The 8th HW of Electrodynamics

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Q1

Two particles with magnetic dipole moment m1 and m2 and are placed at locations with distance R

(a)

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{3(\vec{m}\cdot\widehat{R})\widehat{R}-\vec{m}}{R^3}$$

可知当 $R \perp m_1$, B 与 m_1 方向相反. 平衡时要求 $M = m \times B = 0$, m_2 与 $B(m_1)$ 同向为稳定平衡, 反向时为不稳定平衡, 即 m_1, m_2 方向平行时可以平衡.

(b) 当 m 与 R 平行, $B = \frac{\mu_0}{4\pi}\frac{2\boldsymbol{m}}{R^3}$. 当 m_1, m_2 同向, 达到稳定平衡, 反向则为不稳定平衡.

(c) 平衡时 m_1 产生的磁场与 m_2 平行. 则需要 B_1 平行于 m_2. 设 $\boldsymbol{m}_1 = |m_1|(x_1, y_1, z_1)$, $\boldsymbol{m}_2 = |m_2|(x_2, y_2, z_2)$. 其中 $\sqrt{x^2+y^2+z^2} = 1$. 由于 m_1 与 B_2 平行. 取 $\widehat{R} = (1,0,0)$.

$$m_1 \times B_2 = 0$$

$$m_1 \times \left((m_2\cdot\widehat{R})\widehat{R} - m_2\right) = 0$$

$$\begin{aligned}(x_1,y_1,z_1)\times\{[(x_2,y_2,z_2)\cdot(1,0,0)]\,(1,0,0)-(x_2,y_2,z_2)\} &= 0\\ (x_1,y_1,z_1)\times\{(x_2)(1,0,0)-(x_2,y_2,z_2)\} &= 0\\ (x_1,y_1,z_1)\times(0,y_2,z_2) &= 0\\ (y_1z_2-y_2z_1,-x_1z_2,x_1y_2) &= 0\end{aligned}$$

由对称性, 共有:

$$\begin{cases} y_1z_2 - y_2z_1 = 0 \\ x_1z_2 = x_2z_1 = 0 \\ x_2y_1 = x_1y_2 = 0 \end{cases}$$

有两种情况满足上式,

1. 两个偶极子互相平行, 且垂直于 R.

2. 两个偶极子均平行于 R 方向.

Q2

设镜像偶极子在 d' 处, 偶极矩大小为 m', 空间距球心 R, 与偶极子距离为 r, 夹角为 θ 的一点磁势为

$$\varphi = \frac{m\cos\theta}{4\pi r^2} + \frac{m'\cos\theta'}{4\pi r'^2}$$

$d^2 + r^2 - R^2 = 2dr\cos\theta$, 则 $\frac{\mathrm{d}r}{\mathrm{d}R} = \frac{R}{r-d\cos\theta}$

$$\frac{\partial\varphi}{\partial R} = -\frac{m\cos\theta}{2\pi r^3}\frac{R}{r-d\cos\theta} - \frac{m'\cos\theta'}{2\pi r'^3}\frac{R}{r'-d'\cos\theta'}$$

在 $R = a$ 处, $B_R = -\frac{\partial\varphi}{\partial R} = 0$

$$0 = -\frac{m\cos\theta}{r^3}\frac{1}{r-d\cos\theta} - \frac{m'\cos\theta'}{r'^3}\frac{1}{r'-d'\cos\theta'}$$

$$\implies m' = -(\frac{a}{d})^3 m, \quad z = d' = \frac{a^2}{d}$$