The 3rd HW of Electrodynamics

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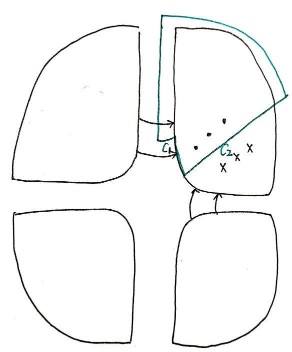
Q1

Find the relation between the current and the field in the gap of a quadrupole magnet.



按如图路径积分, 仅 C_1, C_2 处 $\boldsymbol{B}\mathrm{d}\boldsymbol{l} \neq 0$.

$$\int_{C_1} \vec{H} \cdot d\vec{l} + \int_{C_2} \vec{H} \cdot d\vec{l} = NI$$

由边界条件:

$$B_{\text{ gap }} = B_{\text{ iron }}$$

$$H_{\text{gap}} = \frac{B_{\text{gap}}}{\mu_0}, H_{\text{iron}} = \frac{B_{\text{iron}}}{\mu_0 \mu_{\text{iron}}}$$

设 h 为半个 gap 长.

$$NI=\frac{B_{\text{gap}}h}{\mu_0}+\frac{B_{\text{iron}}l_{\text{iron}}}{\mu_0\mu_{\text{iron}}}$$

$$\Longrightarrow\ NI=\frac{1}{\eta}\frac{B_{\text{gap}}h}{\mu_0},\eta=\frac{\frac{B_{\text{ gap }}h}{\mu_0}}{\frac{B_{\text{ gap }}h}{\mu_0}+\frac{B_{\text{ iron }}l_{\text{ iron }}}{\mu_0\mu_{\text{ iron }}}}$$

通常 $\mu_{\text{ iron }}\approx 1000,\eta\approx 0.99$ 可化简为:

$$NI=\frac{B_x h}{\mu_0}\implies B_x=\frac{NI\mu_0}{h}$$

设边界坐标为 (x,y), 边界方程为 $xy=C$, 则 $y=\frac{C}{x}=\frac{C}{h}$, 代入上式, $B_x=\frac{NI\mu_0}{C}y$. 将积分轨迹变为沿直线 $x=y$ 对称的另一条轨迹, 同理可得 y 方向磁场:$B_y=\frac{NI\mu_0}{C}x$, 即

$$B=(y,x,0)\,\frac{NI\mu_0}{C}.$$

Q2

Find the potential of a uniformly charged ring with radius R and line charge density τ. Find the explicit function of the potential on the symmetry axis.

设线圈在 $r'=R$ 处, 线圈上一点坐标为 $(R,\theta',0)$. 场点 P 在 (r,θ,z) 处.

$$\phi=\oint\frac{1}{4\pi\varepsilon_0}\frac{\tau}{\sqrt{(R\cos\theta'-r\cos\theta)^2+(R\sin\theta'-r\sin\theta)^2+z^2}}R\mathrm{d}\theta'$$

$$\Longrightarrow\ \phi=\frac{\tau R}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{\mathrm{d}\theta'}{\sqrt{R^2+r^2-2Rr\left(\cos\theta'\cos\theta+\sin\theta'+\sin\theta\right)+z^2}}$$

在 z 轴上 $r=0$, 积分可化简:

$$\phi=\frac{\tau R}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{\mathrm{d}\theta'}{\sqrt{R^2+z^2}}=\frac{\tau R}{2\varepsilon_0\sqrt{R^2+z^2}}$$