统计力学第三次作业

肖涵薄 31360164

2019 年 3 月 17 日

2.15

单位时间总能量 $E = 1.35 \times 10^3 \cdot 4\pi \left(1.495 \times 10^{11}\right)^2 = 3.8 \times 10^{26}\ J$. 则根据黑体辐射规律, $E/\left(4\pi\left(6.955 \times 10^8\right)^2\right) = \sigma T^4 \implies T = 5760K$.

2.16

$U = Vu$,

$$
\begin{aligned}
\mathrm{d}Q &= \mathrm{d}\left(Vu\right) + p\mathrm{d}V \\
&= V\mathrm{d}u + u\mathrm{d}V + p\mathrm{d}V \\
&\quad u = aT^4\text{在等温过程不变, } \mathrm{d}u = 0\text{, 再代入}p = \tfrac{1}{3}u \\
&= \tfrac{4}{3}u\mathrm{d}V \\
&= \tfrac{4}{3}\alpha T^4\mathrm{d}V
\end{aligned}
$$

$$\implies Q = \frac{4}{3}\alpha T^4\left(V_2 - V_1\right)$$

2.17

$đQ = T\mathrm{d}S$, 等温过程, 在两个温度下吸热分别为:

$$\Delta Q_1 = T_1\left(S_2 - S_1\right),\ \Delta Q_2 = T_2\left(S_2 - S_1\right)$$

.

绝热 (等熵) 过程中 $\Delta Q=0$, 则

$$\eta=1-\frac{Q_2}{Q_1}=1-\frac{T_2\left(S_2-S_1\right)}{T_1\left(S_2-S_1\right)}=1-\frac{T_2}{T_1}.$$

2.19

对气体体系 $C_p-C_V=\frac{\mathrm{d}(H-U)}{\mathrm{d}T}=T\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_p$ 作代换 $p\to-\mu_0 H,\ V\to m$: $C_p-C_V=-\mu_0 T\left(\frac{\partial H}{\partial T}\right)_m\left(\frac{\partial m}{\partial T}\right)_H$. 再代入 $\left(\frac{\partial m}{\partial T}\right)_H=\left(\frac{\partial H}{\partial T}\right)_m\left(\frac{\partial m}{\partial H}\right)_T$, 可得 $C_p-C_V=\mu_0 T\left(\frac{\partial H}{\partial T}\right)_m^2\left(\frac{\partial m}{\partial H}\right)_T$.

2.20

在等温过程中, $đS\left(T,H\right)=\frac{\partial S}{\partial T}\mathrm{d}T+\frac{\partial S}{\partial H}\mathrm{d}H=\frac{\partial S}{\partial H}\mathrm{d}H$.

$$Q=\int T đS\left(T,H\right)=\int T\frac{\partial S}{\partial H}\mathrm{d}H$$

代入 $\left(\frac{\partial S}{\partial H}\right)_T=\mu_0\left(\frac{\partial m}{\partial T}\right)_H$, $Q=\int\mu_0 T\frac{\partial m}{\partial T}\mathrm{d}H$,
再代入居里定律,

$$Q=-\int\frac{\mu_0 CV}{T}H\mathrm{d}H=-\frac{\mu_0 CV}{2T}H^2$$

3.1

(a) 由热力学第二定律, $\mathrm{d}U=đQ+đW<T\mathrm{d}S-P\mathrm{d}V$, 当 S,V 不变, 即 $\mathrm{d}U<0$. 因此在非平衡态 U 会减小, 稳定时 U 保持不变, 为最小.

(b) $H=U+pV,\ \mathrm{d}H<T\mathrm{d}S+V\mathrm{d}p$, 当 S,p 不变, 即 $\mathrm{d}H<0$. 因此在非平衡态 H 会减小, 稳定时 H 保持不变, 为最小.

(c) 由 (b) 式, $T\mathrm{d}S>\mathrm{d}H-V\mathrm{d}p$, 当 H,p 不变, 即 $\mathrm{d}S>0$. 因此在非平衡态 S 会增大, 稳定时 S 保持不变, 为最大.

(d) $F=U-TS,\ \mathrm{d}F<-S\mathrm{d}T-p\mathrm{d}V$, 当 F,V 不变, 即 $\mathrm{d}T<0$. 因此在非平衡态 T 会减小, 稳定时 T 保持不变, 为最小.

(e) $G=F+pV,\ \mathrm{d}G<-S\mathrm{d}T+V\mathrm{d}p$, 当 G,p 不变, 即 $\mathrm{d}T<0$. 因此在非平衡态 T 会减小, 稳定时 T 保持不变, 为最小.

(f) $\mathrm{d}U < T\mathrm{d}S - p\mathrm{d}V$, 当 U, S 不变, 即 $\mathrm{d}V < 0$. 因此在非平衡态 V 会减小, 稳定时 V 保持不变, 为最小.

(G) $F = U - TS$, $\mathrm{d}F < -S\mathrm{d}T - p\mathrm{d}V$, 当 F, T 不变, 即 $\mathrm{d}V < 0$. 因此在非平衡态 V 会减小, 稳定时 V 保持不变, 为最小.

3.2

$$\delta^2 S\left(U < V\right) = \frac{\partial^2 S}{\partial U^2}\delta^2 U + 2\frac{\partial^2 S}{\partial U \partial V}\delta U \delta V + \frac{\partial^2 S}{\partial V^2}\delta^2 V$$

$\frac{\partial^2 S}{\partial U^2} = \frac{\partial}{\partial U}\left(\frac{1}{T}\right) = -\frac{1}{T^2 C_V}$

$\frac{\partial^2 S}{\partial U \partial V} = \frac{\partial}{\partial V}\left(\frac{1}{T}\right) = \frac{1}{T^2 C_V}[T\frac{\partial p}{\partial T} - p] = \frac{p}{C_V T}\beta - \frac{p}{C_V T^2}$

$\frac{\partial^2 S}{\partial V^2} = \frac{\partial}{\partial V}\left(\frac{p}{T}\right) = \frac{1}{T^2}\left(T\frac{\partial p}{\partial V} - p\frac{\partial T}{\partial V}\right) = -\frac{1}{T}\frac{[T\frac{\partial p}{\partial T} - p] + C_V\frac{\partial T}{\partial V}}{C_V\frac{\partial T}{\partial p}} - \frac{1}{T^2}p\frac{\partial T}{\partial V} = \frac{2p^2\beta}{C_V T} - \frac{p^2}{C_V T^2} - \frac{p^2\beta^2}{C_V} - \frac{1}{TV\kappa_T}$

代入上式即为所需证明的方程.

□

3.4

$$C_p - C_V = \frac{VT\alpha^2}{\kappa_T}$$

由于 $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T > 0$, 因此 $C_p \geq C_V > 0$.

又由于 $\frac{\kappa_S}{\kappa_T} = \frac{\frac{\partial V}{\partial p}}{\frac{\partial V}{\partial p}} = \frac{C_V}{C_p} \leq 1$. 因此 $\frac{\partial V}{\partial p} \leq \frac{\partial V}{\partial p} < 0$.