# ElectroDynamics

#### Lime

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### 1 方程

真空麦克斯韦方程

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$

$$abla imes m{E} = -rac{\partial m{E}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

 $abla \cdot oldsymbol{E} = rac{
ho}{arepsilon_0} \qquad \qquad \oint_S oldsymbol{E} \mathrm{d} oldsymbol{s} = rac{Q}{arepsilon_0}$ 

$$\oint_{S} \boldsymbol{B} \mathrm{d}\boldsymbol{s} = 0$$

$$\oint_L \boldsymbol{E} \mathrm{d} \boldsymbol{l} = - \frac{\mathrm{d} \varphi_L}{\mathrm{d} t}$$

$$abla imes oldsymbol{B} = \mu_0 oldsymbol{J} + \mu_0 arepsilon_0 rac{\partial oldsymbol{E}}{\partial t}$$
 $abla_L oldsymbol{B} \mathrm{d} oldsymbol{l} = \mu_0 I + \mu_0 arepsilon_0 rac{\mathrm{d} arphi_E}{\mathrm{d} t}$ 
安培定律

高斯定律

高斯磁定律

物质内麦克斯韦方程

$$\nabla \cdot \boldsymbol{D} = \rho_f$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$abla imes oldsymbol{E} = -rac{\partial oldsymbol{B}}{\partial t}$$

$$abla imes oldsymbol{H} = oldsymbol{J}_f + rac{\partial oldsymbol{D}}{\partial t}$$

 $\oint_{S} \mathbf{D} \mathrm{d} \mathbf{s} = Q_f$ 

$$\oint_{S} \boldsymbol{B} \mathrm{d} \boldsymbol{s} = 0$$

$$\oint_L m{E} \mathrm{d}m{l} = -rac{\mathrm{d} arphi_B}{\mathrm{d}t}$$

$$\oint_L \boldsymbol{H} d\boldsymbol{l} = I_f + \frac{d\varphi_D}{dt}$$
 安培定律

高斯定律

高斯磁定律

法拉第电磁感应定律

泊松方程

$$\nabla^2 \Phi = -\frac{\rho}{\varepsilon_0}$$

电荷

$$\sigma_{total} = \varepsilon_0 E$$

$$\sigma_{polar} = P$$

$$\sigma_{free} = D$$

边界条件(当无电流和自由电荷)

$$H_{1\parallel} = H_{2\parallel}$$
  $E_{1\parallel} = E_{2\parallel}$   $E_{1\perp} = B_{2\perp}$   $D_{1\perp} = D_{2\perp}$ 

洛伦兹力:

$$F = qE + qv \times B$$

$$oldsymbol{f} = 
ho oldsymbol{E} + oldsymbol{J} imes oldsymbol{B}$$

电磁场:

$$\label{eq:S} \boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$$
 
$$\label{eq:W} \boldsymbol{w} = \frac{1}{2} \left( \boldsymbol{E} \cdot \boldsymbol{D} + \boldsymbol{H} \cdot \boldsymbol{B} \right)$$

电流:

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$
$$J = \sigma E$$

毕奥——萨伐尔定律  $B=\frac{\mu_0}{4\pi}\int \frac{I\mathrm{d} l \times \boldsymbol{e}_r}{r^2}$ ,若 I 为直线, $B=\frac{\mu_0 Il}{4\pi r^2}$ 

电磁波:

$$\nabla \times \boldsymbol{B} - \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t} = 0 \qquad \nabla^2 \boldsymbol{E} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \Box \boldsymbol{E} = 0$$
$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \nabla^2 \boldsymbol{B} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{B}}{\partial t^2} = \Box \boldsymbol{B} = 0$$

磁矢势:

库仑规范:

$$\nabla \cdot \mathbf{A} = 0$$

$$\nabla^2 \varphi = -\frac{\varphi}{\varepsilon_0}$$

$$\Box \mathbf{A} = -\mu_0 \mathbf{J} + \frac{1}{c^2} \nabla \frac{\partial \varphi}{\partial t}$$

洛伦兹规范:

$$\nabla \cdot \boldsymbol{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$
 
$$\Box \varphi = -\frac{\rho}{\varepsilon_0}$$
 
$$\Box \boldsymbol{A} = -\mu_0 \boldsymbol{J}$$

## 2 数学

### 2.1 Cylindrical coordinates $(\rho, \phi, z)$

$$\begin{split} \nabla \varphi &= \widehat{e}_1 \frac{\partial \varphi}{\partial \rho} + \widehat{e}_2 \frac{1}{\rho} \frac{\partial \varphi}{\partial \phi} + \widehat{e}_3 \frac{\partial \varphi}{\partial z} \\ \nabla \cdot \vec{A} &= \frac{1}{\rho} \frac{\partial \left(\rho A_1\right)}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_2}{\partial \phi} + \frac{\partial A_3}{\partial z} \\ \nabla \times \vec{A} &= \widehat{e}_1 \left( \frac{1}{\rho} \frac{\partial A_3}{\partial \phi} - \frac{\partial A_2}{\partial z} \right) + \widehat{e}_2 \left( \frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial \rho} \right) + \widehat{e}_3 \frac{1}{\rho} \left( \frac{\partial \left(\rho A_2\right)}{\partial \rho} - \frac{\partial A_1}{\partial \phi} \right) \\ \nabla^2 \varphi &= \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \varphi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \varphi}{\partial \phi^2} + \frac{\partial^2 \varphi}{\partial z^2} \end{split}$$

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### **2.2** Spherical coordinates $(r, \theta, \varphi)$

$$\nabla \varphi = \widehat{e}_1 \frac{\partial \varphi}{\partial r} + \widehat{e}_2 \frac{1}{r} \frac{\partial \varphi}{\partial \theta} + \widehat{e}_3 \frac{1}{r \sin \theta} \frac{\partial \varphi}{\partial \phi} q$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial r^2 A_1}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta A_2 \right) + \frac{1}{r \sin \theta} \frac{\partial A_3}{\partial \phi}$$

$$\nabla \times \vec{A} = hate_1 \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} \left( \sin \theta A_3 \right) - \frac{\partial A_2}{\partial \phi} \right] + \widehat{e}_2 \left[ \frac{1}{r \sin \theta} \frac{\partial A_1}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} \left( r A_3 \right) \right] + \widehat{e}_3 \frac{1}{r} \left[ \frac{\partial}{\partial r} \left( r A_2 \right) - \frac{\partial A_1}{\partial \theta} \right]$$

$$\nabla^2 \varphi = \frac{1}{r^2 \sin \theta} \left[ \sin \theta \frac{\partial}{\partial r} \left( r^2 \frac{\partial \varphi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \varphi}{\partial \theta} \right) + \frac{1}{\sin \theta} \frac{\partial^2 \varphi}{\partial \phi^2} \right]$$

### 2.3 Vector Trans

$$\begin{split} \nabla \cdot (F \times G) &= (\nabla \times F) \cdot G - F \cdot (\nabla \times G) \\ \vec{A} \times \left( \vec{B} \times \vec{C} \right) &= \left( \vec{A} \cdot \vec{C} \right) \vec{B} - \left( \vec{A} \cdot \vec{B} \right) \vec{C} \\ \left( \vec{A} \times \vec{B} \right) \times \left( \vec{C} \times \vec{D} \right) &= [\vec{A} \cdot \left( \vec{B} \times \vec{D} \right)] \vec{c} - [\vec{A} \cdot \left( \vec{B} \times \vec{C} \right)] \vec{D} \\ \vec{A} \times \left( \vec{B} \times \vec{C} \right) + \vec{B} \times \left( \vec{C} \times \vec{A} \right) + \vec{C} \times \left( \vec{A} \times \vec{B} \right) = 0 \end{split}$$