数值分析第三次作业

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(1)

$$D^{-1} = \begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\alpha & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\Longrightarrow J = I - D^{-1}A = -\begin{pmatrix} 0 & 2/\alpha & 1/\alpha \\ 2/\alpha & 0 & -1/\alpha \\ 2 & 2 & 0 \end{pmatrix}$$

(2) $\lambda = 0, \frac{2}{\alpha}, -\frac{2}{\alpha} \implies rho = \frac{2}{\alpha}$. 当 $rho < 1 \implies 2 > \alpha$ 时收敛.

2

$$(D-L)^{-1} = \begin{pmatrix} 2 & & \\ 1 & 1 & \\ 1 & 1 & -2 \end{pmatrix}^{-1} = -\frac{1}{2}\begin{pmatrix} -1 & & \\ 1 & -2 & \\ 0 & -1 & 1 \end{pmatrix}$$

$$G = (D-L)^{-1}U = \frac{1}{2}\begin{pmatrix} 0 & 1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix},\ f = (D-L)^{-1}b = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

由于 $|G| = 0 < 1$, 收敛.

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(1)

$$(D-L)^{-1} = \begin{pmatrix} 2 & & \\ 2 & 2 & \\ 0 & -1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1/2 & & \\ -1/2 & -1/2 & \\ -1/4 & 1/4 & 1/2 \end{pmatrix}$$

$$G = \begin{pmatrix} 0 & -a/2 & -1/2 \\ -1 & a/2-1 & (1-a)/2 \\ -1/2 & a/4 & -(a-3)/4 \end{pmatrix}$$

$$\begin{aligned} \lambda_1 = & \quad -1 \\ \lambda_2 = & \quad a/8 - ((a+1)(a+25))^{1/2}/8 - 3/8 \\ \lambda_3 = & \quad a/8 + ((a+1)(a+25))^{1/2}/8 - 3/8 \end{aligned}$$

令 $\lambda_3 = 1$, 则 $a = 2$.

(2) 令 $\lambda_3 = 0$, 则 $a = -\frac{1}{2}$.

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迭代公式等价于

$$x = D^{-1}\left(b + (L+U)\begin{pmatrix} x_2 \\ x_1 \end{pmatrix}\right)$$

则收敛的充要条件为: $\rho\left(D^{-1}(L+U)\right) < 1$.

$$D^{-1}(L+U) = \begin{pmatrix} 0 & \frac{a_{12}}{a_{11}} \\ \frac{a_{21}}{a_{22}} & 0 \end{pmatrix}$$

$$\lambda = \pm\frac{a_{12}a_{21}}{a_{11}a_{22}}$$

则收敛条件为:

$$\rho = |\lambda| = \left| \frac{a_{12}a_{21}}{a_{11}a_{22}} \right| < 1$$

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