数值分析第十次作业

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(1) 显式欧拉公式为

$$y_{n+1} = y_n + h(x_n + 1)$$

用右矩形公式, 隐式欧拉公式为

$$y_{n+1} = y_n + h(x_{n+1} + 1)$$

(2) 显式欧拉公式的局部阶段误差为

$$T_{n+1} = y\left(x_{n+1}\right) - y_{n+1} = \frac{y''(\xi)}{2!}h^2 + \mathcal{O}(h^3) \approx \frac{h^2}{2}$$

隐式欧拉公式的局部阶段误差为

$$T_{n+1} = -\frac{h^2}{2}y''\left(x_n\right) + \mathcal{O}\left(h^3\right) \approx -\frac{h^2}{2}$$

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(1) $f(x, y) = -y$. 用显式欧拉公式:

$$y(0.1) = y(0) - 0.1 = 0.9$$

$$y(0.2) = y(0.1) - 0.09 = 0.81$$

用隐式欧拉公式:

$$y(0.1) = y(0) - 0.1 * y(0.1) \implies y(0.1) = \frac{1}{1.1}$$

$$y(0.2) = y(0.1) - 0.1 * y(0.2) \implies y(0.2) = \frac{1}{1.1^2} \approx 0.826$$

用梯形公式:

$$y(0.1) = 1 + \frac{0.1}{2}[-1 - y(0.1)] \implies y(0.1) = \frac{19}{21}$$

$$y(0.2) = \frac{19}{21} + \frac{0.1}{2}[-\frac{19}{21} - y(0.2)] \implies y(0.1) = \frac{19^2}{21^2} = 0.8186$$

(2) 根据第一问所证, 当 $n = 1$, 满足上式, 若 $n = k$ 满足上式, 则对于 $n = k + 1$,

$$y_{k+1} = y_k - \frac{h}{2}[y_k + y_{k+1}] \implies y_{k+1} = \frac{2-h}{2+h}y_k = (\frac{2-h}{2+h})^{k+1}$$

依旧满足上式, 因此得证.

$y_k = y(hk)$.

$$\lim_{k\to 0} y(hk) = \lim_{k\to 0}(\frac{2-h}{2+h})^k$$

设 $x = hk$

$$\lim_{k\to 0} y(x) = \lim_{k\to 0}(1 - \frac{2h}{2+h})^{\frac{x}{h}} = \lim_{k\to 0}(1 - \frac{2h}{2+h})^{\frac{x}{h}}$$

忽略高阶无穷小,

$$= \lim_{k\to 0}(1-h)^{\frac{x}{h}} = \left(\lim_{k\to 0}(1-h)^{-h}\right)^{-x} = e^{-x}$$

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$$T_{n+1} = y_n + hy_n' + \frac{h^2}{2}y_n'' + \frac{h^3}{6}y_n''' - y_n - 2hK_2 + hK_1$$

$$= y_n + hy_n' + \frac{h^2}{2}y_n'' + \frac{h^3}{6}y_n''' - y_n - 2h(y_n' + \frac{h}{4}y_n'' + \frac{h^2}{16}y_n''') + hy_n'$$

$$= \frac{1}{24}h^3y_n'''$$