数值分析第一次作业

肖涵薄 31360164

2019 年 3 月 2 日

1. 误差限为 $\varepsilon = \frac{1}{2} \times 10^{-4} > I^* - I = 0.000012$, 则有效数字为 4.

2.

$$\varepsilon_{rV} = \frac{\frac{\mathrm{d}V}{\mathrm{d}R} e_R}{V} = 3\frac{e_R}{R} \implies \frac{e_R}{R} = 0.33\%$$

3.

$$x_1 = 28 + \sqrt{783} \approx 28 + 27.982 = 55.982$$

$$x_2 = 28 - \sqrt{783} = \frac{1}{28 + \sqrt{783}} \approx \frac{1}{28 + 27.982} \approx 0.017863$$

4.

(1) 都为 6.

(2) 真实值为 $\sqrt{2018} - \sqrt{2017} \approx 0.0111317$. $x - y = 0.0112 \approx 0.011$, 2 位有效数字.

(3) $\frac{1}{x+y} = 1/89.8332 = 0.0111317$. 6 位有效数字.

5. 记 $\delta = \sqrt{783} - 27.982 \approx 0.000137$, 每次计算产生误差 $\delta/100$. 100 次后误差为 δ, 则误差限为 $\varepsilon = 0.5 \times 10^{-3} < \delta$. 有 3 位有效数字.

6. 设 $x = \sqrt{2}$, $e_x = \sqrt{2} - 1.4$.

$$\begin{aligned}
e_1 &= \left|\tfrac{\partial(1+x)^{-6}}{\partial x}\right| e_x & e_3 &= \left|\tfrac{\partial(3+2x)^{-3}}{\partial x}\right| e_x \\
&= 6\,(1+x)^{-7}\, e_x & &= 3\,(3+2x)^{-4}\, e_x \\
&\approx 1.3 \times 10^{-2} e_x & &\approx 2.6 \times 10^{-3} e_x \\
e_2 &= \left|\tfrac{\partial(3-2x)^3}{\partial x}\right| e_x & e_4 &= \left|\tfrac{\partial 99-70x}{\partial x}\right| e_x \\
&= 6\,(3-2x)^2\, e_x & & \\
&\approx 1.2 e_x & &= 70 e_x
\end{aligned}$$

可知 $\frac{1}{(3+2\sqrt{2})^3}$ 式结果最好.