The 5th Homework of Optics

肖涵薄 31360164

2018 年 12 月 24 日

4-5

a. $\frac{I_0}{I} = \left(\sqrt{2}\right)^2 = 2$

b. $\frac{I_0}{I} = \left(\sqrt{2}\right)^2 = 2$

c. $\frac{I_0}{I} = (1/2)^2 = 1/4$

d. $\frac{I_0}{I} = (2 - 2/2)^2 = 1$

e. $\frac{I_0}{I} = \left(\sqrt{2^2 + 1}\right)^2 = 5$

f. $\frac{I_0}{I} = \left(2 \times \frac{3}{4} - 2 + 3/4\right)^2 = 1/16$

4-7

$$I = (50 \times 2 - 1)^2 = 99^2$$

4-9

$$\rho_1^2 = f\lambda \implies \rho = \frac{2\sqrt{5}}{5}mm \approx 0.57mm$$

$$I/I_0 = 1000 = (2k/2)^2 \implies k \approx 32$$

$$\rho_k = \sqrt{32 \times 800 \times 400 \times 10^{-6}} = 3.2mm$$

有效半径为 $3.2mm$

4-10

$$\rho_1^2 = f\lambda \implies 900 \times 30 = 632.8 f' \implies f' = 43cm$$

4-11

(1)

考虑 θ 影响：

$$U = C \int_{-a/2}^{a/2} e^{ik\Delta r}\mathrm{d}x = C\frac{e^{-ikx\sin\theta}}{-ik\sin\theta}\bigg|_{x=-a/2}^{x=a/2}$$

代入 $k\lambda = 2\pi$，

$$U = 2C\frac{\sin\alpha}{\alpha}, \quad \alpha = \frac{\pi a\left(\sin\theta - \sin\theta_0\right)}{\lambda}$$

$$\implies I = U^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2, \quad \alpha = \frac{\pi a\left(\sin\theta - \sin\theta_0\right)}{\lambda}$$

(2)

零级中心即 $\alpha = 0$，即各光线无光程差，根据费马原理，此点即几何光学像点。

(3)

根据 (1) 中式子，暗斑出现在 $\sin\alpha = 0, a \neq 0$ 处，即 $\alpha = k\pi\ (k \neq 0)$。
一级暗斑即零级半角宽，$\frac{\pi a(\sin\theta - \sin\theta_0)}{\lambda} = \pi$

$$\implies \sin\left(\Delta\theta + \theta_0\right) - \sin\theta_0 \approx \lambda/a \implies \cos\theta_0 \cdot \Delta\theta \approx \lambda/a$$

$$\Delta\theta = \frac{\lambda}{\cos\theta_0 a}$$

(4)

在衍射处发生折射，$n = \frac{\sin\theta}{\sin\theta'}$，使用 θ' 代替上式所有 θ 即可，此时：
(1)

$$\implies I = U^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2, \quad \alpha = \frac{\pi a\left(\sin\left(\theta/n\right) - \sin\left(\theta_0/n\right)\right)}{\lambda}$$

(3)

$$\Delta\theta = \frac{\lambda}{\cos\left(\theta_0/n\right)a}$$

4-12

(1)

反射

$$\Delta\theta = \lambda/D = 0.6/10000*180/Pi*60*60 = 12.4''$$

折射

$$\Delta\theta = \lambda/nD = 0.6/10000*180/Pi*60*60 = 8.2''$$

(2)

反射

$$\Delta\theta = \lambda/D = 0.6/10000*180/Pi*60*60/\cos 75 = 47.4''$$

折射

$$\Delta\theta = \lambda/nD = 0.6/10000*180/Pi*60*60/\cos 75 = 10.7''$$

(2)

反射

$$\Delta\theta = \lambda/D = 0.6/10000*180/Pi*60/\cos 89 = 66'40''$$

折射

$$\Delta\theta = \lambda/nD = 0.6/10000*180/Pi*60/\cos 89 = 11'1''$$

4-17

(1)

取 $\lambda = 550nm$

$$\delta y = 0.61\lambda/N.A. = 0.25\mu m$$

(2)

$$V = \frac{0.075}{0.25/1000} = 290$$

(3)

$$V = -\frac{s_0\Delta}{f_O f_E} \implies \Delta = 290 * 1.91 * 50/250 = 111mm$$

4-19

$$\Delta\theta = 1.22\lambda/D = 6.7 \times 10^{-7} \implies \Delta s = s\Delta\theta = 255m$$

4-23

$$U = C \int_{-a/2}^{a/2} \mathrm{d}x \int_{-b/2}^{b/2} e^{ik\Delta r} \mathrm{d}y$$

$$= C \int_{-a/2}^{a/2} e^{-ik\Delta x} \mathrm{d}x \int_{-b/2}^{b/2} e^{-ik\Delta y} \mathrm{d}y$$

$$= abC \frac{e^{-ikx\sin\theta}}{-ik\sin\theta}\bigg|_{x=-a/2}^{x=a/2} \frac{e^{-iky\sin\theta}}{-ik\sin\theta}\bigg|_{y=-b/2}^{y=b/2}$$

令

$$\alpha' = \frac{ka}{2}\left(\sin\theta - \sin\theta_0\right)$$

$$\beta' = \frac{kb}{2}\left(\sin\theta - \sin\theta_0\right)$$

$$\implies I = U^2 = I_0 \left(\frac{\sin\alpha'}{\alpha'}\right)^2 \left(\frac{\sin\beta'}{\beta'}\right)^2$$

4-25

$$U = [U(0) + U(-d) + U(-3d)]\frac{\sin\alpha}{\alpha}$$

$$U(-d) = U(0)\frac{\sin\alpha}{\alpha}, \alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi a d/f}{\lambda}$$

$$U = U_0\frac{\sin\alpha}{\alpha}\sqrt{(1 + \cos\beta + \cos 3\beta)^2 + (\sin\beta + \sin 3\beta)^2}$$

$$I = U^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 [3 + 2(\cos 2\beta + \cos 4\beta + \cos 6\beta)]$$

4-27

(1)

$$\implies I = U^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2$$

(2)

$$\implies I = U^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2$$

(3)

$$\implies I = U^2 = 4I_0\cos^2 2\alpha\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2$$

4-32

(1)

$$k_{max} = d/\lambda = 2$$

$$\delta\lambda = \lambda/kN = 0.05nm$$

(2)

$$1/D_\theta = 1/\frac{k}{d\cos\theta_k} = 0.244nm/(')$$

(3)

$$\theta_b \approx 12°39'$$

与闪耀方向与光栅法线角度相同。

4-34

	光栅	棱镜	F-P 腔
角分辨	3×10^4	3×10^3	6×10^5
角色散	$2.2'/nm$	$0.31'/nm$	$39'/nm$
自由光谱	一级$850nm - 1700nm$	$--$	$\lambda = 550nm : 0.003nm$