Support Vector Machine

1. $\{(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)\}$

x_i are input feature vectors
y_i are labels ($y_i \in \{-1, +1\}$)

2. Hyperplane:

$$w \cdot x + b = 0$$

w = weight vector
b = b is bias term

3. Classification point x:
If $w \cdot x + b > 0$, classify x as $+1$
If $w \cdot x + b \leq 0$, classify x as -1

4. Max. Margin

$$M = \frac{2}{\|w\|}$$

5. Optimization Problem

$$\text{Minimize } \frac{1}{2} \|w\|^2$$

Lagrangian & Dual formulation:

$$\text{Max.} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{i=1}^{n} \alpha_i \alpha_j y_i y_j (x_i \cdot x_j)$$

$$\text{Subject to: } \sum_{i=1}^{n} \alpha_i y_i = 0, \quad 0 \leq \alpha_i \leq C \quad \forall_i$$

Decision Function

$$f(x) = \text{sign}\left(\sum_{i=1}^{n} \alpha_i y_i K(x_i, x) + b \right)$$