The 9th HW of Electrodynamics

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Two independent monochromatic electromagnetic waves with electric fields perpendicular to each otherare traveling in vacuum along the same direction.

设两个波分别为 $E_1 = e^{i(kx-\omega t)}\boldsymbol{e}_1, E_2 = e^{i(kx-\omega t+\varphi)}\boldsymbol{e}_2$.

则合波为:$E = E_1 + E_2 = e^{i(kx-\omega t)}\boldsymbol{e}_1 + e^{i(kx-\omega t+\varphi)}\boldsymbol{e}_2 = (\boldsymbol{e}_1 + e^{i\varphi}\boldsymbol{e}_2)e^{i(kx-\omega t)}$.

当 $\varphi = 0$, 沿 $(\boldsymbol{e}_1 + \boldsymbol{e}_2)$ 方向即 $45°$ 方向.

当 $\varphi = \pi/2$, $E = (\boldsymbol{e}_1 + i\boldsymbol{e}_2)e^{i(kx-\omega t)}$. 沿 $(\boldsymbol{e}_1 + i\boldsymbol{e}_2)$ 方向, 即为顺时针的圆偏光.

当 $\varphi = -\pi/2$, $E = (\boldsymbol{e}_1 - i\boldsymbol{e}_2)e^{i(kx-\omega t)}$. 沿 $(\boldsymbol{e}_1 - i\boldsymbol{e}_2)$ 方向, 即为逆时针的圆偏光.

2

全反射时

$$\sin\theta_T = \frac{n_2}{n_1}$$

对于 p 光, 当 $\theta = \theta_T$, $\theta'' = \pi/2$. 代入

$$\frac{E'}{E} = \frac{\tan(\theta - \theta'')}{\tan(\theta + \theta'')} = 1$$

$$\frac{E''}{E} = \frac{2\cos\theta\sin\theta''}{\sin(\theta + \theta'')\cos(\theta - \theta'')} = \frac{2n_1}{n_2}$$

$$k'_z = k_z$$

$$k''_z = \sqrt{k''^2 - k''^2_x} = \sqrt{\left(k\frac{n_2}{n_1}\right)^2 - k_x^2} = \sqrt{\left(k\frac{n_2}{n_1}\right)^2 - k^2\sin^2\theta_T} = 0$$

当 $\theta > \theta_T$,$k''_Z = \sqrt{k''^2 - k''^2_x} = ik\sqrt{\sin^2\theta - \left(\frac{n_2}{n_1}\right)^2} = i\kappa$ 因此

$$R_s = \frac{\overrightarrow{\overline{S}}_r \cdot \widehat{z}}{\overrightarrow{\overline{S}}_i \cdot \widehat{z}} = \frac{\operatorname{Re}\left(\vec{E}^{**} \cdot \vec{E}'\right)}{\operatorname{Re}\left(\vec{E}^{*} \cdot \vec{E}\right)} = 1$$

$$T_s = \frac{\overrightarrow{\overline{S}}_t \cdot \widehat{z}}{\overrightarrow{\overline{S}}_i \cdot \widehat{z}} = 0$$

3

(a)

当正向移动的波产生相位移动时，单位长度的相位移动应该与介质总长度无关，即相移应该呈指数形式，即

$$E'_+ = E_+\left(z = t_j\right) = E_+(z=0)e^{ik_jt_j} = E_+e^{ik_jt_j}$$

$$E'_- = E_-\left(z =_j\right) = E_-(z=0)e^{-ik_jt_j} = E_-e^{-ik_jt_j}$$

则透射方程可以表示为:

$$T = \begin{bmatrix} e^{ikt} & 0 \\ 0 & e^{-ikt} \end{bmatrix}$$

则

$$T^{-1} = \begin{bmatrix} e^{-ikt} & 0 \\ 0 & e^{ikt} \end{bmatrix}$$

而

$$T^{*} = \begin{bmatrix} e^{-ikt} & 0 \\ 0 & e^{ikt} \end{bmatrix}$$

二者相等.

(b)

边界处有边界条件:

$$E^{\|}: \quad E_+ + E_- = E'_+ + E'_-$$

$$H^{\|}: \quad n_1\left(E_+ - E_-\right) = n_2\left(E'_+ - E'_-\right)$$

设 $n=n_1/n_2$ 则有

$$E'_+ = \frac{1}{2}E_+(1+n)+\frac{1}{2}E_-(1-n)$$
$$E'_- = \frac{1}{2}E_+(1-n)+\frac{1}{2}E_-(1+n)$$

矩阵形式为

$$T_{\text{ interface }}(2,1)=\frac{1}{2}\begin{pmatrix} n+1 & -(n-1) \\ -(n-1) & n+1 \end{pmatrix}$$

(c)

由于透射端只有沿透射方向的波,

$$\begin{pmatrix} E_t \\ 0 \end{pmatrix} = T\begin{pmatrix} E_i \\ E_r \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}\begin{pmatrix} E_i \\ E_r \end{pmatrix}$$

可以解得

$$E_r=-\frac{t_{21}}{t_{22}}E_i, \quad E_t=\frac{t_{11}t_{22}-t_{12}t_{21}}{t_{22}}E_i=\frac{\det(T)}{t_{22}}E_i$$

$\square$

$$w, \omega$$