The 7th Homework of Theoretical Mechanics

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Q1

$$
\begin{gathered}
L = \frac{1}{2} m v_r^2 + \frac{1}{2} m r^2 \dot{\theta}^2 - V(r) \\
\frac{\mathrm{d}}{\mathrm{dt}} \frac{\partial L}{\partial v_r} = m a_r, \quad \frac{\partial L}{\partial r} = m r \dot{\theta}^2 - \frac{\partial V}{\partial r} \\
\frac{\mathrm{d}}{\mathrm{dt}} \frac{\partial L}{\partial \dot{\theta}} = m r \left(r \ddot{\theta} + 2 \dot{r} \dot{\theta} \right), \quad \frac{\partial L}{\partial \theta} = 0 \\
\Longrightarrow m a_r = m r \dot{\theta}^2 - \frac{\partial V}{\partial r}, \quad r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0
\end{gathered}
$$

当满足平方反比，$\frac{\partial V}{\partial r} = -\frac{k}{r^2}$.

Q2

$$
\begin{aligned}
[f, H] &= \frac{\partial f}{\partial q_2} \frac{\partial H}{\partial p_2} - \frac{\partial f}{\partial p_1} \frac{\partial H}{\partial q_1} = 2 q_2 p_1 - 2 p_1 q_2 = 0 \\
[g, H] &= \frac{\partial g}{\partial q_1} \frac{\partial H}{\partial p_1} - \frac{\partial g}{\partial p_2} \frac{\partial H}{\partial q_2} = 2 q_1 p_2 - 2 p_2 q_1 = 0
\end{aligned}
$$

Q3

$$
L = \frac{1}{2} m a^2 \dot{\theta}^2 + m a^2 \dot{\theta} \omega \sin\theta + \frac{1}{2} m \omega^2 a^2 \sin^2\theta - m g a \cos\theta
$$

$$p = \frac{\partial L}{\partial \dot{\theta}} = ma^2\left(\dot{\theta} + \omega \sin\theta\right) \implies \dot{\theta} = \frac{p}{ma^2} - \omega \sin\theta$$

$$L = \frac{1}{2}ma^2\left(\frac{p}{ma^2} - \omega\sin\theta\right)^2 + ma^2\omega\sin\theta\left(\frac{p}{ma^2} - \omega\sin\theta\right) + \frac{1}{2}m\omega^2a^2\sin^2\theta - mga\cos\theta$$

$$H = p\dot{q} - L = \frac{p^2}{2ma^2} - p\omega\sin\theta + mga\cos\theta$$

$$\begin{aligned} \dot{\theta} = \tfrac{\partial H}{\partial p} = &\quad \tfrac{p}{ma^2} - \omega\sin\theta \\ \dot{p} = -\tfrac{\partial H}{\partial \theta} = &\quad p\omega\cos\theta + mga\sin\theta \end{aligned}$$

$$\implies \ddot{\theta} = g\sin\theta/a + \omega^2\sin\theta\cos\theta$$

Q4

$$H = \frac{1}{2}mv_r^2 + \frac{1}{2}mr^2\dot{\theta}^2 - \frac{k}{r}$$

$$p_r = \frac{\partial H}{\partial v_r} = mv_r, \quad p_\theta = mr^2\dot{\theta}$$

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{k}{r}$$

$$\begin{cases} v_r = & \frac{p_r}{m} = v_r \\ \dot{p}_r = & mr\dot{\theta}^2 - \frac{k}{r^2} = ma_r \\ \dot{\theta} = & \frac{p_\theta}{mr^2} \\ \dot{p}_\theta = & 0 \end{cases}$$

$$\implies ma_r = mr\dot{\theta}^2 - \frac{k}{r^2}, \quad r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$$

Q5

$$p = \frac{\partial U}{\partial q} = mgQ \implies Q = p/mg$$

$$P = -\frac{\partial U}{\partial Q} = mg\left(\frac{1}{2}gQ^2 + q\right) = \frac{1}{2}mg^2\frac{p^2}{m^2g^2} + mgq = \frac{p^2}{2m} + mgq = H$$

$$\implies \begin{cases} \dot{Q} = & 1 \implies a = g \\ \dot{P} = & 0 \implies \frac{1}{2}m\dot{q}^2 + mgq = h \end{cases}$$