ElectroDynamics

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1 方程

真空麦克斯韦方程

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \qquad \oint_S \boldsymbol{E} \mathrm{d}\boldsymbol{s} = \frac{Q}{\varepsilon_0} \qquad \text{高斯定律}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \oint_S \boldsymbol{B} \mathrm{d}\boldsymbol{s} = 0 \qquad \text{高斯磁定律}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \oint_L \boldsymbol{E} \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\varphi_B}{\mathrm{d}t} \qquad \text{法拉第电磁感应定律}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \qquad \oint_L \boldsymbol{B} \mathrm{d}\boldsymbol{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{\mathrm{d}\varphi_E}{\mathrm{d}t} \qquad \text{安培定律}$$

物质内麦克斯韦方程

$$\nabla \cdot \boldsymbol{D} = \rho_f \qquad \oint_S \boldsymbol{D} \mathrm{d}\boldsymbol{s} = Q_f \qquad \text{高斯定律}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \oint_S \boldsymbol{B} \mathrm{d}\boldsymbol{s} = 0 \qquad \text{高斯磁定律}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \oint_L \boldsymbol{E} \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\varphi_B}{\mathrm{d}t} \qquad \text{法拉第电磁感应定律}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \oint_L \boldsymbol{H} \mathrm{d}\boldsymbol{l} = I_f + \frac{\mathrm{d}\varphi_D}{\mathrm{d}t} \qquad \text{安培定律}$$

泊松方程

$$\nabla^2 \Phi = -\frac{\rho}{\varepsilon_0}$$

电荷

$$\sigma_{total} = \varepsilon_0 E$$

$$\sigma_{polar} = P$$

$$\sigma_{free} = D$$

边界条件（当无电流和自由电荷）

$$\begin{array}{c|c} H_{1\parallel} = H_{2\parallel} & E_{1\parallel} = E_{2\parallel} \\ B_{1\perp} = B_{2\perp} & D_{1\perp} = D_{2\perp} \end{array}$$

洛伦兹力：

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

$$\boldsymbol{f} = \rho \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}$$

电磁场：

$$\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}$$

$$w=\frac{1}{2}\left(\boldsymbol{E}\cdot\boldsymbol{D}+\boldsymbol{H}\cdot\boldsymbol{B}\right)$$

电流：

$$\nabla\cdot J=-\frac{\partial\rho}{\partial t}$$

$$J=\sigma E$$

毕奥——萨伐尔定律 $B=\frac{\mu_0}{4\pi}\int\frac{I\mathrm{d}l\times\boldsymbol{e}_r}{r^2}$，若 I 为直线，$B=\frac{\mu_0 Il}{4\pi r^2}$

电磁波：

$$\left.\begin{aligned}\boldsymbol{\nabla}\times\boldsymbol{B}-\tfrac{1}{c^2}\tfrac{\partial\boldsymbol{E}}{\partial t}&=0\\ \boldsymbol{\nabla}\times\boldsymbol{E}+\tfrac{\partial\boldsymbol{B}}{\partial t}&=0\end{aligned}\right|\quad\begin{aligned}\boldsymbol{\nabla^2}\boldsymbol{E}-\tfrac{1}{c^2}\tfrac{\partial^2\boldsymbol{E}}{\partial t^2}&=\Box\boldsymbol{E}=0\\ \boldsymbol{\nabla^2}\boldsymbol{B}-\tfrac{1}{c^2}\tfrac{\partial^2\boldsymbol{B}}{\partial t^2}&=\Box\boldsymbol{B}=0\end{aligned}$$

磁矢势：

库仑规范：

$$\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$$

$$\nabla^2\varphi=-\tfrac{\varphi}{\varepsilon_0}$$

$$\boldsymbol{\Box A}=-\mu_0\boldsymbol{J}+\tfrac{1}{c^2}\nabla\tfrac{\partial\varphi}{\partial t}$$

洛伦兹规范：

$$\boldsymbol{\nabla}\cdot\boldsymbol{A}+\tfrac{1}{c^2}\tfrac{\partial\varphi}{\partial t}=0$$

$$\Box\varphi=-\tfrac{\rho}{\varepsilon_0}$$

$$\boldsymbol{\Box A}=-\mu_0\boldsymbol{J}$$

2 数学

2.1 Cylindrical coordinates (ρ,ϕ,z)

$$\nabla\varphi=\widehat{e}_1\frac{\partial\varphi}{\partial\rho}+\widehat{e}_2\frac{1}{\rho}\frac{\partial\varphi}{\partial\phi}+\widehat{e}_3\frac{\partial\varphi}{\partial z}$$

$$\nabla\cdot\vec{A}=\frac{1}{\rho}\frac{\partial\left(\rho A_1\right)}{\partial\rho}+\frac{1}{\rho}\frac{\partial A_2}{\partial\phi}+\frac{\partial A_3}{\partial z}$$

$$\nabla\times\vec{A}=\widehat{e}_1\left(\frac{1}{\rho}\frac{\partial A_3}{\partial\phi}-\frac{\partial A_2}{\partial z}\right)+\widehat{e}_2\left(\frac{\partial A_1}{\partial z}-\frac{\partial A_3}{\partial\rho}\right)+\widehat{e}_3\frac{1}{\rho}\left(\frac{\partial\left(\rho A_2\right)}{\partial\rho}-\frac{\partial A_1}{\partial\phi}\right)$$

$$\nabla^2\varphi=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\varphi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\varphi}{\partial\phi^2}+\frac{\partial^2\varphi}{\partial z^2}$$

2.2 Spherical coordinates (r, θ, φ)

$$\nabla\varphi = \widehat{e}_1 \frac{\partial \varphi}{\partial r} + \widehat{e}_2 \frac{1}{r} \frac{\partial \varphi}{\partial \theta} + \widehat{e}_3 \frac{1}{r \sin\theta} \frac{\partial \varphi}{\partial \phi} q$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial r^2 A_1}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta A_2) + \frac{1}{r \sin\theta} \frac{\partial A_3}{\partial \phi}$$

$$\nabla \times \vec{A} = \ hate_1 \frac{1}{r \sin\theta} \left[\frac{\partial}{\partial \theta} (\sin\theta A_3) - \frac{\partial A_2}{\partial \phi} \right] + \widehat{e}_2 \left[\frac{1}{r \sin\theta} \frac{\partial A_1}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (r A_3) \right] + \widehat{e}_3 \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_2) - \frac{\partial A_1}{\partial \theta} \right]$$

$$\nabla^2 \varphi = \frac{1}{r^2 \sin\theta} \left[\sin\theta \frac{\partial}{\partial r} \left(r^2 \frac{\partial \varphi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial \varphi}{\partial \theta} \right) + \frac{1}{\sin\theta} \frac{\partial^2 \varphi}{\partial \phi^2} \right]$$

2.3 Vector Trans

$$\nabla \cdot (F \times G) = (\nabla \times F) \cdot G - F \cdot (\nabla \times G)$$

$$\vec{A} \times \left(\vec{B} \times \vec{C} \right) = \left(\vec{A} \cdot \vec{C} \right) \vec{B} - \left(\vec{A} \cdot \vec{B} \right) \vec{C}$$

$$\left(\vec{A} \times \vec{B} \right) \times \left(\vec{C} \times \vec{D} \right) = [\vec{A} \cdot \left(\vec{B} \times \vec{D} \right)] \vec{c} - [\vec{A} \cdot \left(\vec{B} \times \vec{C} \right)] \vec{D}$$

$$\vec{A} \times \left(\vec{B} \times \vec{C} \right) + \vec{B} \times \left(\vec{C} \times \vec{A} \right) + \vec{C} \times \left(\vec{A} \times \vec{B} \right) = 0$$