The 14th HW of Electrodynamics

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夫琅禾费衍射式满足

$$\psi(\vec{r}) = -\frac{i\psi_0 e^{ikr}}{4\pi r}\iint_{S_0} e^{i\left(\vec{k}_1-\vec{k}_2\right)\cdot\vec{r'}}\left(\cos\theta_1+\cos\theta_2\right)dS'$$

考虑

$$\theta_1 = 0$$

此时 $\cos\theta_1 = 1, \vec{k}_1\cdot\vec{r'} = 0$, 则

$$\psi(\vec{r}) = -\frac{i\psi_0 e^{ikr}}{4\pi r}\iint_{S_0} e^{-i\vec{k}_2\cdot\vec{r'}}(1+\cos\theta_2)\,\mathrm{d}S'$$

$$\psi(\vec{r}) = -\frac{i\psi_0 e^{ikr}}{4\pi r}(1+\cos\theta_2)\iint_{S_0} e^{-i\vec{k}_2\cdot\vec{r'}}\,\mathrm{d}S' \tag{1}$$

现在需要计算积分

$$\iint_{S_0} e^{-i\vec{k}_2\cdot\vec{r'}}\,\mathrm{d}S' = \int_0^R r\,\mathrm{d}r\int_{-\pi}^{\pi} e^{-ik_2 r\cos\theta}\,\mathrm{d}\theta$$

根据 Bessel 函数的积分表示定义

$$J_\alpha(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(\alpha\tau - x\sin\tau)}d\tau$$

令 $\alpha = 0,\ x = k_2 r,\ \theta = \theta + \pi/2$, 易得

$$\begin{aligned}\iint_{S_0} e^{-i\vec{k}_2\cdot\vec{r'}}\,\mathrm{d}S' &= \int_0^R r\,\mathrm{d}r \int_{-\pi}^{\pi} e^{-ik_2 r\cos\theta}\,\mathrm{d}\theta \\ &= 2\pi\int_0^R rJ_0(k_2 r)\,\mathrm{d}r \\ &= 2\pi\frac{1}{k_2}\int_0^R \mathrm{d}\left(rJ_1(k_2 r)\right) \\ &= 2\pi\frac{1}{k_2} rJ_1(k_2 r)\big|_0^R \\ &= \frac{2\pi}{k_2} RJ_1(k_2 R)\end{aligned}$$

将积分回代到 (1) 中

$$\begin{aligned}\psi(\vec{r}) &= -\frac{i\psi_0 e^{ikr}}{4\pi r}(1+\cos\theta_2)\frac{2\pi}{k_2}RJ_1(k_2 R) \\ &= -\frac{i\psi_0 e^{ikr}R}{2k_2 r}(1+\cos\theta_2)J_1(k_2 R)\end{aligned}$$