The 7th HW of Electrodynamics

肖涵薄 31360164

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Q1

A spherical ball of radius a...

The space have two parts of potential:

$$\Phi_1 = \sum \left(a_n r^n + \frac{b_n}{r^{n+1}} \right) P_n \left(\cos\theta \right), \quad r < a$$

$$\Phi_2 = \sum \left(a'_n r^n + \frac{b'_n}{r^{n+1}} \right) P_n \left(\cos\theta \right), \quad r > a$$

For Φ_1:

$$\Phi_1 \left(0 \right) < \infty \implies b_n = 0$$

And for Φ_2 :

$$\Phi_2 \left(\infty \right) = -H_0 r \cos\theta \implies \begin{cases} a_n = 0, \ n > 1 \\ a_1 = -H_0 \end{cases}$$

Thus

$$\Phi_1 \left(r \right) = \sum a_n r^n P_n \left(\cos\theta \right)$$

$$\Phi_2 \left(r \right) = -H_0 r \cos\theta + \sum \frac{b'_n}{r^{n+1}} P_n \left(\cos\theta \right)$$

Two potentials are equal at $r = a$:

$$a_n a^n = \frac{b'_n}{a^{n+1}} \left(n > 1 \right), \ a_1 a = -H_0 a + \frac{b'_1}{a^2}$$

And for B_n, they are equal:

$$-B_{1n} \left(r \right) = \mu \frac{\partial \Phi_1}{\partial r} = \sum \mu n a_n r^{n-1} P_n \left(\cos\theta \right)$$

$$-B_{2n}\left(r\right)=\mu_{0}\frac{\partial\Phi_{2}}{\partial r}=B_{0}\cos\theta-\sum\mu_{0}\left(n+1\right)\frac{b_{n}'}{r^{n+2}}P_{n}\left(\cos\theta\right)$$

Thus,

$$\mu na_{n}a^{n-1}=-\mu_{0}\left(n+1\right)\frac{b_{n}'}{a^{n+2}},\ n>1$$

$$\mu a_{1}=B_{0}-2\mu_{0}\frac{b_{1}'}{r^{3}}$$

Finally, we have

$$a_{n}a^{2n+1}=-\frac{\mu na_{n}a^{2n+1}}{\mu_{0}\left(n+1\right)}\implies a_{n}=b_{n}=0\left(n>1\right)$$

$$\frac{B_{0}}{\mu_{0}}+\frac{b_{1}'}{a^{3}}=\frac{B_{0}}{\mu}-\frac{2\mu_{0}b_{1}'}{\mu r^{3}}$$

$$\left(\frac{1}{a^{3}}+\frac{2\mu_{0}}{\mu a^{3}}\right)b_{1}'=\left(\frac{1}{\mu}-\frac{1}{\mu_{0}}\right)B_{0}$$

$$b_{1}'=\frac{\left(\mu_{0}-\mu\right)B_{0}a^{3}}{\mu_{0}\mu+2\mu_{0}^{2}}$$

$$a_{1}=\frac{B_{0}}{\mu_{0}}+\frac{\left(\mu_{0}-\mu\right)B_{0}}{\mu_{0}\mu+2\mu_{0}^{2}}=\frac{3B_{0}}{\mu+2\mu_{0}}$$

Thus:

$$B_{1n}\left(r\right)=-\frac{3\mu B_{0}}{\mu+2\mu_{0}}\cos\theta$$

$$B_{2n}\left(r\right)=-B_{0}\cos\theta+2\frac{\left(\mu_{0}-\mu\right)B_{0}a^{3}}{\mu+2\mu_{0}}\frac{1}{r^{3}}\cos\theta$$

$$B_{1t}=-\mu\frac{\partial\Phi_{1}/r}{\partial\theta}=-\mu a_{1}\frac{\partial\cos\theta}{\partial\theta}=\frac{3\mu B_{0}}{\mu+2\mu_{0}}\sin\theta$$

$$B_{2t}=-\mu\frac{\partial\Phi_{2}/r}{\partial\theta}=\frac{\mu B_{0}}{\mu_{0}}\sin\theta+\frac{\mu}{r^{3}}\frac{\left(\mu_{0}-\mu\right)B_{0}a^{3}}{\mu_{0}\mu+2\mu_{0}^{2}}\sin\theta$$

$$\begin{aligned}
B_{1}\left(r,t\right)&=\left[-\frac{3\mu B_{0}}{\mu+2\mu_{0}}\cos\theta,\frac{3\mu B_{0}}{\mu+2\mu_{0}}\sin\theta\right]\\
B_{2}\left(r,t\right)&=\left[-B_{0}\cos\theta+2\frac{\left(\mu_{0}-\mu\right)B_{0}a^{3}}{\mu+2\mu_{0}}\frac{1}{r^{3}}\cos\theta,\frac{\mu B_{0}}{\mu_{0}}\sin\theta+\frac{\mu}{r^{3}}\frac{\left(\mu_{0}-\mu\right)B_{0}a^{3}}{\mu_{0}\mu+2\mu_{0}^{2}}\sin\theta\right]\\
H_{1}\left(r,t\right)&=\left[-\frac{3B_{0}}{\mu+2\mu_{0}}\cos\theta,\frac{3B_{0}}{\mu+2\mu_{0}}\sin\theta\right]\\
H_{2}\left(r,t\right)&=\left[-B_{0}\cos\theta/\mu_{0}+2\frac{\left(\mu_{0}-\mu\right)B_{0}a^{3}}{\mu\mu_{0}+2\mu_{0}^{2}}\frac{1}{r^{3}}\cos\theta,\frac{\mu B_{0}}{\mu_{0}^{2}}\sin\theta+\frac{\mu}{r^{3}}\frac{\left(\mu_{0}-\mu\right)B_{0}a^{3}}{\mu_{0}^{2}\mu+2\mu_{0}^{3}}\sin\theta\right]
\end{aligned}$$

Q2

Let R be the radius of current loop, and r be the distance of current unit and field point. Following the Biot-Savart Law, the magnetic field of one current loop at direction x is:

$$B_0 = \frac{\mu_0 NI}{4\pi r^2} 2\pi R \sin\theta = \frac{\mu_0 NI}{4\pi R^2 / \sin^2\theta} 2\pi R \sin\theta = \frac{\mu_0 NI}{2R} \sin^3\theta$$

and integrate over L:

$$B = \int_L B_0 \, \mathrm{d}x = \int_{\theta_1}^{\pi-\theta_2} B_0 \frac{R \, \mathrm{d}\theta}{\sin^2\theta} = \int_{\theta_1}^{\pi-\theta_2} \frac{\mu_0 NI}{2} \sin\theta \, \mathrm{d}\theta = \frac{\mu_0 NI}{2} (\cos\theta_1 + \cos\theta_2)$$