Лабораторная работа №5

Посторение типовых моделей АРПСС (ARIMA)

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Вариант №19

```
import numpy as np
import numpy.random as rand
import matplotlib.pyplot as plt

import h5py

from statsmodels.tsa import api as tsa
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.tsa.stattools import acf
from statsmodels.tsa.arima_model import ARIMA

%matplotlib inline
import warnings
warnings.filterwarnings('ignore')
```

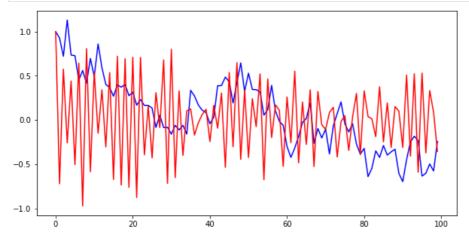
Создадим собственные АРПСС ряды первого и второго порядков для изучения их автокорреляционных функций. Создадим два AP(1) процесса первого порядка:

```
In [45]:     z1 = np.zeros(100)
     z2 = np.zeros(100)

z1[0] = 1
     z2[0] = 1

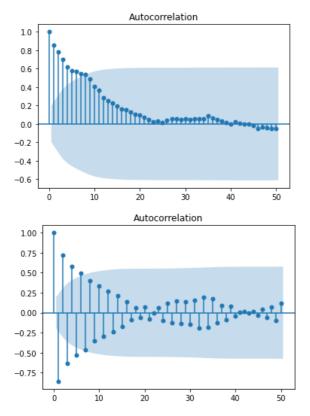
for i in range(1,100):
     z1[i] = 0.8 * z1[i-1] + 0.2 * np.random.randn()
          z2[i] = -0.8 * z2[i-1] + 0.2 * np.random.randn()

     plt.figure(figsize = (10, 5))
     plt.plot(z1, 'b')
     plt.plot(z2, 'r')
     plt.show()
```



Теперь построим для этих рядов функции автокорреляции:

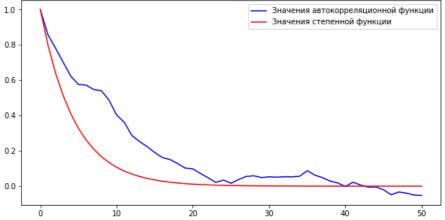
```
In [46]: plt.figure(figsize = (10, 5))
    plot_acf(z1, lags=50)
    plot_acf(z2, lags=50)
    plt.show()
```



Для AP с положительным коэфициентом можно заметить, что зависимость с каждым лагом меняется незначительно. Во втором случае, по автокорреляционной функции можно отметить резкие колебания в BP. Так же можно заметить, что значения автокорреляционных функций при лаге 1 близки к значениям весовых параметров этих процессов.

Удостоверимся, что для модели АР(1) коэффициенты автокорреляции изменяются по степенному закону:

```
In [47]: plt.figure(figsize = (10, 5)) plt.plot(range(51), acf(z1, nlags=50, fft=True), 'b', label='Значения автокорреляционной функции') plt.plot(range(51), np.array([0.8**1 for l in range(51)]), 'r', label='Значения степенной функции') plt.legend() plt.show();
```

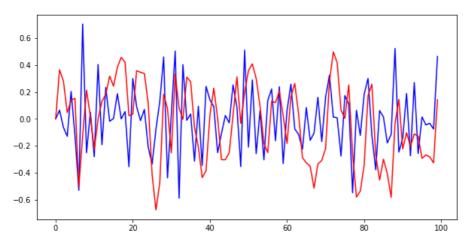


Аналогичным образом построем два СС(1) процесса среднего-скользящего первого порядка:

```
In [48]: z3 = np.zeros(100)
z4 = np.zeros(100)
ar = 0.2 * np.random.randn(100)

for i in range(1, 100):
    z3[i] = ar[i] -0.8 * ar[i -1]
    z4[i] = ar[i] + 0.8 * ar[i -1]

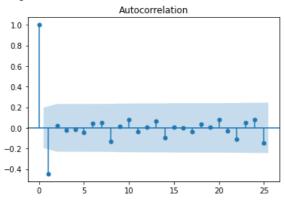
plt.figure(figsize= (10, 5))
plt.plot(z3, 'b')
plt.plot(z4, 'r')
plt.show()
```

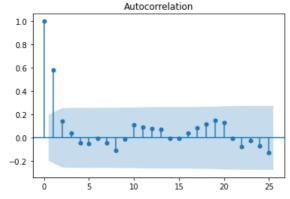


Посторим для этих рядов функции автокорреляции:

```
In [49]: plt.figure(figsize = (10, 5))
    plot_acf(z3, lags=25)
    plot_acf(z4, lags=25)
    plt.show()
```

<Figure size 720x360 with 0 Axes>

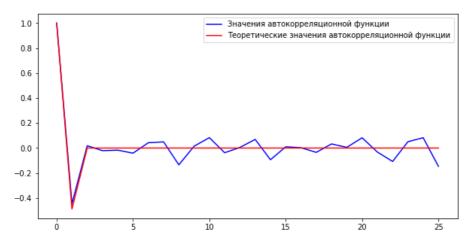




Убежадемся, что значение коэфициента автокорреляционной функции при лаге 1 равна следующему выражению:

```
In [50]: def p1(k):
    if k == 0:
        return 1
    if k == 1:
        return (-0.8) / (1 + (-0.8)**2)
    return 0

plt.figure(figsize = (10, 5))
    plt.plot(range(26), acf(23, nlags=25, fft=True), 'b', label='Значения автокорреляционной функции')
    plt.plot(range(26), np.array([p1(l) for l in range(26)]), 'r', label='Теоретические значения автокорреляционной функци
    plt.legend()
    plt.show();
```



Оценим весовой параметр процесса на основе функции автокорреляции:

```
In [51]:
D = (1/acf(z3, nlags=2, fft=True)[1])**2 - 4
theta1 = (-(1/acf(z3, nlags=2, fft=True)[1]) + np.sqrt(D)) / 2
theta2 = (-(1/acf(z3, nlags=2, fft=True)[1]) - np.sqrt(D)) / 2
print(f"theta1 = {theta1}; theta2 = {theta2} => весовой параметр = {theta2}")
```

theta1 = 1.6290729269934823; theta2 = 0.6138460614194473 => весовой параметр = 0.6138460614194473

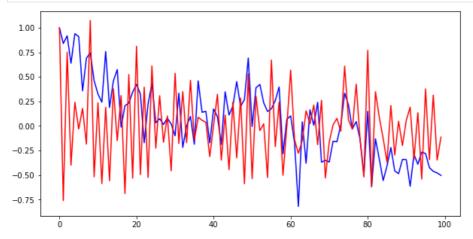
Создадим временной ряд процесса АРСС(1, 1):

```
In [52]:    z5 = np.zeros(100)
    z6 = np.zeros(100)

    z5[0] = 1
    z6[0] = 1
    ar = 0.2 * np.random.randn(100)

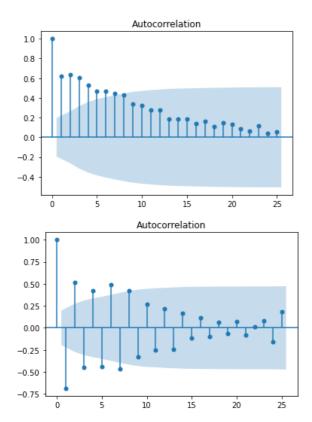
    for i in range(1,100):
        z5[i] = 0.8 * z1[i-1] + ar[i] -0.3 * ar[i-1]
        z6[i] = -0.8 * z2[i-1] + ar[i] -0.3 * ar[i-1]

    plt.figure(figsize= (10, 5))
    plt.plot(z5, 'b')
    plt.plot(z6, 'r')
    plt.show()
```



Построим их автокорреляционные функции:

```
In [53]: plt.figure(figsize = (10, 5))
    plot_acf(z5, lags=25)
    plot_acf(z6, lags=25)
    plt.show()
```



Используем следующую функцию для создания АРСС (2, 2):

```
In [54]: from statsmodels.tsa.arima_process import arma_generate_sample

ar = np.array([0.75, -0.25]) # задаем κοσφομιμεнты AP

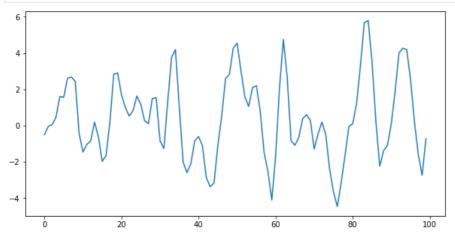
ma = np.array([0.65, 0.35]) # задаем коσφομιμεнты CC

y = arma_generate_sample(np.r_[1, -ar], np.r_[1, ma], 100) # создаем BP для APCC (2, 2) = APПCC (2, 0, 2) из 100 отсче

plt.figure(figsize= (10, 5))

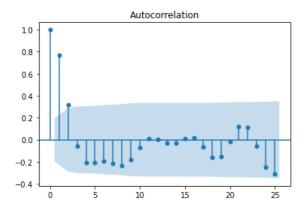
plt.plot(y)

plt.show()
```



Построим ее автокорреляционную функцию:

```
In [55]: plt.figure(figsize = (10, 5))
    plot_acf(y, lags=25)
    plt.show()
```

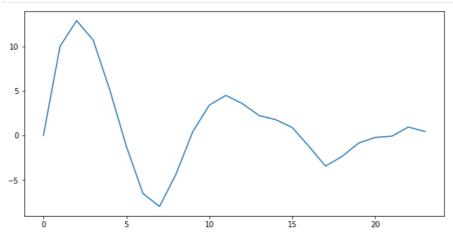


Теперь проведем анализ неизвестного ряда:

```
In [56]: TEST = [0.00, 9.99, 12.89, 10.70, 5.12, -1.21, -6.50, -7.96, -4.30, 0.42, 3.41, 4.50, 3.57, 2.24, 1.78, 0.89, -1.
```

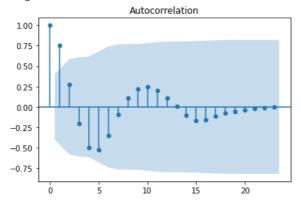
Построим график ВР и его автокорреляционную функцию:

```
In [57]: plt.figure(figsize= (10, 5))
    plt.plot(TEST)
    plt.show()
```



```
In [58]: plt.figure(figsize = (10, 5))
    plot_acf(TEST, lags=len(TEST)-1)
    plt.show()
```

<Figure size 720x360 with 0 Axes>



По нимможно судить, что ВР, в достаточной степени, стационарен, а, так как, эта функция является знакопеременной, то один из членов АР модели имеет отрицательный вес. Создадим три пробные модели АРПСС для проверки ряда на:

```
1. AP(1) = AP\Pi CC(1, 0, 0)
2. AP(2)
```

3. AP(3), без тренда (trend= 'nc')

```
In [59]:

arima1 = ARIMA(TEST, order = (1, 0, 0))# создаем модель
model_fit1 = arima1.fit(disp = False, trend='nc')# подгоняем под ВР
print(model_fit1.summary())# выводим таблицу результатов

arima2 = ARIMA(TEST, order = (2, 0, 0))
model_fit2 = arima2.fit(disp = False, trend='nc')
print(model_fit2.summary())
```

```
arima3 = ARIMA(TEST, order = (3, 0, 0))
model_fit3 = arima3.fit(disp = False, trend='nc')
print(model_fit3.summary())
```

```
ARMA Model Results
Dep. Variable:

Model:

ARMA(1, 0)

Coss-mle

S.D. of innovation
                                No. Observations:
               css-mle S.D. of innovations
Mon, 18 Sep 2023 AIC
10:57:29 BIC
0 HQIC
Time:
Sample:
______
             coef std err z P > |z| [0.025 0.975]
ar.L1.y 0.7426 0.123 6.053
Roots
                              6.053 0.000 0.502 0.983
             Real
                     Imaginary Modulus Frequency
AR.1 1.3465 +0.0000j 1.3465 0.0000
                       ARMA Model Results
_____
Dep. Variable:
                             y No. Observations:
                ARMA(2, 0) Log Likelihood
css-mle S.D. of innovations

Mon, 18 Sep 2023 AIC
10:57:29 BIC
Model:
                                                          -41.543
Method:
                                                           1.201
Date:
                                                           89.086
                             0 HQIC
Sample:
______
            coef std err z P>|z| [0.025 0.975]
ar.L1.y 1.5108 0.056 27.117 0.000 1.402 1.620 ar.L2.y -0.9641 0.035 -27.509 0.000 -1.033 -0.895 Roots
-----
            Real Imaginary Modulus Frequency
AR.1 0.7836 -0.6506j 1.0185 -0.1103
AR.2 0.7836 +0.6506j 1.0185 0.1103
                       ARMA Model Results
_____
Dep. Variable:

Model:

ARMA(3, 0) Log Likelihood

Method:

S. D. of innovations
                                                              24
Model:
                                                          -41.097
               CSS-mle S.D. of innovations

Mon, 18 Sep 2023 AIC

10:57:29 BIC

0 HQIC
Method:
                                                            1.172
Date:
                                                           90.193
                                                           94.906
Time:
Sample:
______
            coef std err z P > |z| [0.025 0.975]

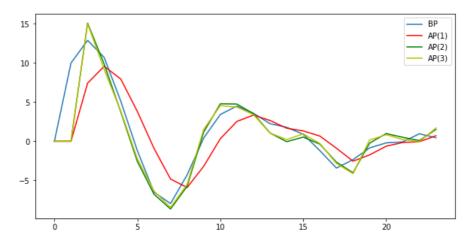
    ar.L1.y
    1.7184
    0.222
    7.755
    0.000
    1.284
    2.153

    ar.L2.y
    -1.2995
    0.345
    -3.770
    0.000
    -1.975
    -0.624

    ar.L3.y
    0.2216
    0.229
    0.968
    0.333
    -0.227
    0.670

                            Roots
______
             Real Imaginary Modulus Frequency
AR.1 0.7786 -0.6645j 1.0236 -0.1124
AR.2 0.7786 +0.6645j 1.0236 0.1124
AR.3 4.3072 -0.0000j 4.3072 -0.0000
plt.figure(figsize= (10, 5))
plt.plot(TEST, label='BP')
plt.plot(model_fit1.fittedvalues, 'r', label='AP(1)')
plt.plot(model_fit2.fittedvalues, 'g', label='AP(2)')
plt.plot(model_fit3.fittedvalues, 'y', label='AP(3)')
```

```
plt.legend()
plt.show()
```



Расчитаем весовые коэффициенты для АР моделей только 1 и 2 порядка самостоятельно:

```
In [61]: acf_coefs = acf(TEST, fft=True)
acf_coefs
```

Весовой коэффициент АР(1)

```
In [62]: theta11 = acf_coefs[1]
    print(theta11)
```

0.7550097626743363

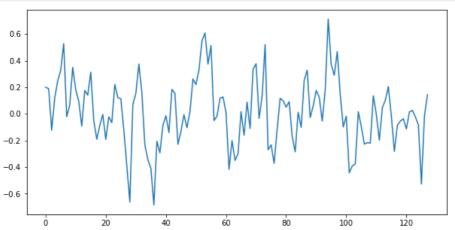
Весовые коэффициенты АР(2)

```
In [63]: theta21 = (acf_coefs[1]*(1 - acf_coefs[2])) / (1 - acf_coefs[1]**2)
theta22 = (acf_coefs[2] - acf_coefs[1]**2) / (1 - acf_coefs[1]**2)
print(theta21, theta22)
```

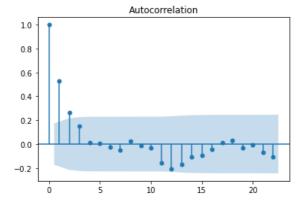
1.2777374692491608 -0.6923456257350362

Несмотря на некоторые различия, полученные веса близки к тем, что были получены с помощью функций Python.

Загрузим временной ряд Z из файла. Построим график BP и его автокорреляционную функцию:



```
In [65]: plt.figure(figsize = (10, 5))
    plot_acf(Z)
    plt.show();
```



Оценим порядок АРССмодели с помощью класса ARIMA.

```
arimaz = ARIMA(Z, order = (1, 0, 0))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())
arimaz = ARIMA(Z, order = (2, 0, 0))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())
arimaz = ARIMA(Z, order = (3, 0, 0))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())
arimaz = ARIMA(Z, order = (0, 0, 1))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())
arimaz = ARIMA(Z, order = (0, 0, 2))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())
arimaz = ARIMA(Z, order = (0, 0, 3))
model_fit = arimaz.fit(disp = False) # подгоняем под ВР
print(model_fit.summary())
                             ARMA Model Results
```

		ARMA Mo	del Resu	ults		
Dep. Variable Model: Method: Date: Time: Sample:		y ARMA(1, 0) css-mle 18 Sep 2023 10:57:36	Log l S.D. AIC BIC	 Dbservations: .ikelihood of innovations	=====	128 15.832 0.214 -25.664 -17.108 -22.188
=========		std err		P> z		0.975]
const ar.L1.y	0.0145 0.5279	0.075	7.070 oots	0.714 0.000	0.382	0.674
	Real		nary	Modulus		Frequency
AR.1	1.8943	+0.0	000j	1.8943		0.0000
		ARMA Mo				
Dep. Variable Model: Method: Date: Time: Sample:		y ARMA(2, 0) css-mle 18 Sep 2023 10:57:36	Log l S.D. AIC BIC	 Dbservations: .ikelihood of innovations	=====	128 15.859 0.214 -23.717 -12.309 -19.082
========	coef			P> z		0.975]
ar.L1.y	0.5387	0.088 0.088	6.109	0.712 0.000 0.818	0.366	
=========	Real	 Imagi	====== nary	Modulus	======	Frequency
AR.1	2.0077	+0.0	 000j	2.0077		0.0000

+0.0000j

-----ARMA Model Results ______ y No. Observations: ARMA(3, 0) Log Likelihood

24.6352

0.0000

128 15.877

AR.2

Model:

Dep. Variable:

24,6352

Method: Date:						
Date:				. of innovat	ions	0.213
	Mon,	, 18 Sep 2				-21.755
Time: Sample:		10:57	7:36 BIC 0 HQI	_		-7.494 -15.961
,						
				0.718		
ar.L1.y	0.5387	0.040	6.111	0.000	0.366	0.712
ar.L2.y	-0.0296	0.101	-0.295	0.768	-0.227	0.167
ar.L3.y	0.0175	0.090	0.194	0.000 0.768 0.846	-0.159	0.194
			Roots			
	1.8400	_	0.0000i	1 1	2400	-0.0000
	-0.0736	-	5.5718j	5.	5723	-0.2521
AR.3	-0.0736	+	-5.5718j	5.!	5723 	0.2521
		ARMA	Model Re	sults		
Dep. Variable:				Observation		 128
Model:				Likelihood		12.562
Method:		CSS-	mle S.D	. of innovat	ions	0.219
Date:	Mon,		2023 AIC			-19.123
Time: Sample:		10:57	7:36 BIC 0 HQI	_		-10.567 -15.647
·						
const ma.L1.y	0.0122	0.029	6.944	0.672 0.000	-0.044 0.348	0.621
			Roots			
MA.1	-2.0637	+	-0.0000j 	2.0	0637 	0.5000
		ARMA	Model Re	sults		
Dep. Variable:	=======			Observation:		 128
Model:		ARMA(0,	2) Log	Likelihood		14.366
Method:		css-	mle S.D	. of innovat	ions	0.216
Date:	Mon,		.023 AIC			-20.732
Time: Sample:		10:57	':36 BIC			-9.324
Sampie.			a HOT	_		
			0 HQI	C		
=========	coef	std err	 z	 P> z	[0.025	-16.097 0.975]
	coef	std err	z	P> z	[0.025	-16.097 0.975]
const	coef 0.0133	std err 0.032	z 0.417	P> z 0.677	[0.025 	-16.097 0.975] 0.076
const ma.L1.y	coef	std err 0.032	z 0.417	P> z 0.677	[0.025	-16.097 0.975] 0.076
const ma.L1.y ma.L2.y	coef 0.0133 0.5154 0.1646	0.032 0.086 0.089	2 0.417 6.011 1.846 Roots	P> z 0.677 0.000 0.065	[0.025 -0.049 0.347 -0.010	-16.097 0.975] 0.076 0.684 0.339
const ma.L1.y ma.L2.y	coef 0.0133 0.5154 0.1646	0.032 0.086 0.089	2 0.417 6.011 1.846 Roots	P> z 0.677 0.000 0.065	[0.025 -0.049 0.347 -0.010	-16.097 0.975] 0.076 0.684 0.339
const ma.L1.y ma.L2.y	coef 0.0133 0.5154 0.1646	0.032 0.086 0.089	0.417 6.011 1.846 Roots	P> z 0.677 0.000 0.065	-0.049 0.347 -0.010	-16.097
const ma.L1.y ma.L2.y	coef 0.0133 0.5154 0.1646	0.032 0.086 0.089	0.417 6.011 1.846 Roots	P> z 0.677 0.000 0.065	-0.049 0.347 -0.010	-16.097
const ma.L1.y ma.L2.y MA.1 MA.2	coef 0.0133 0.5154 0.1646 Real -1.5661	std err 0.032 0.086 0.089	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi	-0.049 0.347 -0.010	-16.097
const ma.l1.y ma.L2.y	coef 0.0133 0.5154 0.1646 Real -1.5661	std err 0.032 0.086 0.089	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi	-0.049 0.347 -0.010	-16.097
const ma.L1.y ma.L2.y	coef 0.0133 0.5154 0.1646 Real -1.5661	0.032 0.086 0.089 In ARMA	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi	-0.049 0.347 -0.010 	-16.097 0.975] 0.076 0.684 0.339 Frequency -0.3596 0.3596
const ma.L1.y ma.L2.y MA.1 MA.2 Dep. Variable: Model: Method:	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661	Std err 0.032 0.086 0.089 In ARMA ARMA(0,	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi 2.4 sults Observation: Likelihood of innovat:	-0.049 0.347 -0.010 	-16.097
const ma.L1.y ma.L2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661	Std err 0.032 0.086 0.089 In ARMA CSS- 18 Sep 2	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi 2.4 sults Observation: Likelihood of innovat:	-0.049 0.347 -0.010 	-16.097
const ma.L1.y ma.L2.y MA.1 MA.2 Dep. Variable: Model: Method: Date: Time:	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661	ARMA(0, CSS-18 Sep 2 10:57	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi 2.4 2.4 Sults Observations Likelihood . of innovats	-0.049 0.347 -0.010 	-16.097
const ma.L1.y ma.L2.y MA.1 MA.2 Dep. Variable: Model: Method: Date: Time:	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661	ARMA(0, CSS-18 Sep 2 10:57	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi 2.4 2.4 Sults Observations Likelihood . of innovats	-0.049 0.347 -0.010 	-16.097
const ma.L1.y ma.L2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661	ARMA(0, css-18 Sep 2 10:57	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi 2 2 sults Observation: Likelihood . of innovat:	[0.025 -0.049 0.347 -0.010 	-16.097
const ma.l1.y ma.l2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661 Mon,	ARMA(0, css-, 18 Sep 2 10:57	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Mod 2.4 2.4 Sults Observation: Likelihood of innovat:	[0.025 -0.049 0.347 -0.010 	-16.097
const ma.l1.y ma.l2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661 Mon,	ARMA(0, css-, 18 Sep 2 10:57	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Mod 2.4 2.4 Sults Observation: Likelihood of innovat:	[0.025 -0.049 0.347 -0.010 	-16.097
const ma.l1.y ma.l2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661 Mon,	ARMA(0, css-, 18 Sep 2 10:57	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Mod 2.4 2.4 Sults Observation: Likelihood of innovat:	[0.025 -0.049 0.347 -0.010 	-16.097
const ma.l1.y ma.l2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661 Mon,	ARMA(0, css-, 18 Sep 2 10:57	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Mod 2.4 2.4 Sults Observation: Likelihood of innovat:	[0.025 -0.049 0.347 -0.010 	-16.097
const ma.L1.y ma.L2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661 coef 0.0143 0.5418 0.2576 0.1659	ARMA(0, css- 18 Sep 2 10:57 20:087 0.098 0.079	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi 2 2 sults Observation: Likelihood of innovat: P> z 0.696 0.000 0.008 0.036	[0.025 -0.049 0.347 -0.010 	-16.097
const ma.L1.y ma.L2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661 Mon, coef 0.0143 0.5418 0.2576 0.1659	ARMA(0, 18 Sep 2 10:57 Std err 0.032 0.086 0.089 In ARMA(0, 55- 18 Sep 2 10:57	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Modi 2.4 2.4 Sults Observation: Likelihood. of innovat: 0.696 0.000 0.008 0.036	[0.025 -0.049 0.347 -0.010 	-16.097
const ma.l1.y ma.L2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661 Mon, coef 0.0143 0.5418 0.2576 0.1659	ARMA(0, 10:57 ARMA(0, 0.037 0.087 0.098 0.079	0.417 6.011 1.846 Roots 1.9037j 1.9037	P> z 0.677 0.000 0.065 Mod 2.4 2.4 Sults Observation: Likelihood. of innovat: 0.696 0.000 0.008 0.036	[0.025 -0.049 0.347 -0.010	-16.097
const ma.L1.y ma.L2.y ===================================	coef 0.0133 0.5154 0.1646 Real -1.5661 -1.5661 Mon, coef 0.0143 0.5418 0.2576 0.1659	ARMA(0, css- 10:57 34 ARMA(0, css- 18 Sep 2 10:57 35 36 37 38 39 30 30 30 30 30 30 30 30 30	0.417 6.011 1.846 Roots 	P> z 0.677 0.000 0.065 Mod 2.4 2.4 Sults Observation: Likelihood. of innovat: 0.696 0.000 0.008 0.036	[0.025 -0.049 0.347 -0.010 	-16.097

Выбираем лучшую модель по параметрам AIC, BIC, HQIC

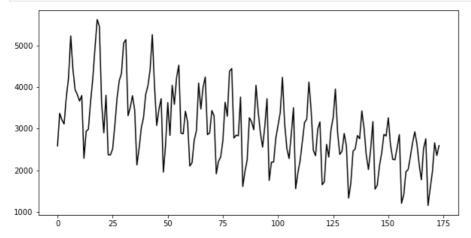
ARMA Model Results

=========			======	=========		========
Dep. Variable:		У	No.	Observations:		128
Model:		ARMA(3, 0)	Log	Likelihood		15.877
Method:		css-mle	S.D.	of innovations		0.213
Date:	Mon	, 18 Sep 2023	AIC			-21.755
Time:		10:57:38	BIC			-7.494
Sample:		0	HQIC			-15.961
==========	=======	=========		==========		========
	coef	std err	Z	P> z	[0.025	0.975]
const				0.718		
,	0.5387			0.000		0.712
ar.L2.y	-0.0296	0.101	-0.295	0.768		
ar.L3.y	0.0175	0.090	0.194	0.846	-0.159	0.194
		R	oots			
==========			======		======	=======
	Real	Imagi	nary	Modulus		Frequency
AR.1	1.8400		000j	1.8400		-0.0000
AR.2	-0.0736	-5.5	9	5.5723		-0.2521
AR.3	-0.0736	+5.5	718j	5.5723		0.2521

Обратимся к прогнозированиюна основе АРПСС моделей. Загрузим ВР из файла:

```
In [68]: file = h5py.File('Fort.mat', 'r')
    data = file.get('Fort')
    Fort = np.array(data)

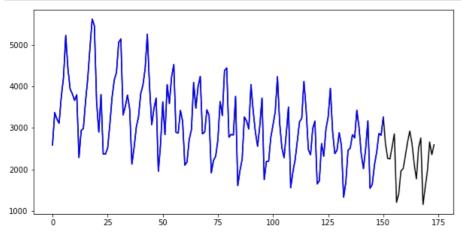
    plt.figure(figsize = (10, 5))
    plt.plot(Fort, 'k')
    plt.show()
```



Будем производить ретроспективный прогноз. Для этого отрежем от данного ряда последние 24 точки (которые мы и будем прогнозировать):

```
In [69]: Z = Fort[:len(Fort)-24+1] # отрезаем последние 24 точки
t = np.arange(0, len(Z), 1) # временная шкала для регрессии
t = t.reshape(-1,1)

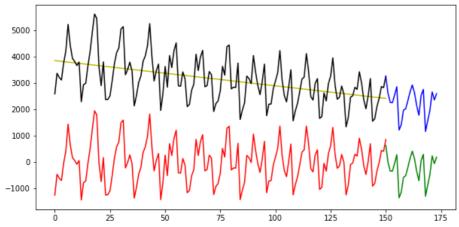
plt.figure(figsize = (10, 5))
plt.plot(Fort, 'k') # исходный ВР
plt.plot(t, Z, 'b') # урезанныйряд
plt.show()
```



Модели АРПСС строятся для рядов с около-нулевым средним, что неверно для заданного временного ряда. Построим линейный тренд прогнозируемого ряда, а затем вычтем его из исходного ряда, приведя его к нулевому среднему значению.

```
In [71]:
          from scipy.optimize import curve_fit
          t0 = np.arange(0, len(Fort), 1) # диапазон полного ряда
          t0 = t0.reshape(-1,1)
          popt, pcov = curve_fit(lambda t, b0, b1: b0 + b1 * t,
                                 t.reshape(1,-1)[0], Z.reshape(1,-1)[0])
          print(f"b0: {popt[0]}; b1: {popt[1]}")
          Zn = Z - (t*popt[1]+popt[0])
          Fort24 = Fort[-24:] - (t[-24:]*popt[1]+popt[0])
          plt.figure(figsize = (10, 5))
          plt.plot(t, (t*popt[1]+popt[0]), 'y')
          plt.plot(t, Z, 'k')
          plt.plot(t, Zn, 'r')
          plt.plot(t0[-24:], Fort24, 'g')
          plt.plot(t0[-24:], Fort[-24:], 'b')
          plt.show()
```

b0: 3849.2388462962995; b1: -9.589498082978835



Рассматривая различные модели АРПСС порядков от 1 до 3, полагаясь на информационные критерии, можно сделать вывод, что по точности все модели примерно одинаковые. Возьмем модель АРПСС(2, 0, 2) как наилучшую модель по информационным критерием с наименьшим количеством параметров.

```
arimaz10 = ARIMA(Zn, order = (1, 0, 0))
In [72]:
          model_fit10 = arimaz10.fit(disp = False) # подгоняем под ВР
          print(model_fit10.summary())
          arimaz20 = ARIMA(Zn, order = (2, 0, 0))
          model_fit20 = arimaz20.fit(disp = False) # подгоняем под ВР
          print(model_fit20.summary())
          arimaz30 = ARIMA(Zn, order = (3, 0, 0))
          model_fit30 = arimaz30.fit(disp = False) # подгоняем под ВР
          print(model_fit30.summary())
          arimaz11 = ARIMA(Zn, order = (1, 0, 1))
          model_fit11 = arimaz11.fit(disp = False) # подгоняем под ВР
          print(model_fit11.summary())
          arimaz12 = ARIMA(Zn, order = (1, 0, 2))
          model_fit12 = arimaz12.fit(disp = False) # подгоняем под ВР
          print(model_fit12.summary())
          arimaz13 = ARIMA(Zn, order = (1, 0, 3))
          model_fit13 = arimaz13.fit(disp = False) # подгоняем под ВР
          print(model_fit13.summary())
          arimaz21 = ARIMA(Zn, order = (2, 0, 1))
          model_fit21 = arimaz21.fit(disp = False) # подгоняем под ВР
          print(model_fit21.summary())
          arimaz22 = ARIMA(Zn, order = (2, 0, 2))
          model_fit22 = arimaz22.fit(disp = False) # подгоняем под ВР
          print(model_fit22.summary())
```

```
arimaz23 = ARIMA(Zn, order = (2, 0, 3))
model_fit23 = arimaz23.fit(disp = False) # подгоняем под ВР
print(model_fit23.summary())
arimaz31 = ARIMA(Zn, order = (3, 0, 1))
model_fit31 = arimaz31.fit(disp = False) # подгоняем под ВР
print(model_fit31.summary())
arimaz32 = ARIMA(Zn, order = (3, 0, 2))
model_fit32 = arimaz32.fit(disp = False) # подгоняем под ВР
print(model_fit32.summary())
                        ARMA Model Results
_____
                                 No. Observations:
Dep. Variable:
                                                          -1193.893
                       ARMA(1, 0) Log Likelihood
               css-mle S.D. of innovations
Mon, 18 Sep 2023 AIC
10:57:44 BIC
0 HQIC
Method:
Date:
Time:
Sample:
          coef std err z P>|z| [0.025 0.975]
      -2.6755 104.070 -0.026 0.979 -206.648 201.297
0.4900 0.072 6.835 0.000 0.349 0.630
Roots
const
ar.L1.v
_____
             Real Imaginary Modulus Frequency
            2.0410
                          +0.0000j
                                           2.0410
                       ARMA Model Results
_____
                              y No. Observations:
Dep. Variable:
Model:
                      ARMA(2, 0) Log Likelihood
               css-mle S.D. of innovations
Mon, 18 Sep 2023 AIC
10:57:44 BIC
0 HQIC
Method:
                                                           646,513
Date:
                                                           2391.275
Time:
                                                           2403.344
                                                           2396.178
Sample:
______
       coef std err z P>|z| [0.025
                                       P>|z| [0.025 0.975]
const -1.7869 87.622 -0.020 0.984 -173.523 169.949 ar.L1.y 0.5738 0.081 7.101 0.000 0.415 0.732 ar.L2.y -0.1728 0.081 -2.141 0.032 -0.331 -0.015 Roots
-----
             Real Imaginary Modulus Frequency
AR.1 1.6605 -1.7408j 2.4058
AR.2 1.6605 +1.7408j 2.4058
                                                           0.1288
-----
                       ARMA Model Results
______
Dep. Variable:

Model:

ARMA(3, 0) Log Likelihood

Method:

S.D. of impositions
                                                               151
                                                          -1191.582
               CSS-mle S.D. of innovations

Mon, 18 Sep 2023 AIC

10:57:44 BIC

0 HQIC
Method:
                                                            646.272
Date:
                                                           2393,164
                                                           2408,251
Time:
                                                           2399.293
Sample:
______
             coef std err z P > |z| [0.025 0.975]

      const
      -1.5406
      85.263
      -0.018
      0.986
      -168.653
      165.571

      ar.L1.y
      0.5688
      0.082
      6.920
      0.000
      0.408
      0.730

      ar.L2.y
      -0.1572
      0.093
      -1.685
      0.092
      -0.340
      0.026

      ar.L3.y
      -0.0273
      0.082
      -0.333
      0.739
      -0.188
      0.133
```

Roots

ARMA Model Results

css-mle S.D. of innovations
Mon, 18 Sep 2023 AIC
10:57:44 BIC
0 HQIC

-2.0308 92.282 -0.022 0.982 -182.901 178.840 0.2386 0.162 1.471 0.141 -0.079 0.557

AR.1 1.4483 -1.4613j 2.0574 AR.2 1.4483 +1.4613j 2.0574 AR.3 -8.6567 -0.0000j 8.6567

Dep. Variable:

Method:

Date:

Time:

Sample:

ar.L1.v

Real Imaginary Modulus Frequency

y No. Observations:
ARMA(1, 1) Log Likelihood

coef std err z P>|z| [0.025 0.975]

-0.1257

0.1257 -0.5000

-1191.710

646.819

2391,421

2403.490

2396.324

ma.L1.y	0.3399	0.162	2.101 coots	0.036	0.023	0.65
	Real	======= Imagi	===== nary	Modulus		Frequency
AR.1 MA.1	4.1915 -2.9422	+0.0 +0.0	000j 000j	4.1915 2.9422		0.0000 0.5000
		ARMA Mo	del Res			
Dep. Variable:		V	No	Ohservations:		15
Model:		ARMA(1, 2)	Log	Likelihood		-1186.09
Method: Date:	Mon	css-mle 18 Sep 2023	S.D.	of innovations		617.44 2382.19
Time:	11011,	10:57:45				2397.27
Sample:		10.37.43		:		2388.32
========		======= std err		P> z		
	-0.5526 0.8027		-0.065 15.107		-17.150 0.699	
•	-0.4065		-5.123		-0.562	
	-0.5934	0.076	-7.782		-0.743	-0.44
========				:======================================		
	Real		nary	Modulus		Frequency
AR.1	1.2457		000j	1.2457		0.0000
MA.1 MA.2	1.0001 -1.6850		000j 000j	1.0001 1.6850		0.0000 0.5000
======= Dep. Variable: Model:	:		======	Observations:	======	 15
Model:		ARMA(1, 3)				-1183.89
Method:				of innovations		612.34
Date:	Mon,	18 Sep 2023				2379.78
Time:		10:57:45	BIC			2397.88
Sample:		0	HQIO			2387.13
========		======= std err		P> z		 0.975
 const	1.3470		0.094		 -26.867	 29.56
			5.383		0.443	0.95
	-0.2834		-2.172		-0.539	-0.02
•	-0.3994		-3.928		-0.599	
ma.L3.y	-0.2484		-2.348 oots	0.019	-0.456	-0.04
========	 Real	 ======== Imagi		Modulus	======	Frequency
AR.1	1.4346		 000j			
MA.1	1.0368		000j	1.4346 1.0368 1.9705		0.0000 -0.0000
	-1.3222		610j	1.9705		-0.3671
MA.3	-1.3222	+1.4	610j	1.9705		0.3671
		ARMA Mo	del Res			
Dep. Variable:		у	No.	Observations:		15
Model: Method:		AKMA(2, 1)	Log	Likelihood of innovations		-1177.43 586.45
Date:	Mon.	18 Sep 2023	ATC	OI IIIIOVACIONS		2364.87
Time:	,	10:57:45	BIC			2379.96
Sample:		0	HQIO			2371.00
				P> z		
				P> z		
ar.L1.v	4.4824 1.3378	0.067	20.202	0.793 0.000	1.207	5/.95 1 46
ar.L2.v	-0.6152	0.064	-9.554	0.000 0.000	-0.741	-0.48
ma.L1.y	-0.9072	0.034 -	27.070 oots	0.000	-0.973	-0.84
			=====	.========== Modulus		
				Modulus		
AR.1 AR.2	1.0873 1.0873		658j 658j	1.2750 1 2750		-0.0874 0.0874
MA.1	1.1023	+0.0	000j	1.2750 1.2750 1.1023		0.0000
		ARMA Mo	del Res	ults		
======= Dep. Variable:		У	No.	Observations:		15
Model:		ARMA(2, 2)	Log	Likelihood		-1156.84
Method:		css-mle	S.D.	of innovations		505.55
Date:	Mon,					2325.68
Time: Sample:		10:57:45 0	HQIC	:		2343.78 2333.03
========		=======		.========		
	coef	std err	z	P> z	[0.025	0.975

ar.12.y	an.12.y	const	3.8869	34.248	0.113	0.910	-63.239	71.01
ar.1.2 y	an.1.y							1.74
Real Imaginary	Real	-	-0.9964	0.004	-226.644	0.000	-1.005	-0.98
Real Imaginary Modulus Frequency	Real Imaginary Modulus Frequency	ma.L1.y	-1.7183	0.034	-50.006	0.000	-1.786	-1.65
Real Imaginary Modulus Frequency	Real Imaginary Modulus Frequency	ma.L2.y	0.9388	0.030		0.000	0.880	0.99
AR. 1 0.8684	AR.1 0.8684	========			======			
AR.2	AR.2							
MA.1	MA.1							
MA.2 0.9151	ARMA Model Results Dep. Variable: V No. Observations: 1: Model: ARMA(2, 3) Log Likelihood -1:158, 2:							
Dep. Variable:	Dep. Variable:	MA.2	0.9151	+0	.4772j	1.0321		0.0765
Dep. Variable:	Dep. Variable: Who dod: ARMA(2, 3) Log Likelishood -1158, 22							
Model: ARMA(2, 3) Log Likelihood -1158.22	Model: ARMA(2, 3) Log Likelihood -1158.2			=======				
Method: css-mle S.D. of innovations 512.91 bate: Mon, 18 Sep 2023 AIC 2330.46 Time: 10:57:46 BIC 2351.57 Sample: 0 HQIC 2339.03 AIC 2350.45 AIR 2350.4	Method: css-mle S.D. of innovations 512.91 Date: Mon, 18 Sep 2023 AIC 2334.61 Time: 10:57:46 BIC 2351.55 Sample: 0 MQIC 2339.02 coef std err z P> z [0.025 0.97] const 3.9227 21.506 0.182 0.855 -38.229 46.02 ar.ll.y 1.6908 0.028 60.292 0.000 1.636 1.74 ar.l2.y -0.9394 0.031 -30.422 0.000 -1.000 -0.83 an.l1.y -1.5360 0.104 -14.755 0.000 -1.740 -1.33 an.l2.y 0.4576 0.224 2.046 0.041 0.019 0.83 an.l3.y 0.2956 0.143 1.439 0.150 -0.074 0.48 an.l3.y 0.2956 0.143 1.439 1.0906 -0.074 0.48 an.l3.y 0.2956 0.143 1.439 1.0696 -0.052 AR.1 0.88999 -0.50465 1.0317 0.0811 AR.2 0.88999 +0.50465 1.0317 0.0811 AR.2 0.8999 +0.50465 1.0317 0.0811 AR.3 -4.2518 -0.0000 4.2518 -0.5000 ARMA Model Results Dep. Variable: y No. Observations: 581.35 Date: Mon, 18 Sep 2023 AIC 2364.31 Date: Mon, 18 Sep 2023 AIC 2371.73 coef std err z P) z [0.025 0.972 const 4.9579 16.909 0.293 0.769 -28.184 38.06 ar.l1.y 1.2441 0.087 14.227 0.000 1.073 1.43 ar.l1.y 1.2441 0.087 14.227 0.000 1.073 1.43 ar.l1.y 0.4154 0.084 -1.003 0.199 -0.301 0.03 ar.l2.y -0.4569 0.128 -3.469 0.000 -0.959 -0.88 AR.3 -5.2451 0.0000 5.2281 1.1864 0.088 AR.3 -5.2451 0.0000 5.2581 0.0000 1.527 0.000 ARMA Model Results Coef std err z P. z [0.025 0.972 const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.11.y 1.7665 0.122 14.451 0.000 1.527 0.000 ARMA Model Results Coef std err z P. z [0.025 0.972 const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.11.y 1.7665 0.122 14.451 0.000 1.527 0.000 ar.12.y 0.0627 0.129 0.486 0.627 -0.190 0.33 ar.12.y 0.0627 0.129 0.486 0.627 -0.190 0.33 ar.11.y 1.7608 0.000 0.729 0.93 ar.12.y 0.0000 0.729 0.93 ar.12.y 0.			ARMA(2.	3) Log	Likelihood		
Date: Mon, 18 Sep 2023 AIC 2336.45 Time: 10:57:46 BIC 2551.5 Sample: 0 HQIC 2339.03 Coef std err	Date: Mon, 18 Sep 2023 AIC 2335.45 Time: 10:57:46 BIC 2351.55 Sample: 0 HQIC 2339.05 Coef std err z P z [0.025 0.97]			css-m	ile S.D.	of innovations		
Time:	Coef std err Z P Z	Date:	Mon,					2330.45
Coef std err Z P3 Z [0.025 0.975	Coef std err Z P ₂ Z [0.025 0.97]	Time:		10:57:	46 BIC			2351.57
Coef std err Z P Z [0.025 0.975	Coef std err z P> z [0.025 0.97]	Sample:			0 HQIC			2339.03
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Dep. Variable: y No. Observations: 15 Model: ARMA(3, 2) Log Likelihood -1158.88 Method: css-mle S.D. of innovations 515.65 Date: Mon, 18 Sep 2023 AIC 2331.76 Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34	Dep. Variable: y No. Observations: 15 Model: ARMA(3, 2) Log Likelihood -1158.88 Method: css-mle S.D. of innovations 515.66 Date: Mon, 18 Sep 2023 AIC 2331.76 Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34	AR.2 AR.3	1.0098 1.0098 -5.2451	-0 +0 -0	.6228j .6228j .0000j	1.1864 1.1864		-0.0880 0.0880 -0.5000
Model: ARMA(3, 2) Log Likelihood -1158.88 Method: css-mle S.D. of innovations 515.65 Date: Mon, 18 Sep 2023 AIC 2331.76 Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34 coef std err z P> z [0.025 0.975 const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.06 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.68 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.51 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roats	Model: ARMA(3, 2) Log Likelihood -1158.88 Method: css-mle S.D. of innovations 515.65 Date: Mon, 18 Sep 2023 AIC 2331.76 Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34 coef std err z P> z [0.025 0.975 const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.06 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.53 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059i 1.0213 -0.0825	AR.2 AR.3 MA.1	1.0098 1.0098 -5.2451 1.1246	-0 +0 -0 +0	.6228j .6228j .0000j .0000j	1.1864 1.1864 5.2451 1.1246		-0.0886 0.0886 -0.5006 0.0006
Method: css-mle S.D. of innovations 515.65 Date: Mon, 18 Sep 2023 AIC 2331.76 Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34	Method: css-mle S.D. of innovations 515.65 Date: Mon, 18 Sep 2023 AIC 2331.76 Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34	AR.2 AR.3 MA.1	1.0098 1.0098 -5.2451 1.1246	-0 +0 -0 +0 ARMA	.6228j .6228j .0000j .0000j Model Res	1.1864 1.1864 5.2451 1.1246		-0.0886 0.0886 -0.5006 0.0006
Date: Mon, 18 Sep 2023 AIC 2331.76 Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34	Date: Mon, 18 Sep 2023 AIC 2331.76 Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34	AR.2 AR.3 MA.1 E===============================	1.0098 1.0098 -5.2451 1.1246	-0 +0 -0 +0 ARMA	0.6228j 0.6228j 0.0000j 0.0000j Model Res y No.	1.1864 1.1864 5.2451 1.1246 ults =		-0.0886 0.0886 -0.5000 0.0000
Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34 coef std err z P> z [0.025 0.975 const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.51 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	Time: 10:57:46 BIC 2352.88 Sample: 0 HQIC 2340.34 coef std err z P> z [0.025 0.975 const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.33 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.55 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1 Dep. Variable: Model:	1.0098 1.0098 -5.2451 1.1246	-0 +0 -0 +0 ARMA 	.6228j .6228j .0000j .0000j Model Res y No. 2) Log	1.1864 1.1864 5.2451 1.1246 		-0.0886 0.0886 -0.5006 0.0006
Sample: 0 HQIC 2340.34 coef std err z P> z [0.025 0.975] const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.06 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.51 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	Coef std err z P> z [0.025 0.975] const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.06 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.55 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1 Dep. Variable: Model: Method:	1.0098 1.0098 -5.2451 1.1246	-0 +0 -0 +0 ARMA ARMA(3,	.6228j .6228j .0000j .0000j Model Res y No. 2) Log le S.D.	1.1864 1.1864 5.2451 1.1246 		-0.0886 0.0886 -0.5006 0.0006
coef std err z P> z [0.025 0.975] const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.51 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	coef std err z P> z [0.025 0.975] const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.Ll.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.33 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.55 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1 Dep. Variable: Model: Method: Date:	1.0098 1.0098 -5.2451 1.1246	-0 +0 -0 +0 ARMA ARMA(3,	.6228j .6228j .0000j .0000j Model Res y No. 2) Log le S.D.	1.1864 1.1864 5.2451 1.1246 		-0.0886 0.0886 -0.5000 0.0000
Const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.Ll.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.51 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	Const 3.4911 22.289 0.157 0.876 -40.195 47.17 ar.Ll.y 1.7665 0.122 14.451 0.000 1.527 2.06 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.51 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1 Dep. Variable: Model: Method: Date: Time:	1.0098 1.0098 -5.2451 1.1246	-0 +0 -0 +0 ARMA ARMA(3,	.6228j .6228j .0000j .0000j 	1.1864 1.1864 5.2451 1.1246 		-0.0886 0.0886 -0.5000 0.0000
ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.51 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots	ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.33 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.55 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots	AR.2 AR.3 MA.1	1.0098 1.0098 -5.2451 1.1246 	-0 +0 -0 +0 	.6228j .6228j .60200j .00000j .00000j 	1.1864 1.1864 5.2451 1.1246 		-0.0886 0.0886 -0.5006 0.0006
ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.31 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.51 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots	ar.L1.y 1.7665 0.122 14.451 0.000 1.527 2.00 ar.L2.y -1.0699 0.207 -5.158 0.000 -1.476 -0.66 ar.L3.y 0.0627 0.129 0.486 0.627 -0.190 0.33 ma.L1.y -1.6988 0.094 -17.982 0.000 -1.884 -1.55 ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots	AR.2 AR.3 MA.1	1.0098 1.0098 -5.2451 1.1246 Mon,	-0 +0 -0 +0 	.6228j .6228j .6028j .0000j .0000j .0000j 	1.1864 1.1864 5.2451 1.1246 	 [0.025	-0.0886 0.0886 -0.5000 0.0006
ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1	1.0098 1.0098 -5.2451 1.1246 	-0 +0 -0 +0 	.6228j .6228j .6028j .0000j .0000j y Model Res y No. 2) Log y No. 2) Log y A IC 46 BIC 0 HQIC	1.1864 1.1864 5.2451 1.1246 	[0.025	-0.0886 0.0886 -0.5006 0.0006
ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1	1.0098 1.0098 -5.2451 1.1246 	-0 +0 -0 +0 	.6228j .6228j .6028j .0000j .0000j y Model Res y No. 2) Log y No. 2) Log y A IC 46 BIC 0 HQIC	1.1864 1.1864 5.2451 1.1246 	[0.025	-0.0886 0.0886 -0.5006 0.0006
ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1	1.0098 1.0098 -5.2451 1.1246 	-0 +0 -0 +0 	.6228j .6228j .6028j .0000j .0000j y Model Res y No. 2) Log y No. 2) Log y A IC 46 BIC 0 HQIC	1.1864 1.1864 5.2451 1.1246 	[0.025	-0.0886 0.0886 -0.5006 0.0006
ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	ma.L2.y 0.8251 0.049 16.766 0.000 0.729 0.92 Roots Real Imaginary Modulus Frequency AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1	1.0098 1.0098 -5.2451 1.1246 	-0 +0 -0 +0 	.6228j .6228j .6028j .0000j .0000j y Model Res y No. 2) Log y No. 2) Log y A IC 46 BIC 0 HQIC	1.1864 1.1864 5.2451 1.1246 	[0.025	-0.0886 0.0886 -0.5006 0.0006
Real Imaginary Modulus Frequency	Real Imaginary Modulus Frequency	AR.2 AR.3 MA.1	1.0098 1.0098 1.0098 -5.2451 1.1246 Mon, coef 3.4911 1.7665 -1.0699 0.0627 -1.6988	-0 +0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0	.6228j .6228j .6228j .0000j .0000j .0000j 	1.1864 1.1864 5.2451 1.1246 	[0.025 -40.195 1.527 -1.476 -0.190 -1.884	-0.0886 0.0886 -0.5006 0.0006
AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.2 AR.3 MA.1	1.0098 1.0098 1.0098 -5.2451 1.1246 Mon, coef 3.4911 1.7665 -1.0699 0.0627 -1.6988	-0 +0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0 -0	0.6228j 0.6228j 0.6028j 0.0000j 0.0000j 	1.1864 1.1864 5.2451 1.1246 	[0.025 -40.195 1.527 -1.476 -0.190 -1.884	-0.0886 0.0886 -0.5006 0.0006
AR.1 0.8872 -0.5059j 1.0213 -0.0825	AR.1 0.8872 -0.5059j 1.0213 -0.0825 AR.2 0.8872 +0.5059j 1.0213 0.0825	AR.2 AR.3 MA.1	1.0098 1.0098 1.0098 -5.2451 1.1246 Mon, coef 3.4911 1.7665 -1.0699 0.0627 -1.6988 0.8251	ARMA(3, CSS-m 18 Sep 20 10:57: std err 22.289 0.122 0.207 0.129 0.094 0.049	.6228j .6228j .6228j .0000j .0000j 	1.1864 1.1864 5.2451 1.1246 	[0.025 -40.195 1.527 -1.476 -0.190 -1.884 0.729	-0.0886 0.0886 -0.5006 0.0006
	AR.2 0.8872 +0.5059j 1.0213 0.0825	AR.2 AR.3 MA.1	1.0098 1.0098 1.0098 -5.2451 1.1246 Mon, coef 3.4911 1.7665 -1.0699 0.0627 -1.6988 0.8251	ARMA(3, css-m 18 Sep 20 10:57:	.6228j .6228j .6228j .0000j .0000j .0000j 	1.1864 1.1864 5.2451 1.1246 ults ====================================	[0.025 -40.195 1.527 -1.476 -0.198 0.729	-0.0880 0.0888 -0.5000 0.0000

```
AR.3 15.2995 -0.0000j 15.2995 -0.0000
MA.1 1.0295 -0.3901j 1.1009 -0.0577
MA.2 1.0295 +0.3901j 1.1009 0.0577
```

```
In [73]: arimaz22 = ARIMA(Zn, order = (2, 0, 2))
model_fit22 = arimaz22.fit(disp = False) # подгоняем под ВР
print(model_fit22.summary())
```

ARMA Model Results

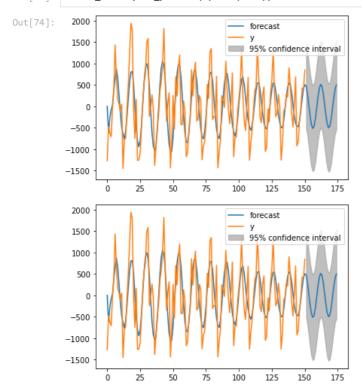
Dep. Variable:	у	No. Observations:	151
Model:	ARMA(2, 2)	Log Likelihood	-1156.842
Method:	css-mle	S.D. of innovations	505.551
Date:	Mon, 18 Sep 2023	AIC	2325.684
Time:	10:57:49	BIC	2343.788
Sample:	0	HQIC	2333.039

	coef	std err	Z	P> z	[0.025	0.975]
const	3.8869	34.248	0.113	0.910	-63.239	71.013
ar.L1.y	1.7305	0.005	337.486	0.000	1.720	1.741
ar.L2.y	-0.9964	0.004	-226.644	0.000	-1.005	-0.988
ma.L1.y	-1.7183	0.034	-50.006	0.000	-1.786	-1.651
ma.L2.y	0.9388	0.030	31.306	0.000	0.880	0.998
			Roots			

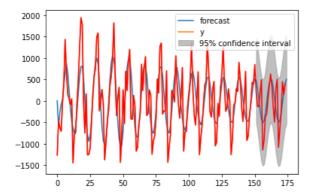
=======	Real Imaginary		Modulus	Frequency
AR.1	0.8684	-0.4995j	1.0018	-0.0831
AR.2	0.8684	+0.4995j	1.0018	0.0831
MA.1	0.9151	-0.4772j	1.0321	-0.0765
MA.2	0.9151	+0.4772j	1.0321	0.0765

Построим график прогноза по данной модели вместе с доверительными интервалами:

```
In [74]: model_fit22.plot_predict(0, len(Fort))
```



Рассмотрим как этот прогноз по АРПСС модели соотносится с исходными известными 24 прогнозными точками. Вычтем из исходного ряда Fort линейный тренд и соотнесем их на одном изображении:



Получим прогнозные значения по модели АРПСС и используем эти значения для оценки точности прогноза:

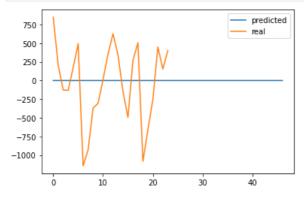
```
In [76]:
          # Средняя ошибка прогноза
          def MAE(pred, real):
            return np.mean(np.abs(pred - real))
          # СКВО прогноза
          def MSD(pred, real):
            return np.sqrt(np.mean((pred - real)**2))
          # Средняя ошибка аппроксимации
          def MAPE(pred, real):
            return np.mean(np.abs((real - pred) / real))
          def res_evaluation(pred, real, title=""):
            print(
                f"Оценка точности прогноза {title}\n"
                f"Средняя ошибка прогноза: {np.round(MAE(pred, real),2)}\n",
                f"CKBO прогноза: {np.round(MSD(pred, real),2)}\n",
                f"Средняя ошибка аппроксимации: {np.round(MAPE(pred, real),2) *100}%\n"
```

```
In [77]: res_evaluation(model_fit22.predict(len(Z), len(Fort)), (Fort-(trend_as_func_of_t0))[-24:])
```

```
Оценка точности прогноза
Средняя ошибка прогноза: 517.42
СКВО прогноза: 642.58
Средняя ошибка аппроксимации: 229.99999999997%
```

Можно заметить, что средняя ошибка аппроксимации для данной модели больше 100 процентов. Это означает, что ошибки намного больше, чем фактические значения. Тем не менее, метрики MAE и MSD все еще достаточно информативны.

```
In [78]: plt.plot(model_fit.predict(len(Z), len(Fort)), label='predicted')
    plt.plot((Fort-(trend_as_func_of_t0))[-24:], label='real')
    plt.legend()
    plt.show()
```



Также попробуем построить АРПСС модель для прогнозирования данного ряда, но без исходного вычитания из него линейного тренда:

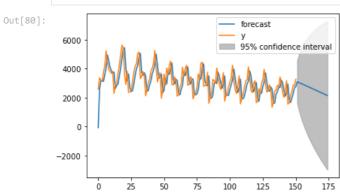
```
In [79]: arimaz22r = ARIMA(Z, order = (2, 0, 2))
model_fit22r = arimaz22r.fit(disp = False) # подгоняем под ВР
print(model_fit22r.summary())
```

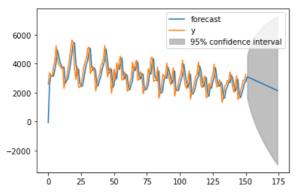
```
ARMA Model Results
Dep. Variable:
                                         No. Observations:
                                         Log Likelihood
                                                                       -1215.319
Model:
                           ARMA(2, 2)
Method:
                              css-mle
                                         S.D. of innovations
                                                                        744.434
                     Mon, 18 Sep 2023
Date:
                                         AIC
                                                                        2442.639
Time:
                             10:58:30
                                         BIC
                                                                        2460.742
Sample:
                                         HQIC
                                                                        2449.993
```

	coef	std err	z	P> z	[0.025	0.975]	
const	-70.7118	2143.390	-0.033	0.974	-4271.678	4130.255	
ar.L1.y	1.9950	nan	nan	nan	nan	nan	
ar.L2.y	-0.9951	nan	nan	nan	nan	nan	
ma.L1.y	-1.2469	0.107	-11.693	0.000	-1.456	-1.038	
ma.L2.y	0.2469	0.106	2.322	0.020	0.039	0.455	
			Roots				

	Real	Imaginary	Modulus	Frequency
AR.1	1.0024	-0.0101j	1.0024	-0.0016
AR.2	1.0024	+0.0101j	1.0024	0.0016
MA.1	1.0000	+0.0000j	1.0000	0.0000
MA.2	4.0498	+0.0000j	4.0498	0.0000

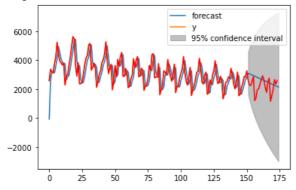
In [80]: model_fit22r.plot_predict(0, len(Fort))





```
In [81]: plt.figure(figsize = (10, 5))
    model_fit22r.plot_predict(0, len(Fort)) # прогноз по АРПСС
    plt.plot(t0, Fort, 'r')
    plt.show()
```

<Figure size 720x360 with 0 Axes>



В данном случае значение метрик MAE и MSD немного хуже, чем для рядов с около-нулевым средним. При этом теперь значение метрики MAPE имеет смысл.

```
In [82]: res_evaluation(model_fit22r.predict(len(Z), len(Fort)), Fort[-24:])
```

Оценка точности прогноза Средняя ошибка прогноза: 546.65 СКВО прогноза: 703.72 Средняя ошибка аппроксимации: 31.0%