统计力学第一次作业

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1-1

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{V}\frac{Nk}{p} = \frac{1}{T}$$

$$\beta = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{p}\frac{Nk}{V} = \frac{1}{T}$$

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = \frac{1}{V}\frac{NkT}{p^2} = \frac{1}{p}$$

1-2

V 可表示为独立变量 T, p 的函数,

$$\mathrm{d}V = \tfrac{\partial V}{\partial T}\mathrm{d}T + \tfrac{\partial V}{\partial p}\mathrm{d}p$$

$$\mathrm{d}V = V\alpha\mathrm{d}T - V\kappa_T\mathrm{d}p$$

$$\mathrm{d}\left(\ln V\right) = \alpha\mathrm{d}T - \kappa_T\mathrm{d}p$$

$$\implies \ln V = \textstyle\int\left(\alpha\mathrm{d}T - \kappa_T\mathrm{d}p\right)$$

代入 $\alpha = \frac{1}{T},\ \kappa_T = \frac{1}{p}$,

$$\ln V = \int\left(\mathrm{d}\left(\ln T\right) - \mathrm{d}\left(\ln p\right)\right) = \ln T - \ln p + C$$

$$\implies \ln\left(\frac{pV}{T}\right) = C \implies pV = k_0 T = nRT$$

1-4

(a)

$$\alpha\Delta T = \kappa_T \Delta p$$

$$\Delta p = \frac{4.85\times 10^{-5}\times 10}{7.8\times 10^{-7}} = 622 p_n$$

(b)

$$\Delta V/V_0 = \alpha\Delta T - \kappa_T\Delta p = 4.85\times 10^{-5}\times 10 - 7.8\times 10^{-7}\times 100 = 4.07\times 10^{-4}$$

1-5

$$\mathrm{d}\mathscr{T} = \frac{\partial \mathscr{T}}{\partial L}\mathrm{d}L + \frac{\partial \mathscr{T}}{\partial T}\mathrm{d}\mathscr{T} = \frac{AE}{L}\mathrm{d}L + \frac{AE}{L}\alpha L\mathrm{d}T$$

当两端固定, $\Delta L = 0$,

$$\mathrm{d}\mathscr{T} = AE\alpha\mathrm{d}T \implies \Delta\mathscr{T} = -EA\alpha\left(T_2 - T_1\right)$$

□

1-6

(a)

$$E = \frac{L}{A}\left(\frac{\partial \mathscr{T}}{\partial L}\right)_T = \frac{L}{A}\times bT\left(\frac{1}{L_0} + 2\frac{L_0^2}{L^3}\right) = \frac{bT}{A}\times\left(\frac{L}{L_0} + \frac{2L_0^2}{L^2}\right)$$

(b) $\mathscr{T}$ 不变, 对物态方程求 T 的偏导:

$$0 = \mathrm{d}\mathscr{T} = \left[\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right) + T\left(-\frac{L}{L_0^2} - 2\frac{L_0}{L^2}\right)\frac{\mathrm{d}L_0}{\mathrm{d}T}\right]\mathrm{d}T + T\left(\frac{1}{L_0} + \frac{2L_0^2}{L^3}\right)\mathrm{d}L$$

$$\implies \alpha T\left(\frac{L^3 + 2L_0^3}{L_0 L^2}\right) = -\frac{L^3 - L_0^3}{L_0 L^2} + T\frac{\mathrm{d}L_0}{\mathrm{d}T}\left(\frac{L}{L_0^2} + 3\frac{2L_0}{L^2}\right)$$

$$\implies \alpha = \frac{1}{L_0}\frac{\mathrm{d}L_0}{\mathrm{d}T} - \frac{1}{T}\frac{L^3/L_0^3 - 1}{L^3/L_0^3 + 2} = \alpha_0 - \frac{1}{T}\frac{L^3/L_0^3 - 1}{L^3/L_0^3 + 2}$$

□

1-8

$$pV = C_1 T,\ pV^n = C_2 \implies TV^{n-1} = C_3 \implies V + (n-1)\,T\frac{\mathrm{d}V}{\mathrm{d}T} = 0 \implies \frac{\mathrm{d}V}{\mathrm{d}T} = \frac{-V}{(n-1)\,T}$$

$$C_n = \frac{\text{đ}Q}{\mathrm{d}T} = C_V + p\frac{\mathrm{d}V}{\mathrm{d}T} = C_V - \frac{pV}{(n-1)\,T} = C_V - \frac{\gamma-1}{n-1}C_V = \frac{n-\gamma}{n-1}C_V$$

□

1-9

$$C_n = \frac{\text{đ}Q}{\mathrm{d}T} = C_V + p\frac{\mathrm{d}V}{\mathrm{d}T} = const, \implies \frac{p\frac{\mathrm{d}V}{\mathrm{d}T}}{C_n - C_V} = 1 = \frac{1}{C_p - C_V}\frac{pV}{T} \implies \mathrm{d}\left(\ln V^{C_p - C_V}\right) = \mathrm{d}\left(\ln T^{C_n - C_V}\right)$$

$$\implies V^{\frac{C_p - C_V}{C_n - C_V}} = const \cdot T = const \cdot pV \implies PV^{1-\frac{C_p - C_V}{C_n - C_V}} = PV^{\frac{C_n - C_p}{C_n - C_V}} = const$$

□

1-10

$$a^2 = \frac{\partial p}{\partial V}\frac{\partial V}{\partial \rho} + \frac{\partial p}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}U}\frac{\mathrm{d}U}{\mathrm{d}\rho} = \frac{NkT}{-V^2}\frac{m}{-(m^2/V^2)} + \frac{Nk}{V}\frac{1}{C_V}\frac{pV^2}{m} = \frac{\gamma NkT}{m}$$

$$u = \int \frac{1}{m}C_V \mathrm{d}T = \frac{C_V T}{m} + u_0 = \frac{NkT}{(\gamma-1)\,m} + u_0 = \frac{a_0^2}{\gamma\,(\gamma-1)} + u_0$$

$$h = u + \frac{pV}{m} = u + \frac{a^2}{\gamma} = \frac{a_0^2}{\gamma - 1} + u_0$$

□

1-12

$$\mathrm{d}\,(\ln VF) = \mathrm{d}\,(\ln V + \ln F) = \mathrm{d}\ln V + \frac{\mathrm{d}\ln T}{(\gamma-1)}$$

$$pV^{\gamma} = const \implies TV^{\gamma-1} = const$$

关于 T 求导:

$$\mathrm{d}T + T\ln V\mathrm{d}\gamma = 0 \implies \mathrm{d}\ln T + \ln V\mathrm{d}\gamma = (\gamma - 1)\left(\mathrm{d}\left(\ln VF\right) - \mathrm{d}\ln V\right) + \ln V\mathrm{d}\gamma = 0$$

$$\implies \mathrm{d}\left(\ln VF\right) = 0 \implies VF = const$$

1-13

等温过程不做功, 绝热过程中, 由于 $TV^{\gamma-1} = const$, $\frac{V_2}{V_1} = \frac{V_3}{V_4}$.

$$\Delta W = -\int_{T_1}^{T_2} p\mathrm{d}V = Nk\left(T_1 - T_2\right)\ln\frac{V_2}{V_1}$$

$$\eta = \frac{W}{Q} = \frac{\left(T_1 - T_2\right)\ln\frac{V_2}{V_1}}{T_1\ln\frac{V_2}{V_1}} = 1 - \frac{T_2}{T_1}$$

□