数值分析第八次作业

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(1)

$$A(h) = \frac{1}{h}\tan(\pi h) = \pi + \frac{1}{3}\pi^3 h^2 + O(h^4)$$
$$A(\frac{h}{2}) = \frac{2}{h}\tan(\pi h/2) = \pi + \frac{1}{12}\pi^3 h^2 + O(h^4)$$

即二者误差均为 $O\left(h^2\right)$

(2)

由第一问可知

$$A(h) - 4A(\frac{h}{2}) = -3\pi + O(h^4)$$

则可通过式 $\frac{4A(\frac{h}{2})-A(h)}{3}$ 计算 π, 误差阶数为 $O(h^4)$.

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$$\begin{cases} \frac{2}{3} = A_1 + A_2 + A_3 \\ 0 = A_1x_1 + A_2x_2 + A_3x_3 \\ \frac{2}{5} = A_1x_1^2 + A_2x_2^2 + A_3x_3^2 \\ 0 = A_1x_1^3 + A_2x_2^3 + A_3x_3^3 \\ \frac{2}{7} = A_1x_1^4 + A_2x_2^4 + A_3x_3^4 \\ 0 = A_1x_1^5 + A_2x_2^5 + A_3x_3^5 \end{cases}$$

$$\begin{cases} \frac{2}{3} = A_1 + A_2 + A_3 \\ 0 = A_1 x_1 - A_3 x_1 \\ \frac{2}{5} = A_1 x_1^2 + A_3 x_1^2 \\ 0 = A_1 x_1^3 + A_3 x_3^3 \\ \frac{2}{7} = A_1 x_1^4 + A_3 x_3^4 \\ 0 = A_1 x_1^5 + A_3 x_3^5 \end{cases} \implies \begin{cases} x_1 = -\sqrt{\frac{5}{7}} \\ x_2 = 0 \\ x_3 = \sqrt{\frac{5}{7}} \\ A_1 = \frac{7}{25} \\ A_2 = \frac{8}{75} \\ A_3 = \frac{7}{25} \end{cases}$$

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(1)

$$\int_a^b mx + 1 \,\mathrm{d}x = 0 \implies \frac{m}{2}(b^2 - a^2) + (b - a) = 0$$

$$m = -\frac{2}{b+a} \implies mx + 1 = 0 \iff x = \frac{a+b}{2}$$

$$\int_a^b f(\frac{a+b}{2}) \,\mathrm{d}x = A = (b-a) f(\frac{a+b}{2})$$

则 $G_0(f) = (b-a) f(\frac{a+b}{2})$.

将 $[a, b]$ n 等分, 得到 G_n. 每个区间为 $I_i(f) = \int_{x_i}^{x_{i+1}} f(x) \,\mathrm{d}x \approx h f(a + ih + \frac{h}{2})$. 则

$$I \approx G_n = \sum_0^{n-1} I_i = h \sum_0^{n-1} f(a + ih + \frac{h}{2})$$

(2)

$$\sum_0^{n-1} I_i = h \sum_0^{n-1} f((a + \frac{h}{2}) + ih)$$

$$T_n(f) = \frac{h}{2} \sum_0^{n-1} (f(a + ih) + f(a + (i+1)h)) = \frac{h}{2} \sum_0^{n-1} (f(a + ih) + f(a + (i+1)h))$$

$$
\begin{aligned}
T_{2n}(f) &= \frac{h}{4}\sum_{0}^{2n-1}(f(a+ih/2)+f(a+(i+1)h/2)) \\
&= \frac{h}{4}\sum_{0}^{2n-2}(f(a+ih/2)+f(a+ih/2+h/2)) \\
&= \frac{h}{4}\sum_{0}^{n-1}(f(a+ih)+2f(a+ih+h/2)+f(a+ih+h)) \\
T_{2n}-\frac{1}{2}T_n &= \frac{h}{4}\sum_{0}^{n-1}(2f(a+ih+h/2)) \\
T_{2n}-\frac{1}{2}T_n &= \frac{h}{2}\sum_{0}^{n-1}f(a+ih+h/2) \\
T_{2n}-\frac{1}{2}T_n &= \frac{1}{2}G_n
\end{aligned}
$$

因此 $\alpha = 2$.