex, Portugal

(Received 26 June 1995; accepted for publication 10 July 1995)

The contribution of the initial plasma to density profiles reconstructed from the $d\phi/df$ vs f curve obtained with reflectometry is studied. It is shown that the initial phase information determines to a great extent the accuracy of the inverted profiles at the edge. As it is difficult to measure the edge contribution models are required to initialize the profile evaluation. A novel method is presented that uses the phase information from the lowest frequency waves to obtain $d\phi/df$ below the first probing frequency F_1, by imposing the continuity with the measured $d\phi/df$ characteristic and its derivative at F_1. An approximate shape of the edge profile is obtained because low-frequency waves are sensitive to the initial plasma where they propagate without reflection. The accuracy of the inverted profiles is thereby improved, as shown by simulation studies performed for profiles with an exponential-like decay and with an edge density plateau (typical of H-mode regimes during ELMs). It is found that the contribution from the initial plasma decreases with density (or frequency); for densities of the order of 10× the first probed density n_{e1} it is reduced to values less than 10% in the case of a profile with a flat edge and 2% for a peaked one. For $n_e > 10n_{e1}$ the profiles can be absolutely calibrated from reflectometry data alone with an accuracy of ±2 mm independent of the initialization model. The numerical study also shows that profile deviations resulting from insufficient phase derivative data, e.g., due to discrete probing, can be more significant than those originated by the initialization process. © *1995 American Institute of Physics.*

I. INTRODUCTION

Reflectometry is a technique specially suited to measure the density profile of thermonuclear plasmas due to its great accuracy and modest access requirements. The profiles are obtained from the integration of the phase derivative versus frequency curve $d\phi/df$ of waves reflected at densities ranging from zero to the highest probed density. The initial part of $d\phi/df$ is difficult to obtain: with O-mode it cannot be measured below $F \approx 16-18$ GHz because the wavelength becomes too long compared to the density gradient scale length; X-mode waves can be reflected from almost zero density but the interpretation of the phase data may be difficult if the edge magnetic field is not accurately known because the cutoff at low densities is dominated by the electron cyclotron frequency. Also, X-mode reflection may be prevented by any unexpected changes of the magnetic field. Due to the lack of initial phase information a model of the edge plasma is needed to complement the reflectometry data. Initialization is a problem common to all reflectometry techniques.

In the following we show that the effect of the initial plasma has a major contribution to the edge of the inverted profiles, changing both its radial position and density gradient. The contribution decreases abruptly to the plasma interior; for a first probed density $n_{e1}=0.1\times10^{19}$ m^{-3} it drops, respectively, below 10% in the case of profiles with a flat edge and 2% for a peaked edge. In this way, errors in the position of the first probed layer x_1 (typically ±1 cm) cause only small deviations Δr at higher densities (for $n_{e2} \geqslant 10n_{e1}$, $\Delta r \leqslant \pm 2$ mm). Above n_{e2} the inverted profiles are dominated by the contribution from the measured $d\phi/df$ curve and can thus be absolutely calibrated from reflectometry data alone, with an accuracy of ±2 mm.

The above study showed the importance of the initialization process to reconstruct the edge profiles. In standard initialization methods the position of the first reflecting layer x_1 is estimated from other edge diagnostics, e.g., Li-beam and Langmuir probes, and either a linear or an exponential-like density profile is assumed below x_1.[1,2] Due to the uncertainty about x_1 and about the shape of the initial plasma, the resulting $d\phi/df$ (below x_1) does not fit to the experimental $d\phi/df$ data leading to significant deviations of the inverted profiles.

We developed an initialization method where the missing part of the phase derivative curve (rather than the initial profile) is estimated by taking into account the first measured $d\phi/df$ data points. An exponentially decaying $d\phi/df$ curve is obtained below the first probing frequency F_1 by imposing the continuity between the modeled and the measured $d\phi/df$ characteristics and its derivatives at F_1. As the phase information of the lowest probing frequencies has a significant contribution from the initial plasma, where the waves propagate without reflection, a good estimate of the nonmeasured profile is obtained and the accuracy of the reconstructed profiles is thereby increased.

Simulation studies performed both with exponential-like profiles and profiles with an edge density plateau (typical of H-mode regimes during ELMs) revealed that the profiles can be reconstructed with great accuracy. In the case of a density plateau it is shown that the flat region can be recovered even when only some part of the plateau is probed. If the plateau is not probed in reflection but it significantly affects the propagation of the lowest frequency waves, the shape of the profile can still be recovered. Finally, the simulation study was applied to discrete phase derivative data resulting from probing the plasma at several fixed frequencies. It was found that the profile deviations can be more significant than those originated by the initialization process.

The remainder of the paper is organized as follows. In Sec. II the contribution of the initial profile is analyzed. In Sec. III the initialization method is explained. The results of simulation studies performed with different density profile shapes are presented and the study is applied to the profile inversion from discrete reflectometry data. The implementation of the initialization method to the routine evaluation of density profiles from reflectometry measurements is discussed in Sec. IV.

II. CONTRIBUTION FROM THE NONREFLECTING PLASMA PROFILE

The phase derivative measured with reflectometry (it is obtained either from the fringe frequency or from the time delay of the reflected signals) depends on the plasma refractive index along the propagation path until reflection occur at the cutoff layer. For the O-mode, $d\phi/df$ varies with the plasma density profile $n_e(r)$ according to

$$\frac{d\phi}{df}=\frac{d}{df}\left[\frac{4\pi F}{c}\int_{r_0}^{r_c}\sqrt{1-\frac{f_{pe}^2}{F^2}}\,dr\right] \tag{1}$$

and the cutoff occurs at $F=f_{pe}(r_c)=(1/2\pi)\times(n_e(r_c)e^2/\epsilon_0 m_e)^{1/2}$. The profile is inverted from the phase derivative using the well-known Abel inversion integral equation,

$$r_c(F)=\frac{c}{2\pi^2}\int_0^F\frac{d\phi}{df}\frac{1}{\sqrt{F^2-f^2}}\,df. \tag{2}$$

Let us assume now that the initial phase derivative $(d\phi/df)_I$ below some frequency F_1 cannot be obtained. Equation (2) can then be written as

$$r_c(F)=\begin{cases}\dfrac{c}{2\pi^2}\displaystyle\int_0^F\frac{d\phi}{df}\frac{1}{\sqrt{F^2-f^2}}\,df=I, & \text{if } F<F_1\\[2ex] \dfrac{c}{2\pi^2}\displaystyle\int_0^{F_1}\frac{d\phi}{df}\frac{1}{\sqrt{F^2-f^2}}\,df+\frac{c}{2\pi^2}\int_{F_1}^F\frac{d\phi}{df}\frac{1}{\sqrt{F^2-f^2}}\,df=I_1+I_2, & \text{if } F>F_1.\end{cases} \tag{3}$$

The integral I in Eq. (3) represents the profile $I=r_c(n_e)$, below the first probed layer, $n_{e1}(F_1)$. For $F>F_1$, I can be split into the contribution I_1 from the plasma below n_{e1} (to be estimated by the initialization method) and I_2, from the plasma above n_{e1}, to be obtained from reflectometry.

In order to study I_1 and I_2 we consider density profiles of the form

$$n_e(r)=n_{em}\left(1-\left(\frac{a-r}{a}\right)^m\right)^n, \tag{4}$$

where a is the plasma radius, n_{em} is the plasma maximum density, and m and n are the shaping parameters.

Figure 1 shows the evolution of I, I_1, and I_2 with frequency (or density) for a profile with $a=0.1m$, $n_{em}=1.5\times10^{19}$ m^{-3}, and $m=n=2$. At the lowest densities both the radial position and density gradient of the inverted profile ($I=I_1+I_2$) are dominated by I_1. The relative contribution of the initial plasma I_1/I_2 decreases rapidly with density (see box in Fig. 1) showing that any uncertainty about the initial plasma result in profile deviations that decrease with density as fast as I_1/I_2 drops to zero. Figure 2 shows the evolution of I_1/I_2 for two profiles with peaked and flat edges and for $F_1=10$ GHz, $n_{e1}\approx0.1\times10^{19}$ m^{-3}. For densities above $n_{e2}\approx1.0\times10^{19}$ m^{-3} the contribution from the initial plasma is rather small: below 2% for the profile with a peaked edge and 10% in the case of the flat one.

The refractive index of waves with $F>3F_1$ ($n_e>10n_{e1}$) is close to unity and I_1 can be approximated by

$$I_1\approx\frac{2d_1}{\pi}\frac{F_1}{F}, \tag{5}$$

where d_1 is the distance between the first plasma layer ($n_{e0}\approx0,x_0=0$) and the first probed density (n_{e1},x_1). Lines (c) and (d) in Fig. 2 represent, for each profile, the value of I_1 [given by Eq. (5)] normalized to I_2, evaluated at $F=30$ GHz, showing that the above approximation is very close to the exact value of I_1/I_2. Above $10n_{e1}$, therefore, the integral I_1 depends only on the distance d_1. This distance can be determined from the location of the antenna x_a and the position x_1. From other edge diagnostics, x_1 can be obtained with an

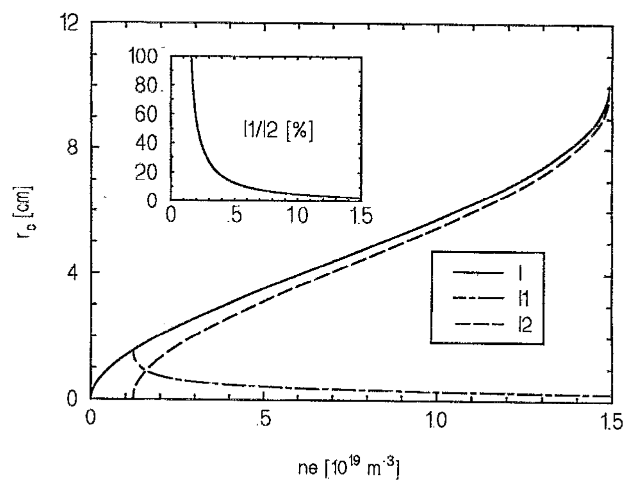

FIG. 1. Contributions to the plasma profile $n_e(r_c)$ or $I=r_c(n_e)$ from the initial (nonreflecting) plasma, I_1, and from the probed plasma, I_2. The decrease of the relative contribution I_1/I_2 to the inverted profile is shown in the enclosed box.

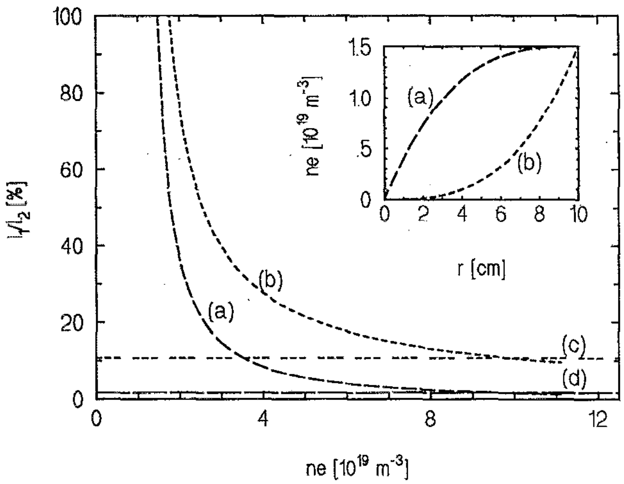

FIG. 2. Relative contribution I_1/I_2 for a first probing frequency $F_1=10$ GHz and for two different profiles: (a) with a peaked edge and (b) with a flat edge. Lines (c) and (d) represent the contribution of I_1 if a vacuum path from the antenna until the first probed layer is assumed.

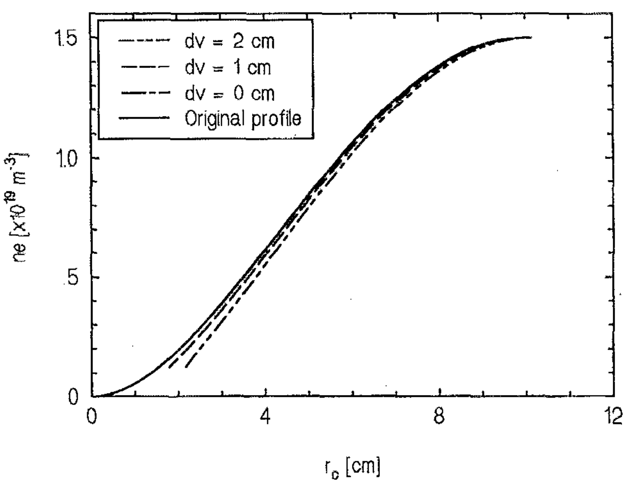

FIG. 3. Simulated (solid line) and reconstructed (dashed lines) profiles above $F_1=10$ GHz for different vacuum distances between the antenna and the first plasma layer showing small deviations (<1 cm) from the original profile. For $d_v=0$ the inverted and original profiles coincide revealing the great accuracy of the initialization method.

accuracy of ± 1 cm. Above $10n_{e1}$ a shift of ± 1 cm results in a much smaller shift of the profile (± 2 mm) due to the decrease of I_1/I_2. This shows that above $10n_{e1}$ the profiles do not depend on the initialization model and can be absolutely calibrated from reflectometry alone. The radial distance between the plasma layers n_{e1} and $10n_{e1}$ can be quite small in peaked edge profiles.

III. INITIALIZATION MODEL

The above study revealed that the accuracy of the edge profiles depends strongly on a good estimate of the plasma profile that cannot be probed in reflection. We develop an initialization model that uses as input data the phase derivative curve $(d\phi/df)_R$ above F_1 (in the experiments this would be measured by reflectometry). The curve is extrapolated down to zero exponentially as

$$\left(\frac{d\phi}{df}\right)_I(f)=A\left(\frac{f}{F_1}\right)^{2/s}. \tag{6}$$

The parameters s and A are determined by imposing the following boundary conditions:

$$\left(\frac{d\phi}{df}\right)_I(F_1)=\left(\frac{d\phi}{df}\right)_R(F_1), \tag{7a}$$

$$\left(\frac{d^2\phi}{df^2}\right)_I(f)\bigg|_{f\to F_1^-}=\frac{2}{sF_1}A=\left(\frac{d^2\phi}{df^2}\right)_R(f)\bigg|_{f\to F_1^+}, \tag{7b}$$

which correspond to equal the values of the modeled $(d\phi/df)_I$ and original (or measured) $(d\phi/df)_R$ phase derivatives and its slopes at $F=F_1$. From Eq. (7a) we obtain $A=(d\phi/df)_R(F_1)$. The value of s that satisfies Eq. (7b) is determined by iteratively matching the slope of the two curves until some *a priori* fixed difference (typically 10^{-6} rad GHz^{-2}) is obtained.

In order to simulate as close as possible the experiments, all the phase contributions of the reflected signals were considered: (i) $(d\phi/df)_\mu$ from the microwave circuit, (ii) $(d\phi/df)_v$ due to the vacuum path between the antenna mouth and the first plasma layer, and (iii) $(d\phi/df)_p$ originated at the plasma. The contribution from the microwave circuit is determined from *a priori* calibration with a metallic mirror inside the machine and can be subtracted. The vacuum term is evaluated from the distance between the antenna position x_a and the position of the first plasma layer, x_0. As the exact location x_0 is not known, a term was added to the model, to account for different vacuum distances (which will also change the shape of the initial profile), as follows:

$$\left(\frac{d\phi}{df}\right)_I(f)=\frac{4\pi}{c}d_v+\left[\left(\frac{d\phi}{df}\right)_R(F_1)-\frac{4\pi}{c}dv\right]\left(\frac{f}{F_1}\right),$$

$$0<f<F_1, \tag{8a}$$

$$\left(\frac{d^2\phi}{df^2}\right)_I(f)\bigg|_{f\to F_1^-}=\frac{2}{sF_1}\left[\left(\frac{d\phi}{df}\right)_R(F_1)-\frac{4\pi}{c}d_v\right]$$

$$=\left(\frac{d^2\phi}{df^2}\right)_R(f)\bigg|_{f\to F_1^+}. \tag{8b}$$

A. Profiles with an exponential-like decay

Let us consider a density profile with $m=n=2$, $a=0.1$ m, and $n_{em}=1.5\times10^{19}$ m^{-3}. We assume the first probing frequency to be $F_1=10$ GHz and three vacuum distances $d_{v1}=0$ cm, $d_{v2}=1$ cm, and $d_{v3}=2$ cm ($d_{v1}=0$ cm corresponds to the exact value of x_0). The $(d\phi/df)_R$ curve for $F>F_1$ is given to the initialization algorithm and three best-fitting s parameters (corresponding to different initial plasma shapes) are obtained.

Figure 3 shows the original (solid line) and the reconstructed profiles; the corresponding phase derivatives are depicted in Fig. 4. For d_{v1} the inverted profile coincides with

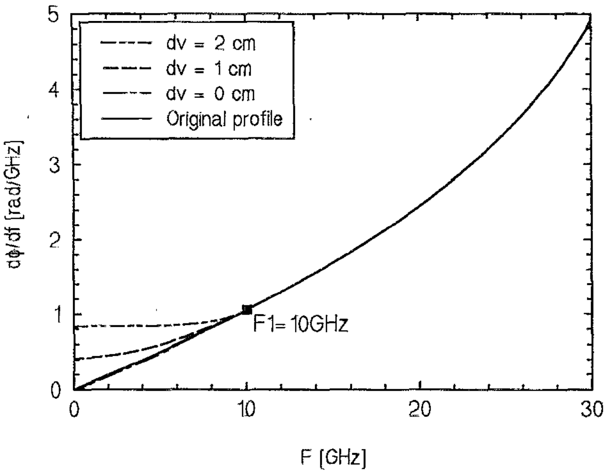

FIG. 4. Phase derivative vs frequency curves corresponding to the original (solid line) and to the reconstructed (dashed lines) profiles shown in Fig. 3.

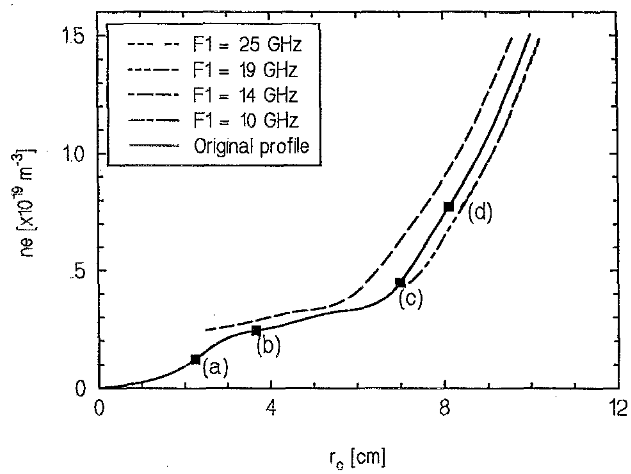

FIG. 5. A density profile with an edge plateau (solid line) is probed with four different first frequencies (10, 14, 19, and 25 GHz) corresponding to the density cutoff layers marked as (a)–(d). The inverted profiles (dashed lines) show that the original profile can be reconstructed with only small deviations with the novel initialization method. For $F_1=10$ GHz the inverted and original profiles coincide. The larger deviations are obtained for $F_1=14$ GHz when the extrapolated $d\phi/df$ curve is always below the original one (see Fig. 6). The profile for $F_1=25$ GHz cannot be seen because it is coincident with the profile for $F_1=19$ GHz, above $n_e \approx 7.3\times10^{19}$ m^{-3}.

the original one showing the great accuracy of the initialization method. For $d_v \neq 0$ the profiles exhibit deviations that decrease to the plasma interior, as a consequence of the decrease of the relative contribution I_1/I_2 (see Sec. II). It should be noted that an error of 2 cm in the location of the first plasma layer ($d_{v2}=2$ cm) produces a radial shift of the profile that is always below 1 cm. Errors in d_v are limited between $d_v=0$ (correct location) and $d_v=(c/4\pi)\times(d\phi/df)_R(F_1)$, which corresponds to assume a zero slope of the phase derivative between $F=0$ and $F=F_1$. A negative slope is not acceptable because it would result in a profile with an initial backward evolution. In the example, the uncertainty in d_v is below 2.5 cm. In the experiments d_1 can be obtained with an accuracy of ±1 cm from the locations of the antenna and the first probed layer (as given by other edge diagnostics).

B. Profiles with an edge density plateau

In tokamaks based on the divertor configuration, the plasma density profile can form a flat region at the edge, e.g., in H-mode regimes during ELMs. We simulate a density profile with such a shape (solid line in Fig. 5) and assume that the profile is probed above four different initial frequencies, indicated by points (a)–(d) in Fig. 5. The corresponding phase derivative (for $F>F_1$) is complemented in each case with a $d\phi/df$ curve obtained from the initialization algorithm, as represented in Fig. 6. The reconstructed profiles (dashed lines in Fig. 5) show that the smaller deviation occurs for $F_1=10$ GHz, when a major part of the plateau is probed and the larger one corresponds to $F_1=14$ GHz when the plateau is only partially probed. For $F_1=19$ and 25 GHz, the plateau is not probed and the errors are small. The observed behavior is due to the degree of compensation of the contributions to the inversion integral, represented by the areas limited by the original $d\phi/df$ curve and the curve given by the initialization model.

It should be noted that for $F_1=19$ GHz, although the density plateau is not probed in reflection, the phase derivative curve has a significant deformation resulting from the propagation in the flat region; this causes the slightly negative slope in the initial part of the phase derivative curve. Such a shape has been observed in different reflectometry experiments.[2,3] To account for these new situations the algorithm was modified: for initial positive slopes boundary conditions (7a) and (7b) are used; for negative slopes boundary condition (7b) is dropped and a linear extrapolation of $d\phi/df$ down to zero is assumed. This straight line will be both above and below the original $d\phi/df$ curve and therefore the errors in the integral contributions to Eq. (2) can be compensated in the profile evaluation.

The radial shift in plasma position and the density gra-

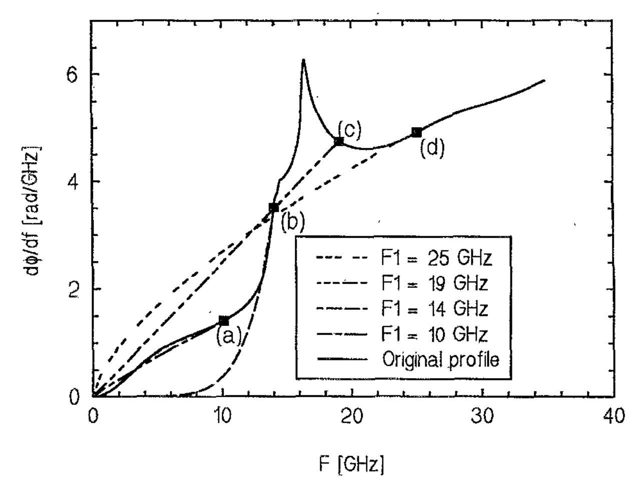

FIG. 6. The solid line represents the phase derivative corresponding to the original profile of Fig. 5. The dashed lines display the extrapolated $d\phi/df$ curves for the different probed ranges (below F_1). The extrapolated curve is (a) close to the original one for $F_1=10$ GHz, (b) both above and below for $F_1=19$ and 25 GHz, and (c) always below for $F_1=14$ GHz.

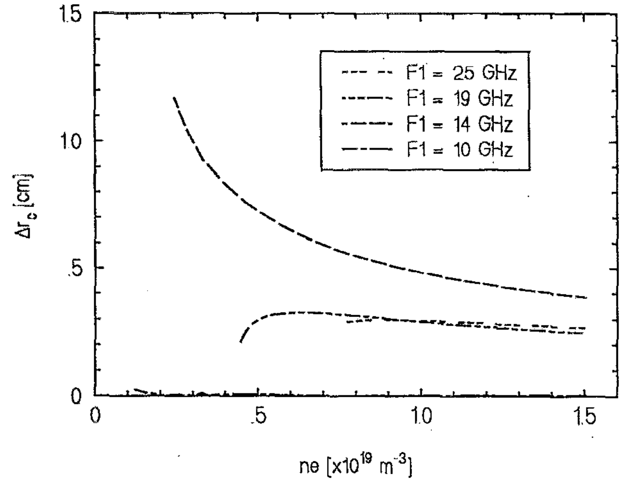

FIG. 7. Radial shifts of the plasma position for the inverted profiles of Fig. 5. Typical shifts are between 3 and 5 mm.

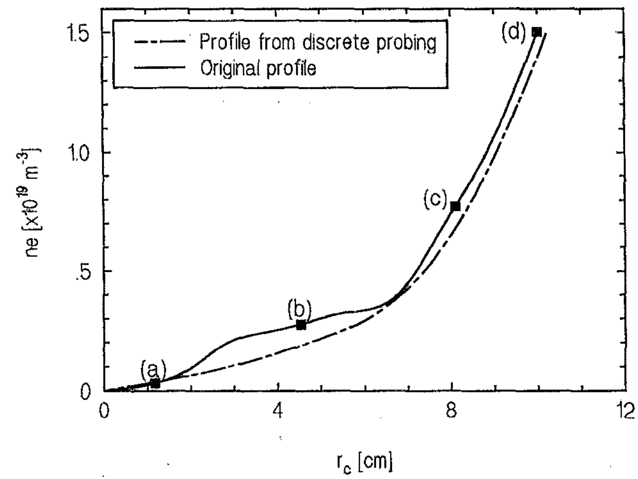

FIG. 9. Density profile (solid line) with a flat region "probed" at four density layers, marked as (a)–(d). The profile reconstructed from a spline interpolation between the four $d\phi/df(F_i)$ points is represented by the dashed line. The density plateau cannot be recovered due to the insufficient discrete probing.

dient relative errors corresponding to the reconstruction of the profile from data obtained above the four different frequencies are presented in Figs. 7 and 8, respectively. As predicted by the evolution of I_1/I_2 the errors are significant only at densities close to the first probed density and decrease rapidly with increasing density. In the example, the errors in the radial position and density gradient are, respectively, below 1 cm and 10% for densities of the order of $2n_{e1}$.

C. Discrete plasma probing

The above simulation study can also be used to analyze the errors due to the lack of phase derivative data in any plasma region. In the case of discrete probing at several fixed frequencies the $d\phi/df$ data is missing between the probed density layers and has to be interpolated. Let us consider that the plasma is probed with four discrete frequencies: 5, 15, 25, and 35 GHz. The phase derivative data between the "measured" points is obtained by a spline interpolation. As

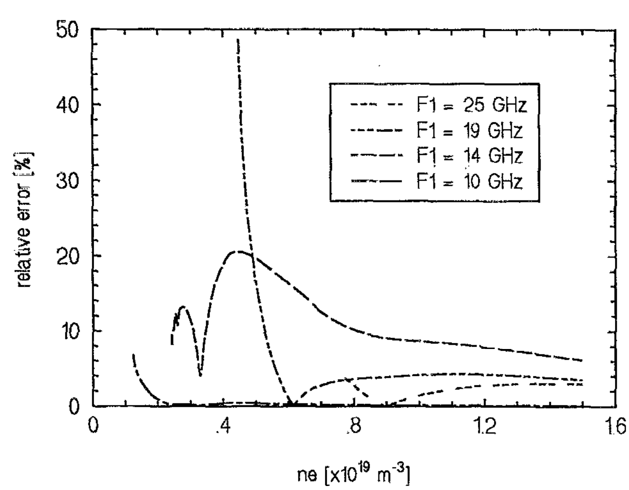

FIG. 8. Relative errors of the profile gradient for the inverted profiles of Fig. 5. Significant errors are obtained at the edge for F_1=19 GHz but only in a very narrow density range [n_e: (4.5–5.5)$\times 10^{19}$ m^{-3}].

shown in Fig. 9 (dashed line) the density plateau cannot be recovered due to the insufficient probing. In an experimental situation the deviations would be much larger due to the errors associated to each data point caused by the plasma fluctuations.

IV. DISCUSSION

As reported in the paper the contribution from the initial plasma dominates the edge of the profiles reconstructed from reflectometry. The X-mode can be used to probe the most outer plasma edge but in some cases the local B_{tot} cannot be known with sufficient accuracy or, due to changes of the plasma regimes, the X-mode is not reflected from the edge. Initialization of the profiles is needed whenever it is not possible to measure $d\phi/df$ (or τ) down to zero plasma density. An initialization model was developed that enables one to improve the accuracy of the inverted profiles. The application of the method to experimental results requires clear phase derivative data at the initial frequency range. Fluctuations, as they can distort or mask the measured phase characteristic, will be the main limitation to the accuracy of the inverted profiles. Data from other diagnostics can be used, in some situations, to cross check the initial profile resulting from the model. Also, the plotting for each profile of the contributions from the initial profile I_1 and from reflectometry I_2 indicates the density range where the profile is affected by the initial plasma and where its influence is negligible. Above densities of the order of 10$\times$ the first probed density reflectometry can be used to calibrate other density diagnostics because the contribution from the initial model is negligible.

The novel initialization method is being used in the routine evaluation of density profiles in ASDEX Upgrade.[4] A systematic comparison with density data from Li-beam and Langmuir probes will test the accuracy of the reconstructed edge profiles.

ACKNOWLEDGMENTS

This work was supported by Instituto Superior Técnico, Junta Nacional de Investigação Científica e Tecnológica and European Atomic Energy Community. The authors also acknowledge the fruitful discussions with Dr. F. Serra and Dr. A. Kallenbach.

[1] M. E. Manso, F. Nunes, A. Silva, and P. Varela, Proceedings of the IAEA Technical Committee Meeting on Reflectometry, Abingdon, 1992.

[2] E. de la Luna, G. Hanson, J. Wilsen, J. Sanchez, V. Zhuralev, B. Kaita, J. Harris, and the PBX-M team, Proceedings of the IAEA Technical Meeting on Microwave Reflectometry for Fusion Plasma Diagnostics, Princeton, New Jersey, 1994, Vol. II.

[3] G. R. Hanson, J. B. Wilsen, T. S. Bigelow, I. Collazo, D. A. Rasmussen, D. B. Batchelor, D. J. Hoffman, E. F. Jaeger, R. Majeski, M. Murakami, H. J. Park, J. H. Rogers, P. M. Ryan, D. C. Stallings, and J. R. Wilson, Proceedings of the IAEA Technical Meeting on Microwave Reflectometry for Fusion Plasma Diagnostics, Princeton, New Jersey, 1994, Vol. II.

[4] A. Silva, L. Cupido, M. Manso, F. Serra, F. X. Soeldner, P. Varela, C. Correia, C. Fernandes, C. Loureiro, A. Moreira, J. Neves, R. Pereira, J. Santos, and the ASDEX Upgrade team, *Proceedings of the 17th Symposium on Fusion Technology, Rome, 1992*, edited by C. Ferro, M. Gasparotto, and H. Knoepfel (North-Holland, Amsterdam, 1992), Vol. 1, pp. 747–750.