ElectroDynamics

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1 方程

真空麦克斯韦方程

$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$	$\oiint_S \boldsymbol{E} \mathrm{d}\boldsymbol{s} = \frac{Q}{\varepsilon_0}$	高斯定律
$\nabla \cdot \boldsymbol{B} = 0$	$\oiint_S \boldsymbol{B} \mathrm{d}\boldsymbol{s} = 0$	高斯磁定律
$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	$\oint_L \boldsymbol{E} \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$	法拉第电磁感应定律
$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$	$\oint_L \boldsymbol{B} \mathrm{d}\boldsymbol{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$	安培定律

物质内麦克斯韦方程

$\nabla \cdot \boldsymbol{D} = \rho_f$	$\oiint_S \boldsymbol{D} \mathrm{d}\boldsymbol{s} = Q_f$	高斯定律
$\nabla \cdot \boldsymbol{B} = 0$	$\oiint_S \boldsymbol{B} \mathrm{d}\boldsymbol{s} = 0$	高斯磁定律
$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	$\oint_L \boldsymbol{E} \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$	法拉第电磁感应定律
$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t}$	$\oint_L \boldsymbol{H} \mathrm{d}\boldsymbol{l} = I_f + \frac{\mathrm{d}\Phi_D}{\mathrm{d}t}$	安培定律

边界条件（当无电流和自由电荷）

$$\begin{array}{c|c} H_{1\parallel} = H_{2\parallel} & E_{1\parallel} = E_{2\parallel} \\ B_{1\perp} = B_{2\perp} & D_{1\perp} = D_{2\perp} \end{array}$$

洛伦兹力：

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

$$\boldsymbol{f} = \rho \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}$$

电磁场：

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$$

$$w = \frac{1}{2}(\boldsymbol{E} \cdot \boldsymbol{D} + \boldsymbol{H} \cdot \boldsymbol{B})$$

电流：

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

$$J = \sigma E$$

毕奥——萨伐尔定律 $B = \frac{\mu_0}{4\pi}\int\frac{I\mathrm{d}l\times\boldsymbol{e}_r}{r^2}$，若 I 为直线，$B = \frac{\mu_0 Il}{4\pi r^2}$

电磁波：

$$\begin{array}{r|l} \boldsymbol{\nabla}\times\boldsymbol{B} - \frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t} = 0 & \boldsymbol{\nabla^2}\boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{E}}{\partial t^2} = \square\boldsymbol{E} = 0 \\ \boldsymbol{\nabla}\times\boldsymbol{E} + \frac{\partial\boldsymbol{B}}{\partial t} = 0 & \boldsymbol{\nabla^2}\boldsymbol{B} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{B}}{\partial t^2} = \square\boldsymbol{B} = 0 \end{array}$$

磁矢势：

库仑规范：

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} = 0$$

$$\nabla^2\varphi = -\frac{\varphi}{\varepsilon_0}$$

$$\square\boldsymbol{A} = -\mu_0\boldsymbol{J} + \frac{1}{c^2}\nabla\frac{\partial\varphi}{\partial t}$$

洛伦兹规范：

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$$

$$\square\varphi = -\frac{\rho}{\varepsilon_0}$$

$$\square\boldsymbol{A} = -\mu_0\boldsymbol{J}$$

2 数学

$$\nabla^2\psi = \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\varphi^2}\right]$$

$$\nabla\cdot(F\times G) = (\nabla\times F)\cdot G - F\cdot(\nabla\times G)$$