Лабораторная работа №5

Постороение типовых моделей АРПСС (ARIMA)

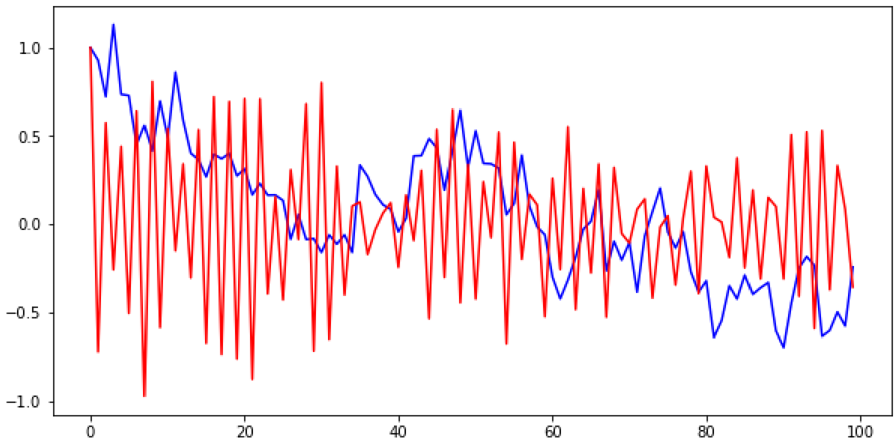
РИ-681223 Черепанов Александр

Вариант №19

In [44]:
```
import numpy as np
import numpy.random as rand
import matplotlib.pyplot as plt

import h5py

from statsmodels.tsa import api as tsa
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.tsa.stattools import acf
from statsmodels.tsa.arima_model import ARIMA

%matplotlib inline

import warnings
warnings.filterwarnings('ignore')
```

Создадим собственные АРПСС ряды первого и второго порядков для изучения их автокорреляционных функций. Создадим два АР(1) процесса первого порядка:

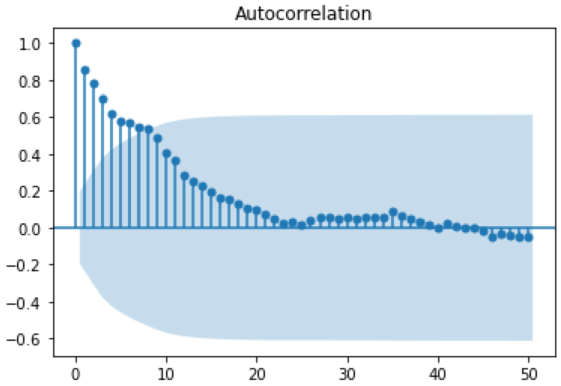
In [45]:
```
z1 = np.zeros(100)
z2 = np.zeros(100)

z1[0] = 1
z2[0] = 1

for i in range(1,100):
  z1[i] = 0.8 * z1[i-1] + 0.2 * np.random.randn()
  z2[i] = -0.8 * z2[i-1] + 0.2 * np.random.randn()

plt.figure(figsize = (10, 5))
plt.plot(z1, 'b')
plt.plot(z2, 'r')
plt.show()
```

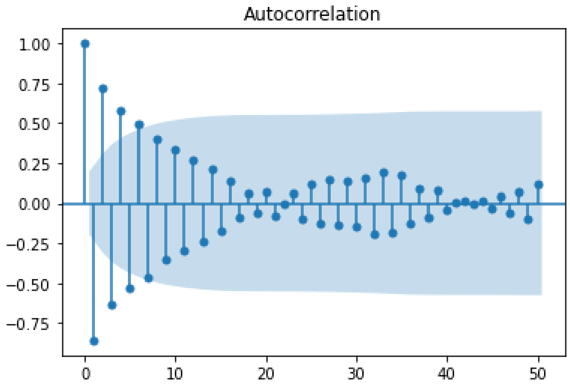


Теперь построим для этих рядов функции автокорреляции:

In [46]:
```
plt.figure(figsize = (10, 5))
plot_acf(z1, lags=50)
plot_acf(z2, lags=50)
plt.show()
```

```
<Figure size 720x360 with 0 Axes>
```

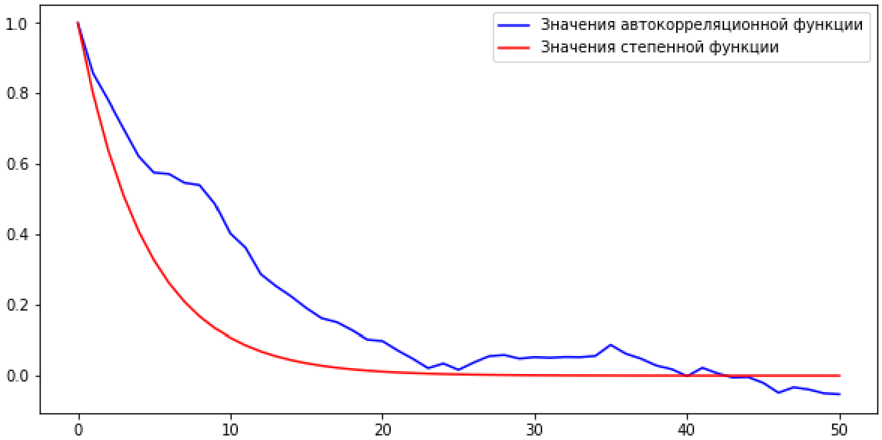




Для АР с положительным коэфициентом можно заметить, что зависимость с каждым лагом меняется незначительно. Во втором случае, по автокорреляционной функции можно отметить резкие колебания в ВР. Так же можно заметить, что значения автокорреляционных функций при лаге 1 близки к значениям весовых параметров этих процессов.

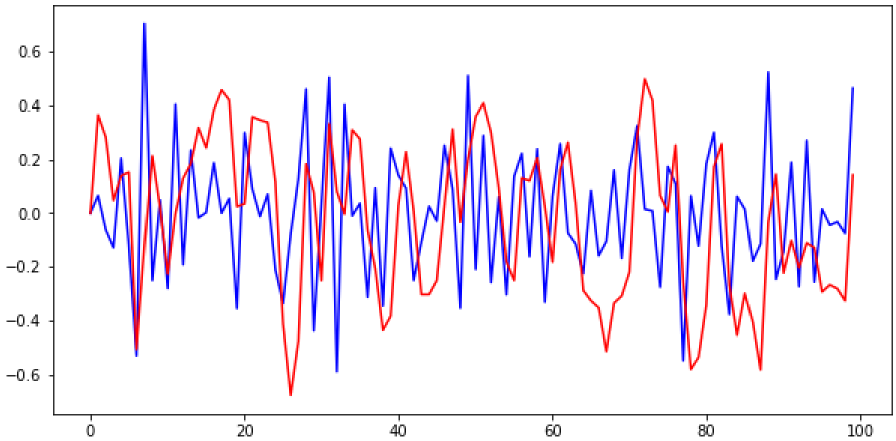
Удостоверимся, что для модели АР(1) коэффициенты автокорреляции изменяются по степенному закону:

In [47]:
```
plt.figure(figsize = (10, 5))
plt.plot(range(51), acf(z1, nlags=50, fft=True), 'b', label='Значения автокорреляционной функции')
plt.plot(range(51), np.array([0.8**l for l in range(51)]), 'r', label='Значения степенной функции')
plt.legend()
plt.show();
```



Аналогичным образом построем два СС(1) процесса среднего-скользящего первого порядка:

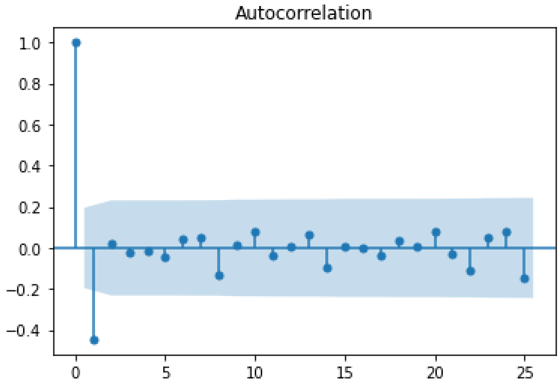
In [48]:
```
z3 = np.zeros(100)
z4 = np.zeros(100)
ar = 0.2 * np.random.randn(100)

for i in range(1, 100):
  z3[i] = ar[i] -0.8 * ar[i -1]
  z4[i] = ar[i] + 0.8 * ar[i -1]

plt.figure(figsize= (10, 5))
plt.plot(z3, 'b')
plt.plot(z4, 'r')
plt.show()
```

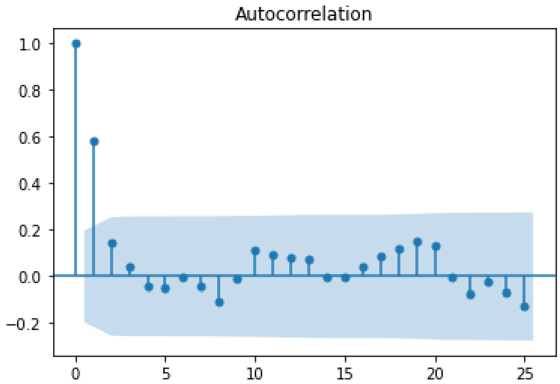


Посторим для этих рядов функции автокорреляции:

In [49]:
```
plt.figure(figsize = (10, 5))
plot_acf(z3, lags=25)
plot_acf(z4, lags=25)
plt.show()
```

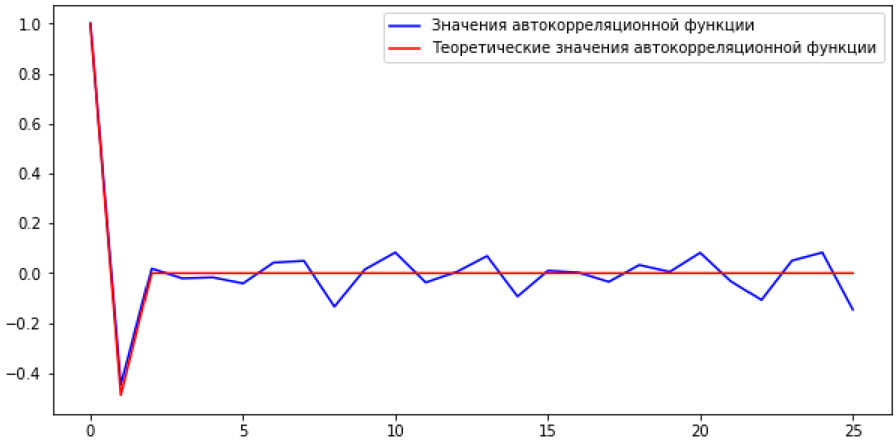
```
<Figure size 720x360 with 0 Axes>
```





Убежадемся, что значение коэфициента автокорреляционной функции при лаге 1 равна следующему выражению:

In [50]:
```
def p1(k):
  if k == 0:
    return 1
  if k == 1:
    return (-0.8) / (1 + (-0.8)**2)
  return 0

plt.figure(figsize = (10, 5))
plt.plot(range(26), acf(z3, nlags=25, fft=True), 'b', label='Значения автокорреляционной функции')
plt.plot(range(26), np.array([p1(l) for l in range(26)]), 'r', label='Теоретические значения автокорреляционной функци
plt.legend()
plt.show();
```



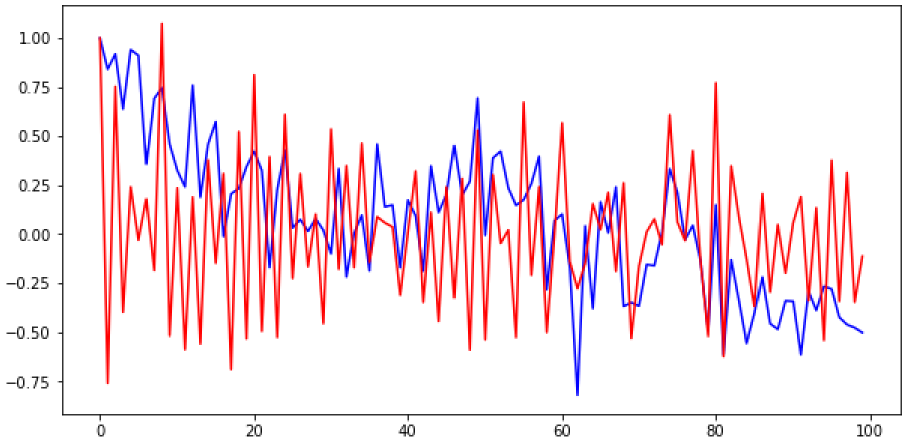
Оценим весовой параметр процесса на основе функции автокорреляции:

In [51]:
```
D = (1/acf(z3, nlags=2, fft=True)[1])**2 - 4
theta1 = (-(1/acf(z3, nlags=2, fft=True)[1]) + np.sqrt(D)) / 2
theta2 = (-(1/acf(z3, nlags=2, fft=True)[1]) - np.sqrt(D)) / 2
print(f"theta1 = {theta1}; theta2 = {theta2} => весовой  параметр = {theta2}")
```

```
theta1 = 1.6290729269934823; theta2 = 0.6138460614194473 => весовой  параметр = 0.6138460614194473
```

Создадим временной ряд процесса АРСС(1, 1):

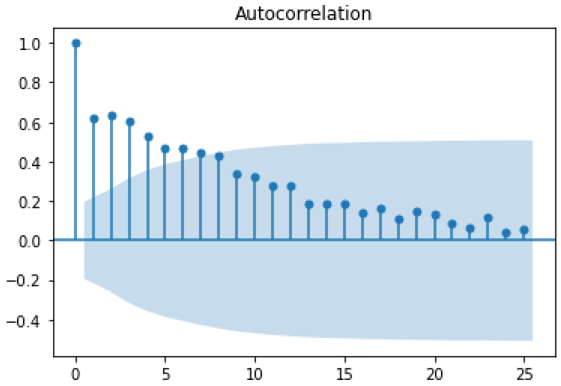
In [52]:
```
z5 = np.zeros(100)
z6 = np.zeros(100)

z5[0] = 1
z6[0] = 1
ar = 0.2 * np.random.randn(100)

for i in range(1,100):
  z5[i] = 0.8 * z1[i-1] + ar[i] -0.3 * ar[i-1]
  z6[i] = -0.8 * z2[i-1] + ar[i] -0.3 * ar[i -1]

plt.figure(figsize= (10, 5))
plt.plot(z5, 'b')
plt.plot(z6, 'r')
plt.show()
```

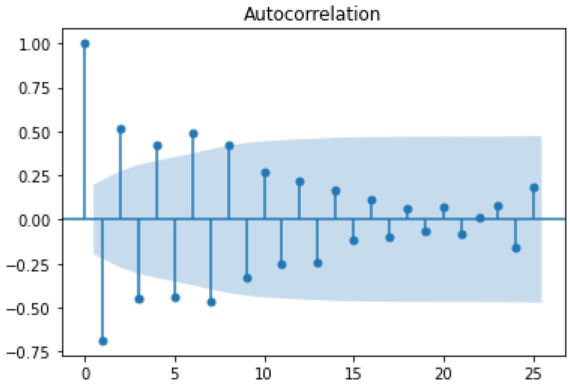


Построим их автокорреляционные функции:

In [53]:
```
plt.figure(figsize = (10, 5))
plot_acf(z5, lags=25)
plot_acf(z6, lags=25)
plt.show()
```

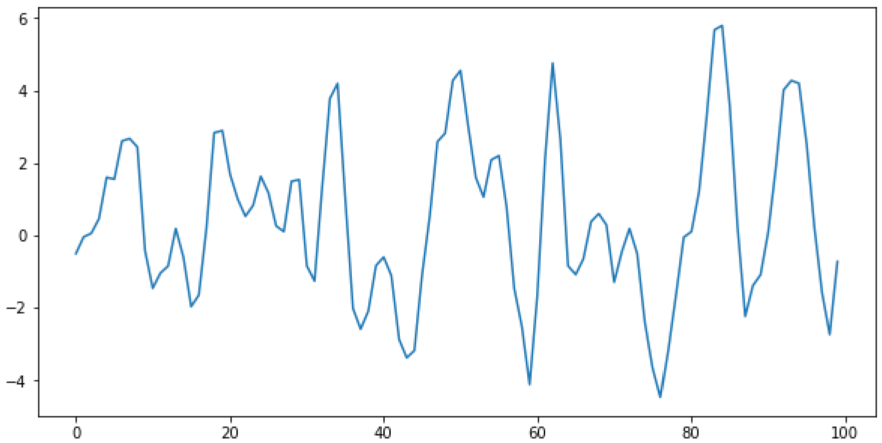
```
<Figure size 720x360 with 0 Axes>
```





Используем следующую функцию для создания АРСС (2, 2):

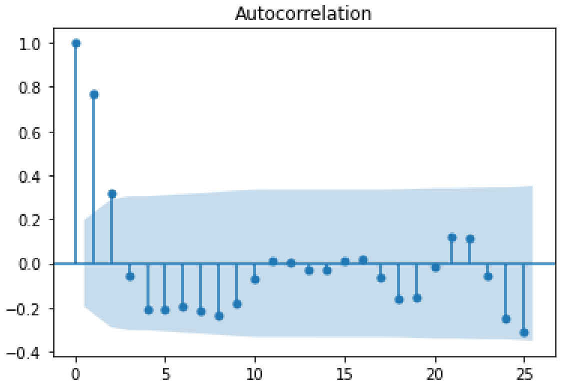
In [54]:
```
from statsmodels.tsa.arima_process import arma_generate_sample

ar = np.array([0.75, -0.25]) # задаем коэффициенты АР
ma = np.array([0.65, 0.35])  # задаем коэффициенты СС
y = arma_generate_sample(np.r_[1, -ar], np.r_[1, ma], 100) # создаем ВР для АРСС (2, 2) = АРПСС (2, 0, 2) из 100 отсче

plt.figure(figsize= (10, 5))
plt.plot(y)
plt.show()
```



Построим ее автокорреляционную функцию:

In [55]:
```
plt.figure(figsize = (10, 5))
plot_acf(y, lags=25)
plt.show()
```

```
<Figure size 720x360 with 0 Axes>
```

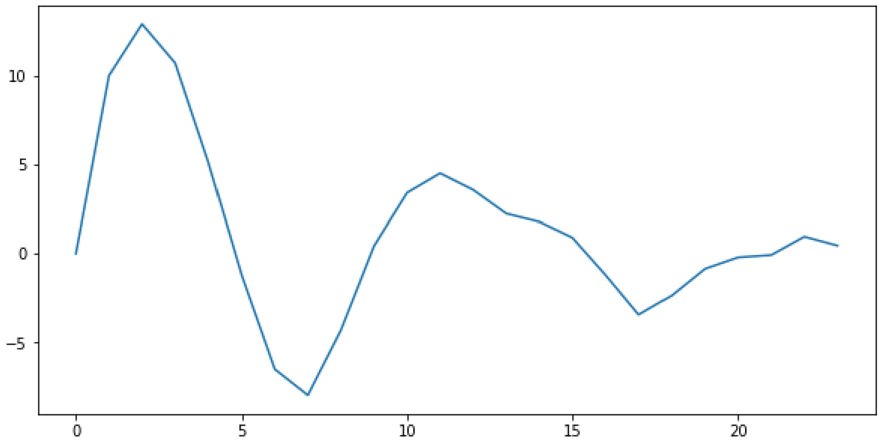


Теперь проведем анализ неизвестного ряда:

In [56]:
```
TEST = [0.00, 9.99, 12.89, 10.70, 5.12, -1.21, -6.50, -7.96, -4.30, 0.42, 3.41, 4.50,  3.57,  2.24,  1.78,  0.89, -1.
```

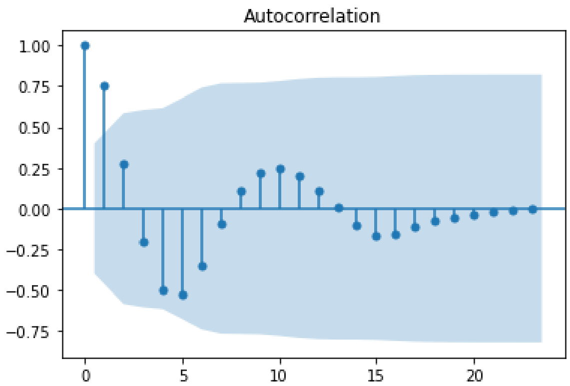
Построим график ВР и его автокорреляционную функцию:

In [57]:
```
plt.figure(figsize= (10, 5))
plt.plot(TEST)
plt.show()
```



In [58]:
```
plt.figure(figsize = (10, 5))
plot_acf(TEST, lags=len(TEST)-1)
plt.show()
```

```
<Figure size 720x360 with 0 Axes>
```



По нимможно судить, что ВР, в достаточной степени, стационарен, а, так как, эта функция является знакопеременной, то один из членов АР модели имеет отрицательный вес. Создадим три пробные модели АРПСС для проверки ряда на:

1. АР(1) = АРПСС(1, 0, 0)
2. АР(2)
3. АР(3), без тренда (trend= ‘nc’)

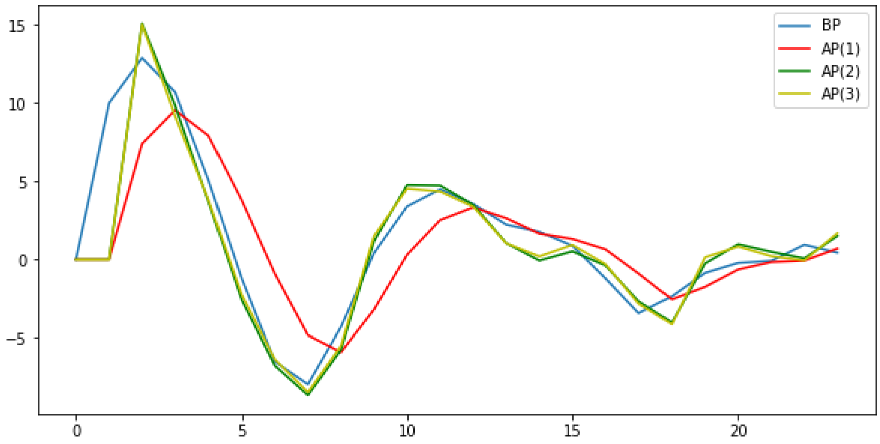
In [59]:
```
arima1 = ARIMA(TEST, order = (1, 0, 0))# создаем модель
model_fit1 = arima1.fit(disp = False, trend='nc')# подгоняем под ВР
print(model_fit1.summary())# выводим таблицу результатов

arima2 = ARIMA(TEST, order = (2, 0, 0))
model_fit2 = arima2.fit(disp = False, trend='nc')
print(model_fit2.summary())
```

```
arima3 = ARIMA(TEST, order = (3, 0, 0))
model_fit3 = arima3.fit(disp = False, trend='nc')
print(model_fit3.summary())
```

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                   24
Model:                     ARMA(1, 0)   Log Likelihood                 -62.425
Method:                       css-mle   S.D. of innovations              3.207
Date:                Mon, 18 Sep 2023   AIC                            128.850
Time:                        10:57:29   BIC                            131.206
Sample:                             0   HQIC                           129.475

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1.y        0.7426      0.123      6.053      0.000       0.502       0.983
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.3465           +0.0000j            1.3465            0.0000
-----------------------------------------------------------------------------
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                   24
Model:                     ARMA(2, 0)   Log Likelihood                 -41.543
Method:                       css-mle   S.D. of innovations              1.201
Date:                Mon, 18 Sep 2023   AIC                             89.086
Time:                        10:57:29   BIC                             92.620
Sample:                             0   HQIC                            90.024

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1.y        1.5108      0.056     27.117      0.000       1.402       1.620
ar.L2.y       -0.9641      0.035    -27.509      0.000      -1.033      -0.895
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            0.7836           -0.6506j            1.0185           -0.1103
AR.2            0.7836           +0.6506j            1.0185            0.1103
-----------------------------------------------------------------------------
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                   24
Model:                     ARMA(3, 0)   Log Likelihood                 -41.097
Method:                       css-mle   S.D. of innovations              1.172
Date:                Mon, 18 Sep 2023   AIC                             90.193
Time:                        10:57:29   BIC                             94.906
Sample:                             0   HQIC                            91.443

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1.y        1.7184      0.222      7.755      0.000       1.284       2.153
ar.L2.y       -1.2995      0.345     -3.770      0.000      -1.975      -0.624
ar.L3.y        0.2216      0.229      0.968      0.333      -0.227       0.670
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            0.7786           -0.6645j            1.0236           -0.1124
AR.2            0.7786           +0.6645j            1.0236            0.1124
AR.3            4.3072           -0.0000j            4.3072           -0.0000
-----------------------------------------------------------------------------
```

In [60]:

```
plt.figure(figsize= (10, 5))
plt.plot(TEST, label='BP')
plt.plot(model_fit1.fittedvalues, 'r', label='AP(1)')
plt.plot(model_fit2.fittedvalues, 'g', label='AP(2)')
plt.plot(model_fit3.fittedvalues, 'y', label='AP(3)')
plt.legend()
plt.show()
```



Расчитаем весовые коэффициенты для АР моделей только 1 и 2 порядка самостоятельно:

In [61]:
```
acf_coefs = acf(TEST, fft=True)
acf_coefs
```

Out[61]:
```
array([ 1.        ,  0.75500976,  0.27235864, -0.2066116 , -0.49581347,
       -0.52955567, -0.35044634, -0.0963739 ,  0.10437004,  0.21646702,
        0.24520358,  0.19968012,  0.11140263,  0.0036976 , -0.10470012,
       -0.16946472, -0.16160427, -0.10778955, -0.07271434, -0.05285789,
       -0.03431416, -0.01650911, -0.01101053,  0.00157628])
```

Весовой коэффициент АР(1)

In [62]:
```
theta11 = acf_coefs[1]
print(theta11)
```

```
0.7550097626743363
```

Весовые коэффициенты АР(2)

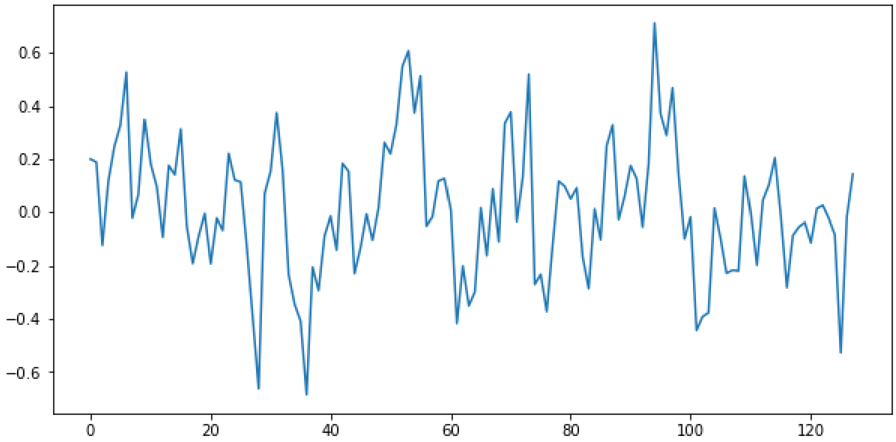
In [63]:
```
theta21 = (acf_coefs[1]*(1 - acf_coefs[2])) / (1 - acf_coefs[1]**2)
theta22 = (acf_coefs[2] - acf_coefs[1]**2) / (1 - acf_coefs[1]**2)
print(theta21, theta22)
```

```
1.2777374692491608 -0.6923456257350362
```

Несмотря на некоторые различия, полученные веса близки к тем, что были получены с помощью функций Python.

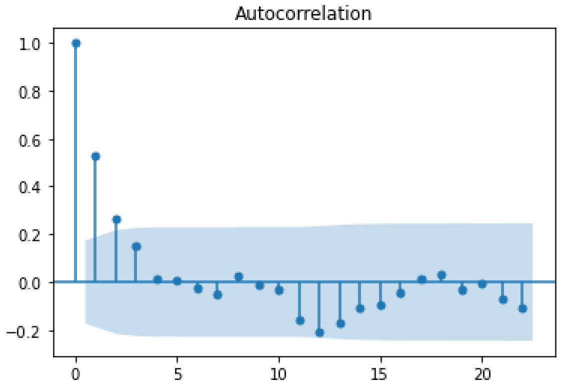
Загрузим временной ряд Z из файла. Построим график ВР и его автокорреляционную функцию:

In [64]:
```
file = h5py.File('19.mat', 'r')
data = file.get('z19')
Z = np.array(data)

plt.figure(figsize = (10, 5))
plt.plot(Z)
plt.show()
```



In [65]:
```
plt.figure(figsize = (10, 5))
plot_acf(Z)
plt.show();
```

```
<Figure size 720x360 with 0 Axes>
```



Оценим порядок АРССмодели с помощью класса ARIMA.

In [66]:
```
arimaz = ARIMA(Z, order = (1, 0, 0))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())

arimaz = ARIMA(Z, order = (2, 0, 0))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())

arimaz = ARIMA(Z, order = (3, 0, 0))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())

arimaz = ARIMA(Z, order = (0, 0, 1))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())

arimaz = ARIMA(Z, order = (0, 0, 2))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())

arimaz = ARIMA(Z, order = (0, 0, 3))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())
```

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  128
Model:                     ARMA(1, 0)   Log Likelihood                  15.832
Method:                       css-mle   S.D. of innovations              0.214
Date:                Mon, 18 Sep 2023   AIC                            -25.664
Time:                        10:57:36   BIC                            -17.108
Sample:                             0   HQIC                           -22.188

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0145      0.040      0.366      0.714      -0.063       0.092
ar.L1.y        0.5279      0.075      7.070      0.000       0.382       0.674
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.8943           +0.0000j            1.8943            0.0000
-----------------------------------------------------------------------------
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  128
Model:                     ARMA(2, 0)   Log Likelihood                  15.859
Method:                       css-mle   S.D. of innovations              0.214
Date:                Mon, 18 Sep 2023   AIC                            -23.717
Time:                        10:57:36   BIC                            -12.309
Sample:                             0   HQIC                           -19.082

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0144      0.039      0.370      0.712      -0.062       0.091
ar.L1.y        0.5387      0.088      6.109      0.000       0.366       0.711
ar.L2.y       -0.0202      0.088     -0.230      0.818      -0.193       0.152
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            2.0077           +0.0000j            2.0077            0.0000
AR.2           24.6352           +0.0000j           24.6352            0.0000
-----------------------------------------------------------------------------
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  128
Model:                     ARMA(3, 0)   Log Likelihood                  15.877
```

```
Method:                       css-mle   S.D. of innovations              0.213
Date:                Mon, 18 Sep 2023   AIC                            -21.755
Time:                        10:57:36   BIC                             -7.494
Sample:                             0   HQIC                           -15.961
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0143      0.040      0.362      0.718      -0.063       0.092
ar.L1.y        0.5387      0.088      6.111      0.000       0.366       0.712
ar.L2.y       -0.0296      0.101     -0.295      0.768      -0.227       0.167
ar.L3.y        0.0175      0.090      0.194      0.846      -0.159       0.194
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.8400           -0.0000j            1.8400           -0.0000
AR.2           -0.0736           -5.5718j            5.5723           -0.2521
AR.3           -0.0736           +5.5718j            5.5723            0.2521
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  128
Model:                     ARMA(0, 1)   Log Likelihood                  12.562
Method:                       css-mle   S.D. of innovations              0.219
Date:                Mon, 18 Sep 2023   AIC                            -19.123
Time:                        10:57:36   BIC                            -10.567
Sample:                             0   HQIC                           -15.647
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0122      0.029      0.424      0.672      -0.044       0.068
ma.L1.y        0.4846      0.070      6.944      0.000       0.348       0.621
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
MA.1           -2.0637           +0.0000j            2.0637            0.5000
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  128
Model:                     ARMA(0, 2)   Log Likelihood                  14.366
Method:                       css-mle   S.D. of innovations              0.216
Date:                Mon, 18 Sep 2023   AIC                            -20.732
Time:                        10:57:36   BIC                             -9.324
Sample:                             0   HQIC                           -16.097
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0133      0.032      0.417      0.677      -0.049       0.076
ma.L1.y        0.5154      0.086      6.011      0.000       0.347       0.684
ma.L2.y        0.1646      0.089      1.846      0.065      -0.010       0.339
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
MA.1           -1.5661           -1.9037j            2.4651           -0.3596
MA.2           -1.5661           +1.9037j            2.4651            0.3596
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  128
Model:                     ARMA(0, 3)   Log Likelihood                  16.425
Method:                       css-mle   S.D. of innovations              0.213
Date:                Mon, 18 Sep 2023   AIC                            -22.850
Time:                        10:57:36   BIC                             -8.590
Sample:                             0   HQIC                           -17.056
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0143      0.037      0.390      0.696      -0.058       0.086
ma.L1.y        0.5418      0.087      6.225      0.000       0.371       0.712
ma.L2.y        0.2576      0.098      2.640      0.008       0.066       0.449
ma.L3.y        0.1659      0.079      2.097      0.036       0.011       0.321
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
MA.1           -1.7077           -0.0000j            1.7077           -0.5000
MA.2            0.0773           -1.8774j            1.8790           -0.2435
MA.3            0.0773           +1.8774j            1.8790            0.2435
-----------------------------------------------------------------------------
```

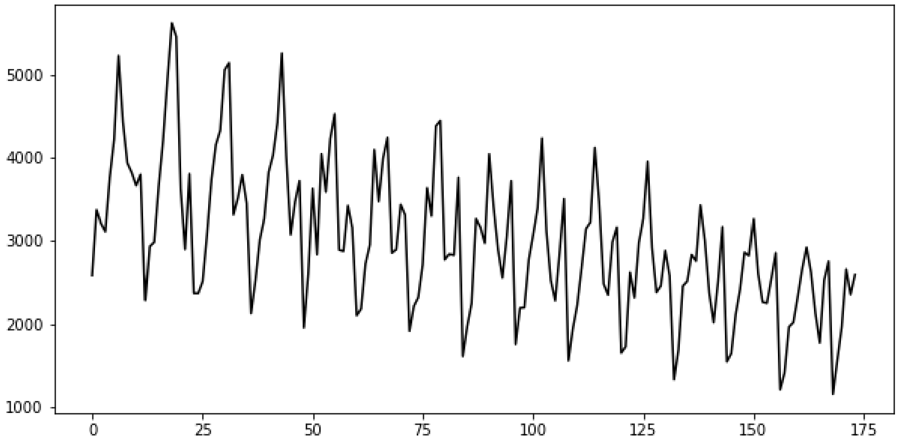
Выбираем лучшую модель по параметрам AIC, BIC, HQIC

In [67]:
```
arimaz = ARIMA(Z, order = (3, 0, 0))
model_fit = arimaz.fit(disp = False)
print(model_fit.summary())
```

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  128
Model:                     ARMA(3, 0)   Log Likelihood                  15.877
Method:                       css-mle   S.D. of innovations              0.213
Date:                Mon, 18 Sep 2023   AIC                            -21.755
Time:                        10:57:38   BIC                             -7.494
Sample:                             0   HQIC                           -15.961

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0143      0.040      0.362      0.718      -0.063       0.092
ar.L1.y        0.5387      0.088      6.111      0.000       0.366       0.712
ar.L2.y       -0.0296      0.101     -0.295      0.768      -0.227       0.167
ar.L3.y        0.0175      0.090      0.194      0.846      -0.159       0.194
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.8400           -0.0000j            1.8400           -0.0000
AR.2           -0.0736           -5.5718j            5.5723           -0.2521
AR.3           -0.0736           +5.5718j            5.5723            0.2521
-----------------------------------------------------------------------------
```

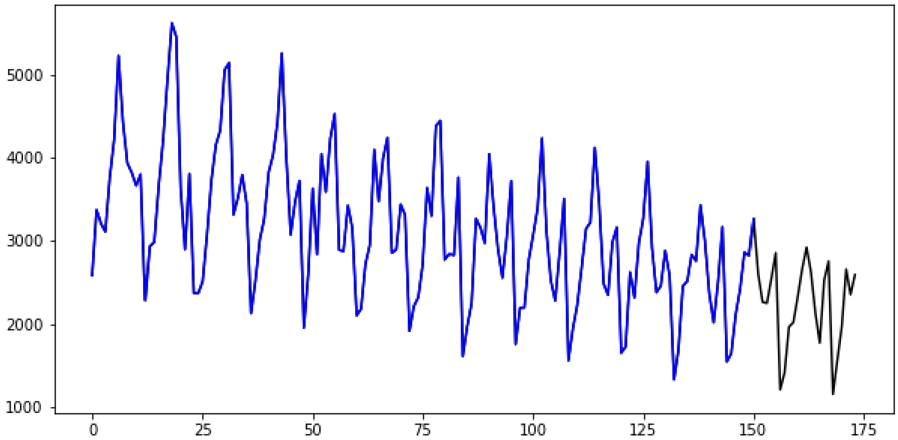
Обратимся к прогнозированиюна основе АРПСС моделей. Загрузим ВР из файла:

In [68]:
```
file = h5py.File('Fort.mat', 'r')
data = file.get('Fort')
Fort = np.array(data)

plt.figure(figsize = (10, 5))
plt.plot(Fort, 'k')
plt.show()
```



Будем производить ретроспективный прогноз. Для этого отрежем от данного ряда последние 24 точки (которые мы и будем прогнозировать):

In [69]:
```
Z = Fort[:len(Fort)-24+1] # отрезаем последние 24 точки
t = np.arange(0, len(Z), 1) # временная шкала для регрессии
t = t.reshape(-1,1)

plt.figure(figsize = (10, 5))
plt.plot(Fort, 'k') # исходный ВР
plt.plot(t, Z, 'b') # урезанныйряд
plt.show()
```



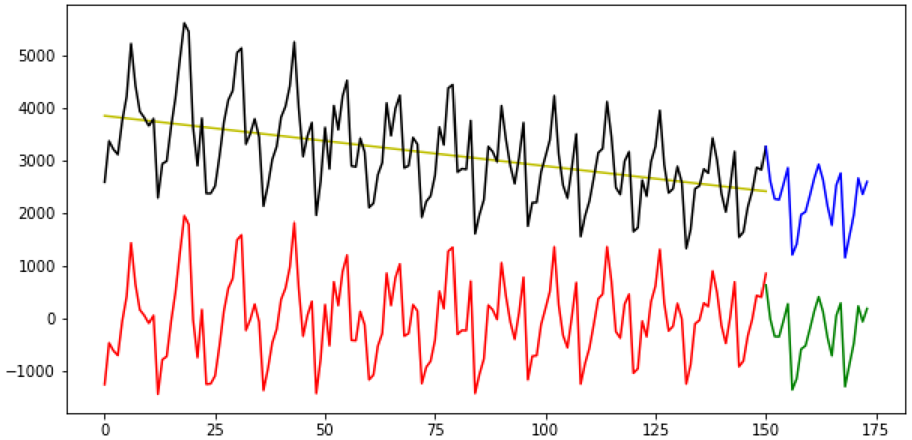
In [70]:
```
Fort.mean()
```

Out[70]: 3010.183908045977

Модели АРПСС строятся для рядов с около-нулевым средним, что неверно для заданного временного ряда. Построим линейный тренд прогнозируемого ряда, а затем вычтем его из исходного ряда, приведя его к нулевому среднему значению.

In [71]:
```
from scipy.optimize import curve_fit

t0 = np.arange(0, len(Fort), 1) # диапазон полного ряда
t0 = t0.reshape(-1,1)

popt, pcov = curve_fit(lambda t, b0, b1: b0 + b1 * t,
                       t.reshape(1,-1)[0], Z.reshape(1,-1)[0])

print(f"b0: {popt[0]}; b1: {popt[1]}")

Zn = Z - (t*popt[1]+popt[0])
Fort24 = Fort[-24:] - (t[-24:]*popt[1]+popt[0])


plt.figure(figsize = (10, 5))
plt.plot(t, (t*popt[1]+popt[0]), 'y')
plt.plot(t, Z, 'k')
plt.plot(t, Zn, 'r')
plt.plot(t0[-24:], Fort24, 'g')
plt.plot(t0[-24:], Fort[-24:], 'b')
plt.show()
```

b0: 3849.2388462962995; b1: -9.589498082978835



Рассматривая различные модели АРПСС порядков от 1 до 3, полагаясь на информационные критерии, можно сделать вывод, что по точности все модели примерно одинаковые. Возьмем модель АРПСС(2, 0, 2) как наилучшую модель по информационным критерием с наименьшим количеством параметров.

In [72]:
```
arimaz10 = ARIMA(Zn, order = (1, 0, 0))
model_fit10 = arimaz10.fit(disp = False) # подгоняем под ВР
print(model_fit10.summary())

arimaz20 = ARIMA(Zn, order = (2, 0, 0))
model_fit20 = arimaz20.fit(disp = False) # подгоняем под ВР
print(model_fit20.summary())

arimaz30 = ARIMA(Zn, order = (3, 0, 0))
model_fit30 = arimaz30.fit(disp = False) # подгоняем под ВР
print(model_fit30.summary())

arimaz11 = ARIMA(Zn, order = (1, 0, 1))
model_fit11 = arimaz11.fit(disp = False) # подгоняем под ВР
print(model_fit11.summary())

arimaz12 = ARIMA(Zn, order = (1, 0, 2))
model_fit12 = arimaz12.fit(disp = False) # подгоняем под ВР
print(model_fit12.summary())

arimaz13 = ARIMA(Zn, order = (1, 0, 3))
model_fit13 = arimaz13.fit(disp = False) # подгоняем под ВР
print(model_fit13.summary())

arimaz21 = ARIMA(Zn, order = (2, 0, 1))
model_fit21 = arimaz21.fit(disp = False) # подгоняем под ВР
print(model_fit21.summary())

arimaz22 = ARIMA(Zn, order = (2, 0, 2))
model_fit22 = arimaz22.fit(disp = False) # подгоняем под ВР
print(model_fit22.summary())
```

```python
arimaz23 = ARIMA(Zn, order = (2, 0, 3))
model_fit23 = arimaz23.fit(disp = False) # подгоняем под ВР
print(model_fit23.summary())

arimaz31 = ARIMA(Zn, order = (3, 0, 1))
model_fit31 = arimaz31.fit(disp = False) # подгоняем под ВР
print(model_fit31.summary())

arimaz32 = ARIMA(Zn, order = (3, 0, 2))
model_fit32 = arimaz32.fit(disp = False) # подгоняем под ВР
print(model_fit32.summary())
```

```
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(1, 0)   Log Likelihood               -1193.893
Method:                       css-mle   S.D. of innovations            656.374
Date:                Mon, 18 Sep 2023   AIC                           2393.787
Time:                        10:57:44   BIC                           2402.838
Sample:                             0   HQIC                          2397.464
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -2.6755    104.070     -0.026      0.979    -206.648     201.297
ar.L1.y        0.4900      0.072      6.835      0.000       0.349       0.630
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            2.0410           +0.0000j            2.0410            0.0000
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(2, 0)   Log Likelihood               -1191.638
Method:                       css-mle   S.D. of innovations            646.513
Date:                Mon, 18 Sep 2023   AIC                           2391.275
Time:                        10:57:44   BIC                           2403.344
Sample:                             0   HQIC                          2396.178
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -1.7869     87.622     -0.020      0.984    -173.523     169.949
ar.L1.y        0.5738      0.081      7.101      0.000       0.415       0.732
ar.L2.y       -0.1728      0.081     -2.141      0.032      -0.331      -0.015
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.6605           -1.7408j            2.4058           -0.1288
AR.2            1.6605           +1.7408j            2.4058            0.1288
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(3, 0)   Log Likelihood               -1191.582
Method:                       css-mle   S.D. of innovations            646.272
Date:                Mon, 18 Sep 2023   AIC                           2393.164
Time:                        10:57:44   BIC                           2408.251
Sample:                             0   HQIC                          2399.293
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -1.5406     85.263     -0.018      0.986    -168.653     165.571
ar.L1.y        0.5688      0.082      6.920      0.000       0.408       0.730
ar.L2.y       -0.1572      0.093     -1.685      0.092      -0.340       0.026
ar.L3.y       -0.0273      0.082     -0.333      0.739      -0.188       0.133
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.4483           -1.4613j            2.0574           -0.1257
AR.2            1.4483           +1.4613j            2.0574            0.1257
AR.3           -8.6567           -0.0000j            8.6567           -0.5000
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(1, 1)   Log Likelihood               -1191.710
Method:                       css-mle   S.D. of innovations            646.819
Date:                Mon, 18 Sep 2023   AIC                           2391.421
Time:                        10:57:44   BIC                           2403.490
Sample:                             0   HQIC                          2396.324
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -2.0308     92.282     -0.022      0.982    -182.901     178.840
ar.L1.y        0.2386      0.162      1.471      0.141      -0.079       0.557
```

```
ma.L1.y        0.3399      0.162      2.101      0.036       0.023       0.657
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            4.1915           +0.0000j            4.1915            0.0000
MA.1           -2.9422           +0.0000j            2.9422            0.5000
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(1, 2)   Log Likelihood               -1186.096
Method:                       css-mle   S.D. of innovations            617.448
Date:                Mon, 18 Sep 2023   AIC                           2382.191
Time:                        10:57:45   BIC                           2397.278
Sample:                             0   HQIC                          2388.320
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.5526      8.468     -0.065      0.948     -17.150      16.045
ar.L1.y        0.8027      0.053     15.107      0.000       0.699       0.907
ma.L1.y       -0.4065      0.079     -5.123      0.000      -0.562      -0.251
ma.L2.y       -0.5934      0.076     -7.782      0.000      -0.743      -0.444
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.2457           +0.0000j            1.2457            0.0000
MA.1            1.0001           +0.0000j            1.0001            0.0000
MA.2           -1.6850           +0.0000j            1.6850            0.5000
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(1, 3)   Log Likelihood               -1183.892
Method:                       css-mle   S.D. of innovations            612.344
Date:                Mon, 18 Sep 2023   AIC                           2379.784
Time:                        10:57:45   BIC                           2397.888
Sample:                             0   HQIC                          2387.139
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.3470     14.395      0.094      0.925     -26.867      29.561
ar.L1.y        0.6971      0.129      5.383      0.000       0.443       0.951
ma.L1.y       -0.2834      0.130     -2.172      0.030      -0.539      -0.028
ma.L2.y       -0.3994      0.102     -3.928      0.000      -0.599      -0.200
ma.L3.y       -0.2484      0.106     -2.348      0.019      -0.456      -0.041
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.4346           +0.0000j            1.4346            0.0000
MA.1            1.0368           -0.0000j            1.0368           -0.0000
MA.2           -1.3222           -1.4610j            1.9705           -0.3671
MA.3           -1.3222           +1.4610j            1.9705            0.3671
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(2, 1)   Log Likelihood               -1177.439
Method:                       css-mle   S.D. of innovations            586.452
Date:                Mon, 18 Sep 2023   AIC                           2364.878
Time:                        10:57:45   BIC                           2379.964
Sample:                             0   HQIC                          2371.007
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.4824     17.079      0.262      0.793     -28.992      37.957
ar.L1.y        1.3378      0.067     20.009      0.000       1.207       1.469
ar.L2.y       -0.6152      0.064     -9.554      0.000      -0.741      -0.489
ma.L1.y       -0.9072      0.034    -27.070      0.000      -0.973      -0.841
                                    Roots                                    
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.0873           -0.6658j            1.2750           -0.0874
AR.2            1.0873           +0.6658j            1.2750            0.0874
MA.1            1.1023           +0.0000j            1.1023            0.0000
-----------------------------------------------------------------------------
                              ARMA Model Results                              
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(2, 2)   Log Likelihood               -1156.842
Method:                       css-mle   S.D. of innovations            505.551
Date:                Mon, 18 Sep 2023   AIC                           2325.684
Time:                        10:57:45   BIC                           2343.788
Sample:                             0   HQIC                          2333.039
                                                                              
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
```

```
------------------------------------------------------------------------------
const          3.8869     34.248      0.113      0.910     -63.239      71.013
ar.L1.y        1.7305      0.005    337.486      0.000       1.720       1.741
ar.L2.y       -0.9964      0.004   -226.644      0.000      -1.005      -0.988
ma.L1.y       -1.7183      0.034    -50.006      0.000      -1.786      -1.651
ma.L2.y        0.9388      0.030     31.306      0.000       0.880       0.998
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            0.8684           -0.4995j            1.0018           -0.0831
AR.2            0.8684           +0.4995j            1.0018            0.0831
MA.1            0.9151           -0.4772j            1.0321           -0.0765
MA.2            0.9151           +0.4772j            1.0321            0.0765
-----------------------------------------------------------------------------
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(2, 3)   Log Likelihood               -1158.225
Method:                       css-mle   S.D. of innovations            512.912
Date:                Mon, 18 Sep 2023   AIC                           2330.450
Time:                        10:57:46   BIC                           2351.571
Sample:                             0   HQIC                          2339.031

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.9227     21.506      0.182      0.855     -38.229      46.074
ar.L1.y        1.6908      0.028     60.292      0.000       1.636       1.746
ar.L2.y       -0.9394      0.031    -30.422      0.000      -1.000      -0.879
ma.L1.y       -1.5360      0.104    -14.755      0.000      -1.740      -1.332
ma.L2.y        0.4576      0.224      2.046      0.041       0.019       0.896
ma.L3.y        0.2056      0.143      1.439      0.150      -0.074       0.486
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            0.8999           -0.5046j            1.0317           -0.0813
AR.2            0.8999           +0.5046j            1.0317            0.0813
MA.1            1.0131           -0.3430j            1.0696           -0.0520
MA.2            1.0131           +0.3430j            1.0696            0.0520
MA.3           -4.2518           -0.0000j            4.2518           -0.5000
-----------------------------------------------------------------------------
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(3, 1)   Log Likelihood               -1176.190
Method:                       css-mle   S.D. of innovations            581.382
Date:                Mon, 18 Sep 2023   AIC                           2364.381
Time:                        10:57:46   BIC                           2382.484
Sample:                             0   HQIC                          2371.735

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.9579     16.909      0.293      0.769     -28.184      38.099
ar.L1.y        1.2441      0.087     14.227      0.000       1.073       1.416
ar.L2.y       -0.4369      0.128     -3.409      0.001      -0.688      -0.186
ar.L3.y       -0.1354      0.084     -1.603      0.109      -0.301       0.030
ma.L1.y       -0.8892      0.035    -25.087      0.000      -0.959      -0.820
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.0098           -0.6228j            1.1864           -0.0880
AR.2            1.0098           +0.6228j            1.1864            0.0880
AR.3           -5.2451           -0.0000j            5.2451           -0.5000
MA.1            1.1246           +0.0000j            1.1246            0.0000
-----------------------------------------------------------------------------
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(3, 2)   Log Likelihood               -1158.884
Method:                       css-mle   S.D. of innovations            515.657
Date:                Mon, 18 Sep 2023   AIC                           2331.768
Time:                        10:57:46   BIC                           2352.889
Sample:                             0   HQIC                          2340.349

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.4911     22.289      0.157      0.876     -40.195      47.178
ar.L1.y        1.7665      0.122     14.451      0.000       1.527       2.006
ar.L2.y       -1.0699      0.207     -5.158      0.000      -1.476      -0.663
ar.L3.y        0.0627      0.129      0.486      0.627      -0.190       0.315
ma.L1.y       -1.6988      0.094    -17.982      0.000      -1.884      -1.514
ma.L2.y        0.8251      0.049     16.766      0.000       0.729       0.922
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            0.8872           -0.5059j            1.0213           -0.0825
AR.2            0.8872           +0.5059j            1.0213            0.0825
```

```
AR.3           15.2995           -0.0000j          15.2995           -0.0000
MA.1            1.0295           -0.3901j           1.1009           -0.0577
MA.2            1.0295           +0.3901j           1.1009            0.0577
-----------------------------------------------------------------------------
```

In [73]:

```python
arimaz22 = ARIMA(Zn, order = (2, 0, 2))
model_fit22 = arimaz22.fit(disp = False) # подгоняем под ВР
print(model_fit22.summary())
```

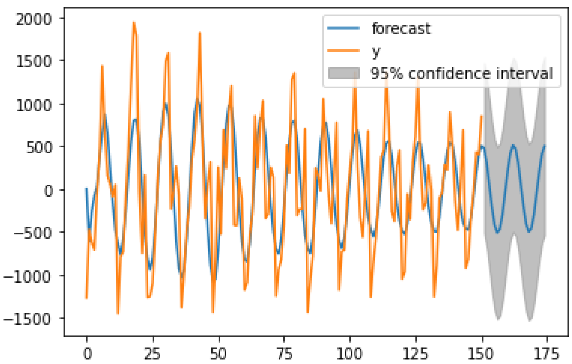
```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(2, 2)   Log Likelihood               -1156.842
Method:                       css-mle   S.D. of innovations            505.551
Date:                Mon, 18 Sep 2023   AIC                           2325.684
Time:                        10:57:49   BIC                           2343.788
Sample:                             0   HQIC                          2333.039

==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.8869     34.248      0.113      0.910     -63.239      71.013
ar.L1.y        1.7305      0.005    337.486      0.000       1.720       1.741
ar.L2.y       -0.9964      0.004   -226.644      0.000      -1.005      -0.988
ma.L1.y       -1.7183      0.034    -50.006      0.000      -1.786      -1.651
ma.L2.y        0.9388      0.030     31.306      0.000       0.880       0.998
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            0.8684           -0.4995j           1.0018           -0.0831
AR.2            0.8684           +0.4995j           1.0018            0.0831
MA.1            0.9151           -0.4772j           1.0321           -0.0765
MA.2            0.9151           +0.4772j           1.0321            0.0765
-----------------------------------------------------------------------------
```

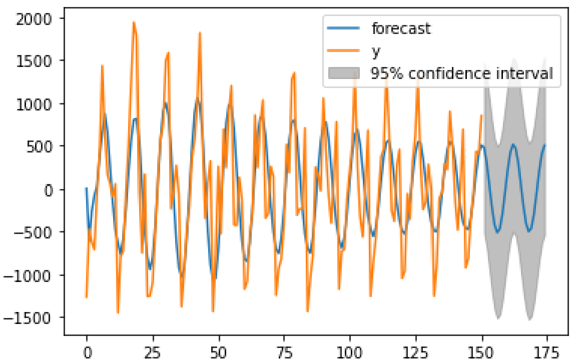
Построим график прогноза по данной модели вместе с доверительными интервалами:

In [74]:

```python
model_fit22.plot_predict(0, len(Fort))
```

Out[74]:



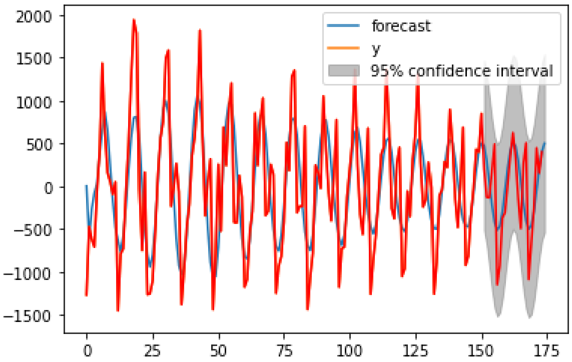


Рассмотрим как этот прогноз по АРПСС модели соотносится с исходными известными 24 прогнозными точками. Вычтем из исходного ряда Fort линейный тренд и соотнесем их на одном изображении:

In [75]:

```python
popt, pcov = curve_fit(lambda t, b0, b1: b0 + b1 * t,
                       t.reshape(1,-1)[0], Z.reshape(1,-1)[0])

trend_as_func_of_t0 = t0*popt[1]+popt[0]

plt.figure(figsize = (10, 5))
model_fit22.plot_predict(0, len(Fort)) # прогноз по АРПСС
plt.plot(t0, Fort-(trend_as_func_of_t0), 'r') # исходный ВР минус тренд
plt.show();
```

```
<Figure size 720x360 with 0 Axes>
```



Получим прогнозные значения по модели АРПСС и используем эти значения для оценки точности прогноза:

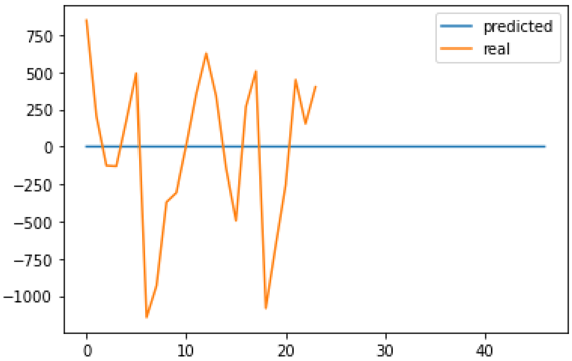
In [76]:
```
# Средняя ошибка прогноза
def MAE(pred, real):
  return np.mean(np.abs(pred - real))

# СКВО прогноза
def MSD(pred, real):
  return np.sqrt(np.mean((pred - real)**2))

# Средняя ошибка аппроксимации
def MAPE(pred, real):
  return np.mean(np.abs((real - pred) / real))

def res_evaluation(pred, real, title=""):
  print(
      f"Оценка точности прогноза {title}\n"
      f"Средняя ошибка прогноза: {np.round(MAE(pred, real),2)}\n",
      f"СКВО прогноза: {np.round(MSD(pred, real),2)}\n",
      f"Средняя ошибка аппроксимации: {np.round(MAPE(pred, real),2) *100}%\n"
      )
```

In [77]:
```
res_evaluation(model_fit22.predict(len(Z), len(Fort)), (Fort-(trend_as_func_of_t0))[-24:])
```

```
Оценка точности прогноза
Средняя ошибка прогноза: 517.42
 СКВО прогноза: 642.58
 Средняя ошибка аппроксимации: 229.99999999999997%
```

Можно заметить, что средняя ошибка аппроксимации для данной модели больше 100 процентов. Это означает, что ошибки намного больше, чем фактические значения. Тем не менее, метрики MAE и MSD все еще достаточно информативны.

In [78]:
```
plt.plot(model_fit.predict(len(Z), len(Fort)), label='predicted')
plt.plot((Fort-(trend_as_func_of_t0))[-24:], label='real')
plt.legend()
plt.show()
```



Также попробуем построить АРПСС модель для прогнозирования данного ряда, но без исходного вычитания из него линейного тренда:

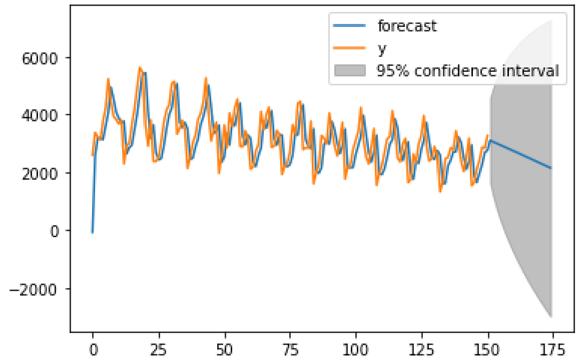
In [79]:
```
arimaz22r = ARIMA(Z, order = (2, 0, 2))
model_fit22r = arimaz22r.fit(disp = False) # подгоняем под ВР
print(model_fit22r.summary())
```

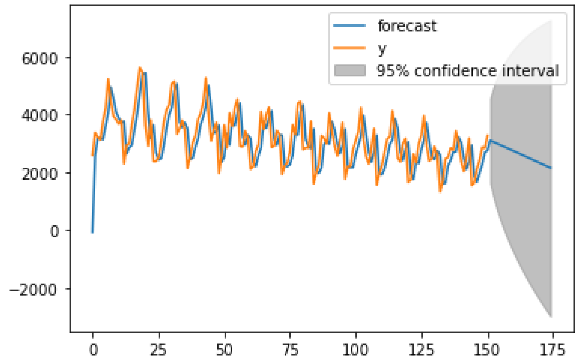
```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  151
Model:                     ARMA(2, 2)   Log Likelihood               -1215.319
Method:                       css-mle   S.D. of innovations            744.434
Date:                Mon, 18 Sep 2023   AIC                           2442.639
Time:                        10:58:30   BIC                           2460.742
Sample:                             0   HQIC                          2449.993
```

```
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const        -70.7118   2143.390     -0.033      0.974   -4271.678    4130.255
ar.L1.y        1.9950        nan        nan        nan         nan         nan
ar.L2.y       -0.9951        nan        nan        nan         nan         nan
ma.L1.y       -1.2469      0.107    -11.693      0.000      -1.456      -1.038
ma.L2.y        0.2469      0.106      2.322      0.020       0.039       0.455
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            1.0024           -0.0101j            1.0024           -0.0016
AR.2            1.0024           +0.0101j            1.0024            0.0016
MA.1            1.0000           +0.0000j            1.0000            0.0000
MA.2            4.0498           +0.0000j            4.0498            0.0000
-----------------------------------------------------------------------------
```

In [80]:
```python
model_fit22r.plot_predict(0, len(Fort))
```

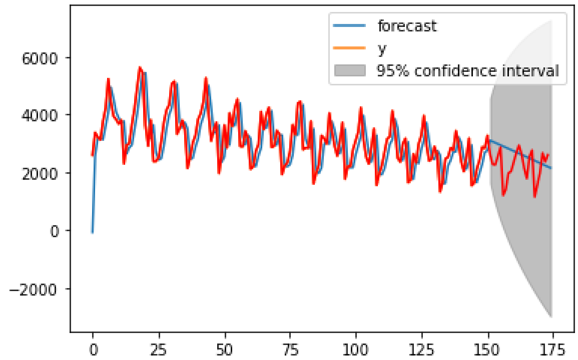
Out[80]:





In [81]:
```python
plt.figure(figsize = (10, 5))
model_fit22r.plot_predict(0, len(Fort)) # прогноз по АРПСС
plt.plot(t0, Fort, 'r')
plt.show()
```

```
<Figure size 720x360 with 0 Axes>
```



В данном случае значение метрик MAE и MSD немного хуже, чем для рядов с около-нулевым средним. При этом теперь значение метрики MAPE имеет смысл.

In [82]:
```python
res_evaluation(model_fit22r.predict(len(Z), len(Fort)), Fort[-24:])
```

```
Оценка точности прогноза
Средняя ошибка прогноза: 546.65
 СКВО прогноза: 703.72
 Средняя ошибка аппроксимации: 31.0%
```