ElectroDynamics

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1 方程

真空麦克斯韦方程

$$
\begin{array}{lll}
\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} & \oiint_S \boldsymbol{E} \mathrm{d}\boldsymbol{s} = \frac{Q}{\varepsilon_0} & \text{高斯定律} \\
\nabla \cdot \boldsymbol{B} = 0 & \oiint_S \boldsymbol{B} \mathrm{d}\boldsymbol{s} = 0 & \text{高斯磁定律} \\
\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} & \oint_L \boldsymbol{E} \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\varphi_B}{\mathrm{d}t} & \text{法拉第电磁感应定律} \\
\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} & \oint_L \boldsymbol{B} \mathrm{d}\boldsymbol{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\mathrm{d}\varphi_E}{\mathrm{d}t} & \text{安培定律}
\end{array}
$$

物质内麦克斯韦方程

$$
\begin{array}{lll}
\nabla \cdot \boldsymbol{D} = \rho_f & \oiint_S \boldsymbol{D} \mathrm{d}\boldsymbol{s} = Q_f & \text{高斯定律} \\
\nabla \cdot \boldsymbol{B} = 0 & \oiint_S \boldsymbol{B} \mathrm{d}\boldsymbol{s} = 0 & \text{高斯磁定律} \\
\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} & \oint_L \boldsymbol{E} \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\varphi_B}{\mathrm{d}t} & \text{法拉第电磁感应定律} \\
\nabla \times \boldsymbol{H} = \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t} & \oint_L \boldsymbol{H} \mathrm{d}\boldsymbol{l} = I_f + \frac{\mathrm{d}\varphi_D}{\mathrm{d}t} & \text{安培定律}
\end{array}
$$

边界条件（当无电流和自由电荷）

$$
\begin{array}{c|c}
H_{1\parallel} = H_{2\parallel} & E_{1\parallel} = E_{2\parallel} \\
B_{1\perp} = B_{2\perp} & D_{1\perp} = D_{2\perp}
\end{array}
$$

洛伦兹力：

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

$$\boldsymbol{f} = \rho \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}$$

电磁场：

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$$

$$w = \frac{1}{2}(\boldsymbol{E} \cdot \boldsymbol{D} + \boldsymbol{H} \cdot \boldsymbol{B})$$

电流：

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

$$J = \sigma E$$

毕奥——萨伐尔定律 $B = \frac{\mu_0}{4\pi}\int \frac{I\mathrm{d}l\times \boldsymbol{e}_r}{r^2}$，若 I 为直线，$B = \frac{\mu_0 Il}{4\pi r^2}$

电磁波：

$$\begin{array}{r|l} \boldsymbol{\nabla}\times\boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = 0 & \boldsymbol{\nabla}^2\boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \Box\boldsymbol{E} = 0 \\ \boldsymbol{\nabla}\times\boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 & \boldsymbol{\nabla}^2\boldsymbol{B} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} = \Box\boldsymbol{B} = 0 \end{array}$$

磁矢势：

库仑规范：

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} = 0$$

$$\nabla^2\varphi = -\frac{\varphi}{\varepsilon_0}$$

$$\Box\boldsymbol{A} = -\mu_0\boldsymbol{J} + \frac{1}{c^2}\nabla\frac{\partial\varphi}{\partial t}$$

洛伦兹规范：

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$$

$$\Box\varphi = -\frac{\rho}{\varepsilon_0}$$

$$\Box\boldsymbol{A} = -\mu_0\boldsymbol{J}$$

2 数学

2.1 Cylindrical coordinates (ρ, ϕ, z)

$$\nabla\varphi = \widehat{e}_1\frac{\partial\varphi}{\partial\rho} + \widehat{e}_2\frac{1}{\rho}\frac{\partial\varphi}{\partial\phi} + \widehat{e}_3\frac{\partial\varphi}{\partial z}$$

$$\nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_1)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_2}{\partial\phi} + \frac{\partial A_3}{\partial z}$$

$$\nabla\times\vec{A} = \widehat{e}_1\left(\frac{1}{\rho}\frac{\partial A_3}{\partial\phi} - \frac{\partial A_2}{\partial z}\right) + \widehat{e}_2\left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial\rho}\right) + \widehat{e}_3\frac{1}{\rho}\left(\frac{\partial(\rho A_2)}{\partial\rho} - \frac{\partial A_1}{\partial\phi}\right)$$

$$\nabla^2\varphi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\varphi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\varphi}{\partial\phi^2} + \frac{\partial^2\varphi}{\partial z^2}$$

2.2 Spherical coordinates (r, θ, φ)

$$\nabla\varphi = \widehat{e}_1\frac{\partial\varphi}{\partial r} + \widehat{e}_2\frac{1}{r}\frac{\partial\varphi}{\partial\theta} + \widehat{e}_3\frac{1}{r\sin\theta}\frac{\partial\varphi}{\partial\phi}q$$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial r^2 A_1}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_2) + \frac{1}{r\sin\theta}\frac{\partial A_3}{\partial\phi}$$

$$\nabla\times\vec{A} = hate_1\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_3) - \frac{\partial A_2}{\partial\phi}\right] + \widehat{e}_2\left[\frac{1}{r\sin\theta}\frac{\partial A_1}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_3)\right] + \widehat{e}_3\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_2) - \frac{\partial A_1}{\partial\theta}\right]$$

$$\nabla^2\varphi = \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2\varphi}{\partial\phi^2}\right]$$

2.3 Vector Trans

$$\nabla \cdot (F \times G) = (\nabla \times F) \cdot G - F \cdot (\nabla \times G)$$

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \left(\vec{A} \cdot \vec{C}\right)\vec{B} - \left(\vec{A} \cdot \vec{B}\right)\vec{C}$$

$$\left(\vec{A} \times \vec{B}\right) \times \left(\vec{C} \times \vec{D}\right) = [\vec{A} \cdot \left(\vec{B} \times \vec{D}\right)]\vec{c} - [\vec{A} \cdot \left(\vec{B} \times \vec{C}\right)]\vec{D}$$

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) + \vec{B} \times \left(\vec{C} \times \vec{A}\right) + \vec{C} \times \left(\vec{A} \times \vec{B}\right) = 0$$