数值分析第六次作业

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(1)

$f(-1) = 1/e$, $f(0) = 1$, $f(1) = e$. 则为连接相邻两点的直线

$$S(x) = \begin{cases} (1 - 1/e)\,x + 1, & x \in [-1, 0] \\ (e - 1)\,x + 1, & x \in [0, 1] \end{cases}$$

(2)

即求 $I(x)_{min} = min\left\{\int_{-1}^{0} (S_1(x) - e^x)^2 \,\mathrm{d}x + \int_{0}^{1} (S_2(x) - e^x)^2 \,\mathrm{d}x\right\}$. 设 $S_1(x) = k_1 x + b$, $S_2(x) = k_2 x + b$.

$I(x) = \dfrac{k_1^2 x^3}{3} + k_1 b x^2 + b^2 x - 2(k_1 x + b) e^x + 2k_1 e^x + \dfrac{1}{2} e^{2x}\Big|_{-1}^{0} + \dfrac{k_1^2 x^3}{3} + k_2 b x^2 + b^2 x - 2(k_2 x + b) e^x + 2k_2 e^x + \dfrac{1}{2} e^{2x}\Big|_{0}^{1}$

$I = 2b^2 - bk_1 + \dfrac{2(b - 2k_1)}{e} + b(k_2 - 2e + 2) - 2b + \dfrac{k_1^2}{3} + 2k_1 + \dfrac{1}{6}\left(2k_2^2 - 12k_2 + 3e^2 - 3\right) + \dfrac{1}{2} - \dfrac{1}{2e^2}$

且满足

$$\frac{\partial I}{\partial b} = \frac{\partial I}{\partial k_1} = \frac{\partial I}{\partial k_2} = 0$$

$$\Longrightarrow S(x) = \begin{cases} \frac{3\left(4-4e+e^2\right)}{e} x + \frac{2\left(2-3e+e^2\right)}{e}, & x \in [-1, 0] \\ \frac{3\left(2-4e+e^2\right)}{e} x + \frac{2\left(2-3e+e^2\right)}{e}, & x \in [0, 1] \end{cases}$$

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基函数为 $1, x^2$, 即要求

$$
\begin{aligned}
(1,1)\,a + \left(1,x^2\right) b &= \left(f\left(x\right),1\right) \\
\left(x^2,1\right) a + \left(x^2,x^2\right) b &= \left(f\left(x\right),x^2\right)
\end{aligned}
$$

代入 $(1,1) = 1, \left(1,x^2\right) = 1/3, \left(x^2,x^2\right) = 1/5$, 可解得

$$
\left.\begin{aligned}
a + b/3 &= \pi/4 \\
a/3 + b/5 &= 1 - \pi/4
\end{aligned}\right\} \implies a = -\frac{9\pi}{4}, b = \frac{15\pi}{2}
$$

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$Ax* = b \implies A^TAx^* = A^Tb \implies x^* = \left(A^TA\right)^{-1}A^Tb.$

$$
A^TA = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix} \implies \left(A^TA\right)^{-1} = \frac{1}{6}\begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}
$$

$$
A^Tb = \begin{pmatrix} -3 \\ -5 \end{pmatrix}
$$

$$
\implies x^* = -\frac{1}{6}\begin{pmatrix} 5 \\ 4 \end{pmatrix}
$$