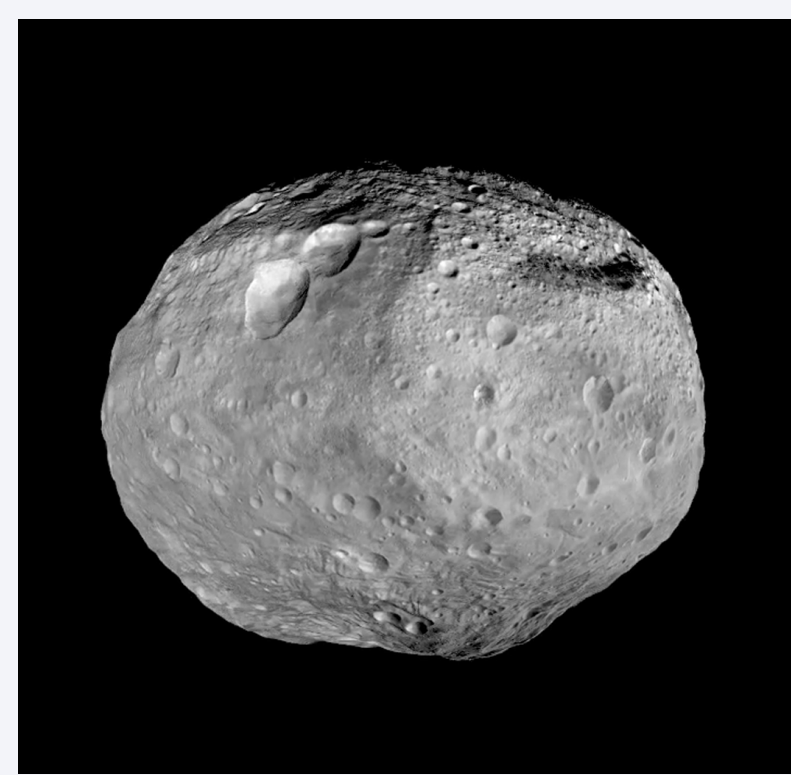


## Asteroids in the Solar System

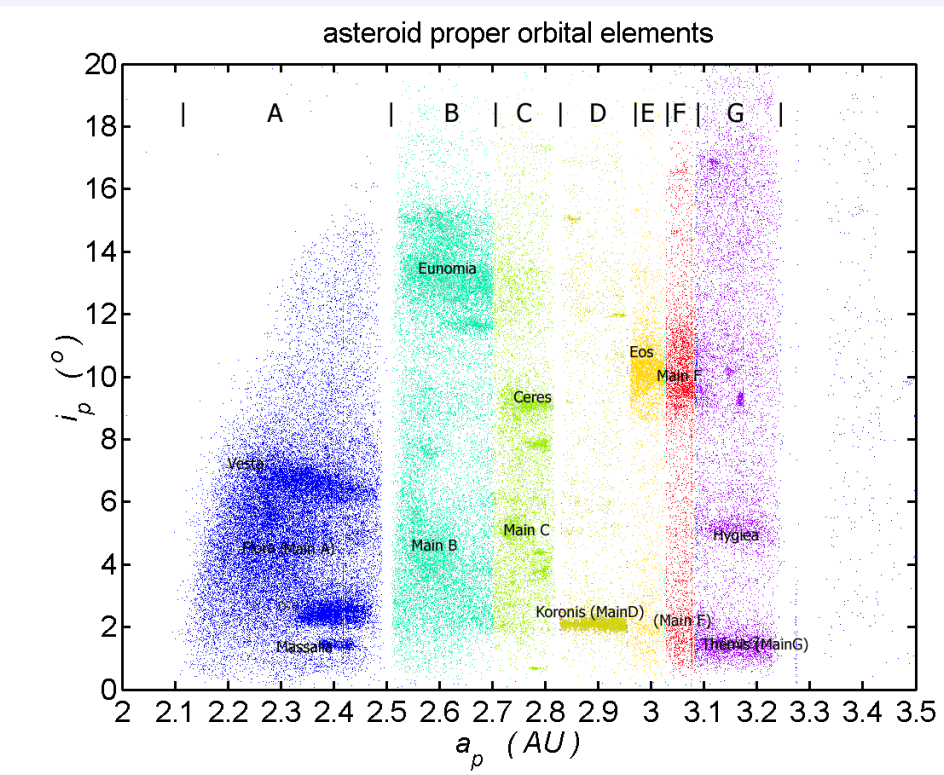
**Asteroids** is the most numerous and also most interesting group of bodies in the **Solar System**. The first asteroid was discovered in 1801 and more than half a million asteroids are known today.

In the **main asteroid belt** between *Mars* and *Jupiter*, asteroids form **families** — groupings created by a initial **breakup** of the same parent body, caused by a collision with another body. In our work, we focus on a large family called *Eunomia*, located in the middle main belt.

By studying collisional families, we can find out more about the creation of the Solar System and its **dynamical structure** [1], for example we can support the **Late Heavy Bombardment** theory [2].



(a) Asteroid (4) Vesta — second largest and most massive body of the main belt.



(b) Main asteroid belt in the space of **proper orbital elements** — proper semi-major axis  $a_p$  and proper inclination  $\sin i_p$ .

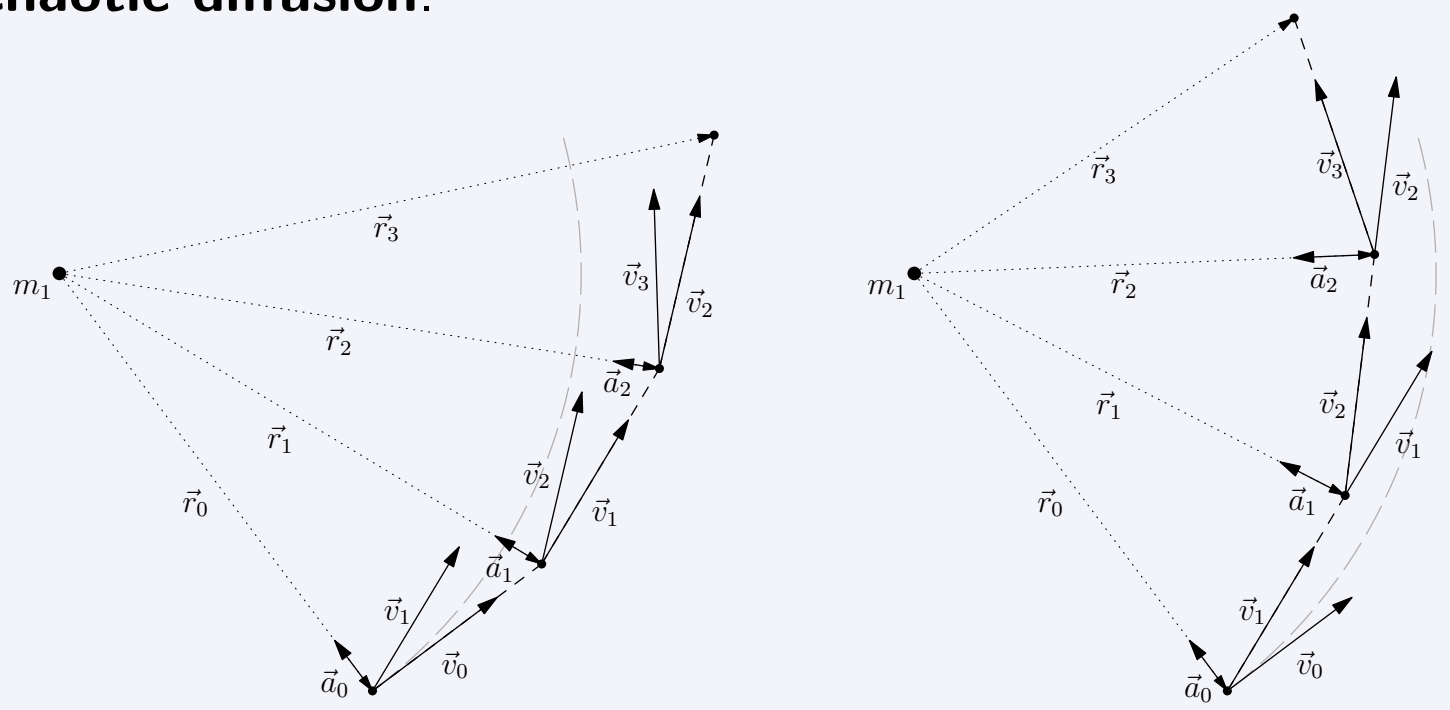
## Methods of celestial mechanics

The fundamental problem of celestial mechanics is the **N-body problem** — calculate the position of bodies, that are gravitationally bound together according Newton's law of universal gravitation, at any time.

$$\vec{F}_i = m_i \vec{a}_i = - \sum_{j=1}^N G \frac{m_i m_j}{|\vec{r}_i - \vec{r}_j|^3} (\vec{r}_i - \vec{r}_j), \quad \text{pro } i \in \{1, 2, \dots, N\}$$

For simulation of orbital evolution, we use a **numerical integrator SWIFT**, which counts with

- **Yarkovsky effect**,
- **YORP effect**,
- **random collisions**,
- **chaotic diffusion**.

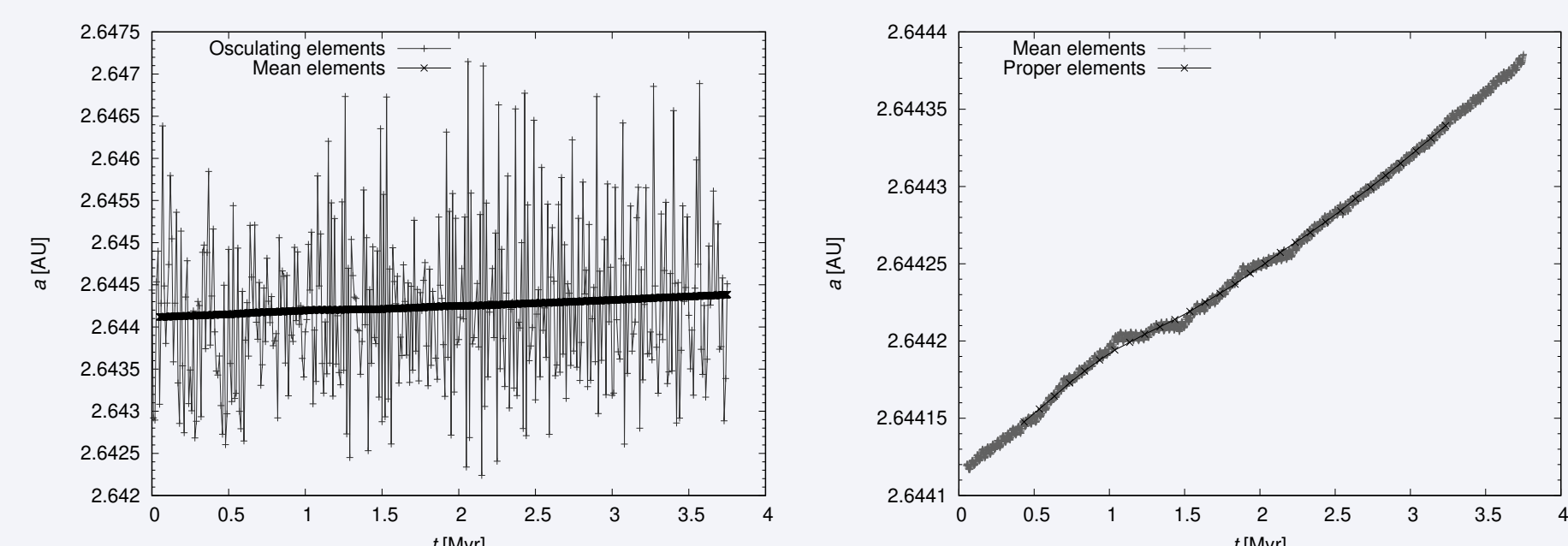


Obrázek: Illustration of a simpler integration method — **Euler's method** — which is in principle similar to ours.

The orbit around the Sun of an asteroid can be described with **orbital elements**:

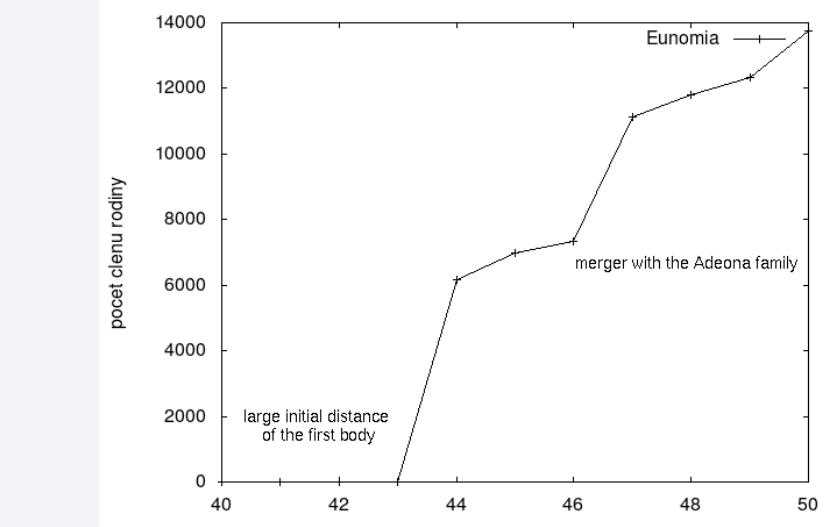
- **semi-major axis**  $a$
- **eccentricity**  $e$
- **inclination**  $i$  (or also  $\sin i$ )

They are subject to change by **perturbations** (e.g. gravitational forces of other planets); we can thus „average“ them over long periods to **mean** and to **proper orbital elements**, where the latter are not subject to any periodical forces.



Obrázek: Comparison of **osculating** (actual) and **mean** semi-major axis (left), and **mean** and **proper** semi-major axis (right) for one particle simulated for 3.76 million years.

For identification of members of a family, we use the **hierarchical clustering method (HCM)**: in the phase space ( $a_p, e_p, \sin i_p$ ) we choose a cut-off „distance“ of bodies  $v_{\text{cutoff}}$  (with units of velocity), according to which, beginning with the parent body (15) *Eunomia*, we then determine the members.

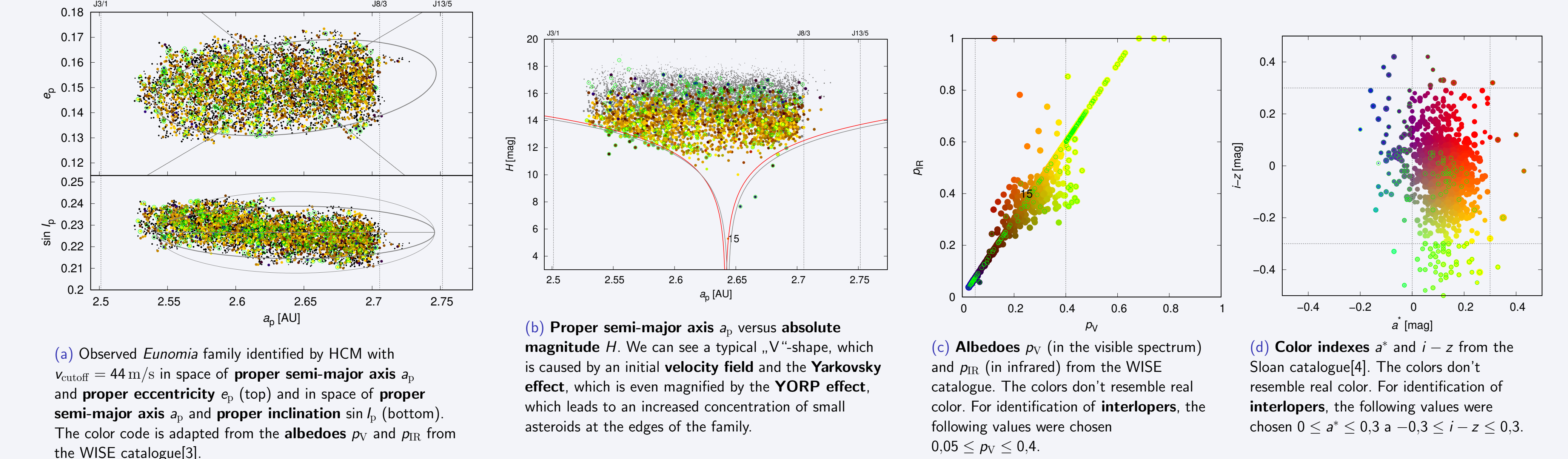


Obrázek: Dependence of the number of the members of the family *Eunomia* on the chosen cut-off velocity  $v_{\text{cutoff}}$  while using the HCM.

$$v_{\text{cutoff}} = na_p \sqrt{C_a \left( \frac{\Delta a_p}{a_p} \right)^2 + C_e (\Delta e_p)^2 + C_i (\Delta \sin i_p)^2}$$

## Identification of members of the Eunomia family

For determining the *Eunomia* family, we used the clustering algorithm. Then, we removed **interlopers** using the relationship between **semi-major axis drift**  $\Delta a_p$  and **absolute magnitude**  $H$ , and using two spectroscopic methods — the relationship of **albedoes**  $p_V$  a  $p_{IR}$  and the relationship of **color indexes**  $a^* - i - z$ . Before the removal, the member count was 6503; after using all the mentioned methods it was 6184.



(a) Observed *Eunomia* family identified by HCM with  $v_{\text{cutoff}} = 44 \text{ m/s}$  in space of **proper semi-major axis**  $a_p$  and **proper eccentricity**  $e_p$  (top) and in space of **proper semi-major axis**  $a_p$  and **proper inclination**  $\sin i_p$  (bottom). The color code is adapted from the **albedoes**  $p_V$  and  $p_{IR}$  from the WISE catalogue[3].

(b) **Proper semi-major axis**  $a_p$  versus **absolute magnitude**  $H$ . We can see a typical „V“-shape, which is caused by an initial **velocity field** and the **Yarkovsky effect**, which is even magnified by the **YORP effect**, which leads to an increased concentration of small asteroids at the edges of the family.

(c) **Albedoes**  $p_V$  (in the visible spectrum) and  $p_{IR}$  (in infrared) from the WISE catalogue. The colors don't resemble real color. For identification of **interlopers**, the following values were chosen  $0.05 \leq p_V \leq 0.4$ .

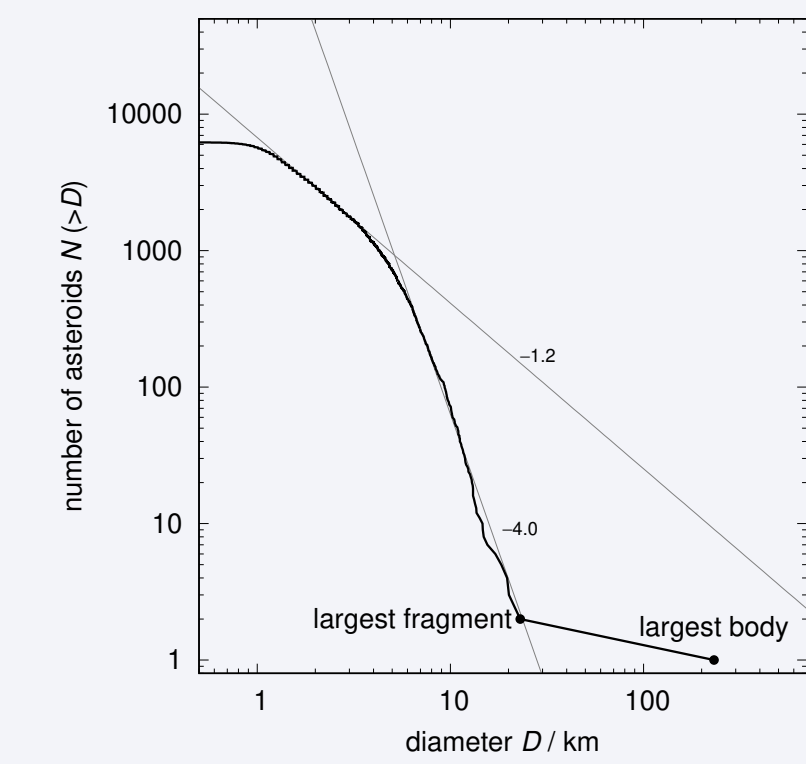
(d) **Color indexes**  $a^* - i - z$  from the Sloan catalogue[4]. The colors don't resemble real color. For identification of **interlopers**, the following values were chosen  $0 \leq a^* \leq 0.3$  a  $-0.3 \leq i - z \leq 0.3$ .

## Simulation of orbital evolution

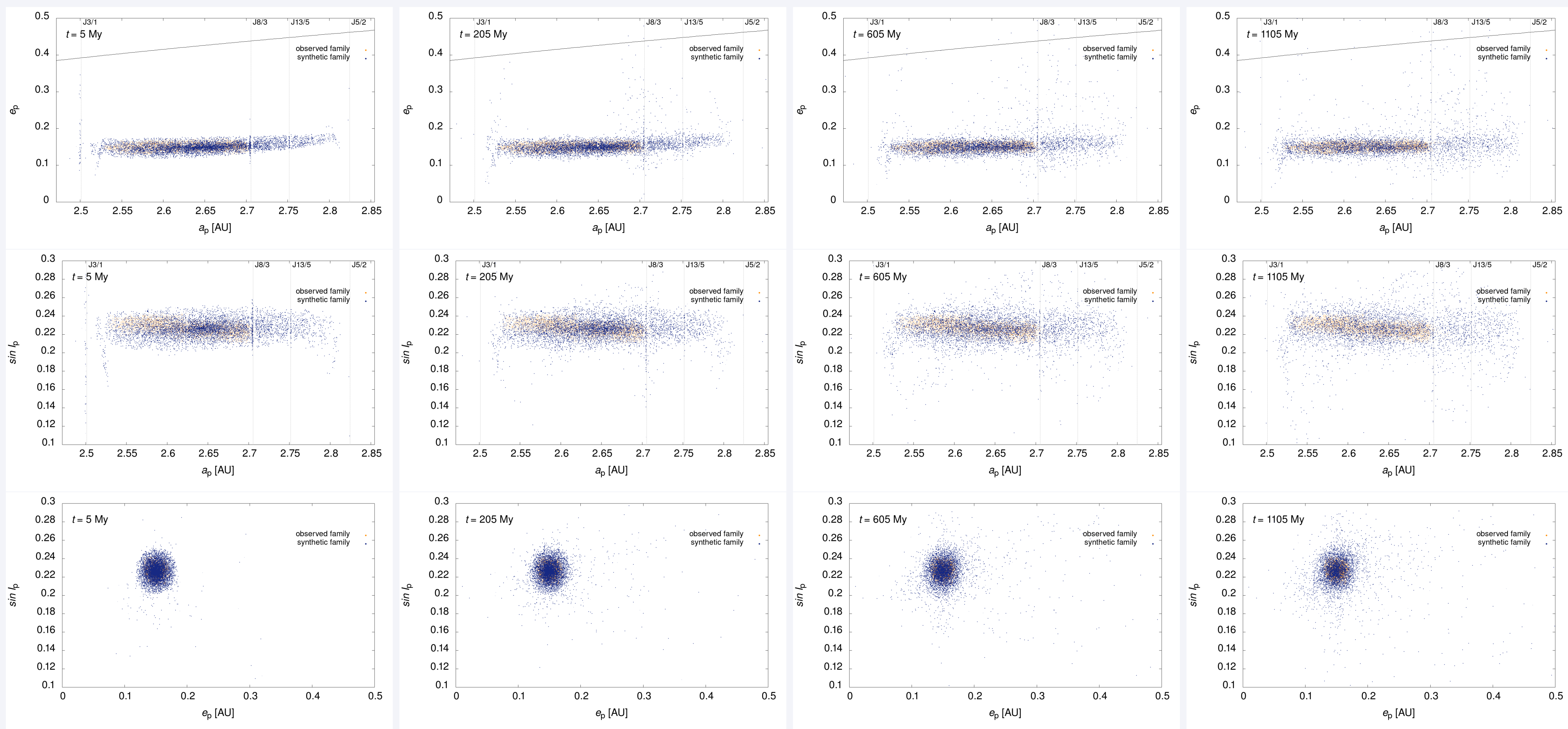
When creating the **synthetic** population of asteroids, we assigned the following properties to the particles

- **diameters** (from observed data — we took the **size-frequency distribution** into account),
- **albedoes** (from observed data),
- **rotational axis orientations** (randomly; influence on the **Yarkovsky effect**),
- **initial velocities** (simulating an **isotropic breakup** at the location on the orbit with values  $f = 90^\circ$  and  $\omega + f = 50^\circ$ ).

We simulated a population of **6210 particles** for **1.3 billion years**. The computation was run on a **server of the Astronomical Institute of Charles University**; it took around **50000 CPU hours** and the total amount of **binary data** was **3GB**.

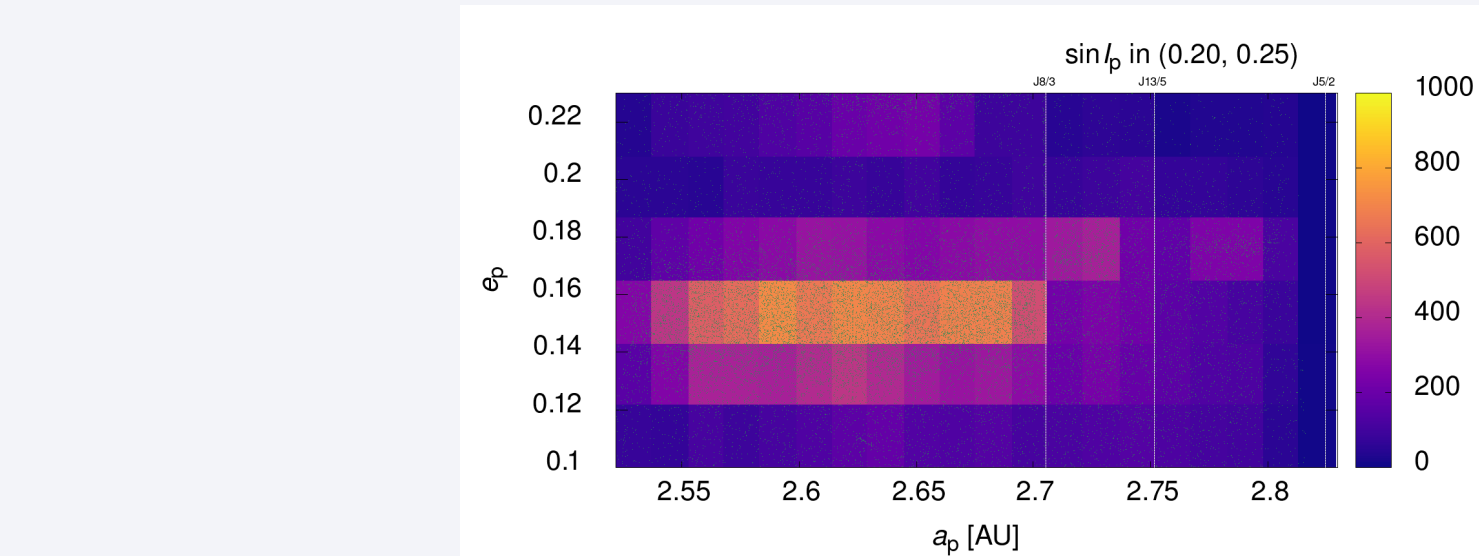


Obrázek: **Size-frequency distribution** of the *Eunomia* asteroids.

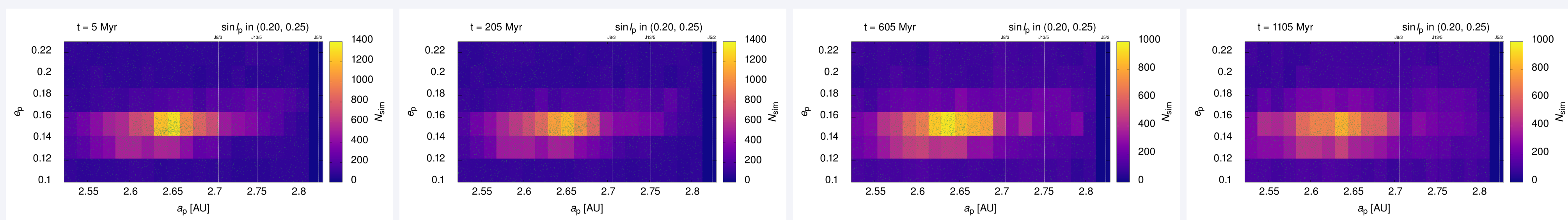


Obrázek: Results of the simulation in space of ( $a_p, e_p$ ), ( $a_p, \sin i_p$ ) and ( $e_p, \sin i_p$ ) at times  $t = 5, 205, 605, 1105$  million year. The labels J3/1, J8/3, J13/5 a J5/2 indicate the most significant **resonances** with *Jupiter*. The black line at the top indicates the edge of the region, where the asteroid's orbit crosses *Mars*'. A similar border exists for **Jupiter** as well, but it is located outside these graphs (at  $e = 0.65$ ). The purple rectangle labels the region chosen for a sample for the **background** population.

- Due to the specific **proper elements** calculation process from initial velocities, at  $t = 5$  million years, we can see a slightly unsymmetrical shape of the simulated family.
- The mechanism, through which the asteroids **leave** the family is the following: due to the **Yarkovsky effect**, the asteroid gets close to a **resonance**, the eccentricity of its orbit **increases** until it starts to **cross the orbit** of *Mars* or *Jupiter*, whereat due to a **close encounter** it gets swung out of its orbit.
- The asteroids initially located near the J5/2 resonance, were very quickly diffused, thus they are not present at the  $t = 5 \text{ My}$  graph.
- **Resonances** J8/3 and J13/5 clearly divide the family into three parts, which have different widths, and thus the asteroids in them get diffused at different rates.
- It is confirmed, that the J8/3 **resonance** is stronger than the J13/5 **resonance** (asteroids near the J8/3 resonance at  $t = 205 \text{ My}$  got diffused into a region of width  $0.05 < e_p < 0.5$ , while near the J13/5 resonance, they reached only  $0.1 < e_p < 0.23$ )
- At the ( $a_p, \sin i_p$ ) graph, we can observe a slight „tilt“ of the observer family (the part under  $a \approx 2.62 \text{ AU}$  has a higher inclination  $i_p$ ), which we can unfortunately not yet spot on the simulated family.
- With time, the concentration of asteroids in space **decreases**, which is caused by **all the present resonances**.



Obrázek: Graph ( $a_p, e_p$ ) for the observed *Eunomia* family. The color code indicates the number of particles in the given **box**.



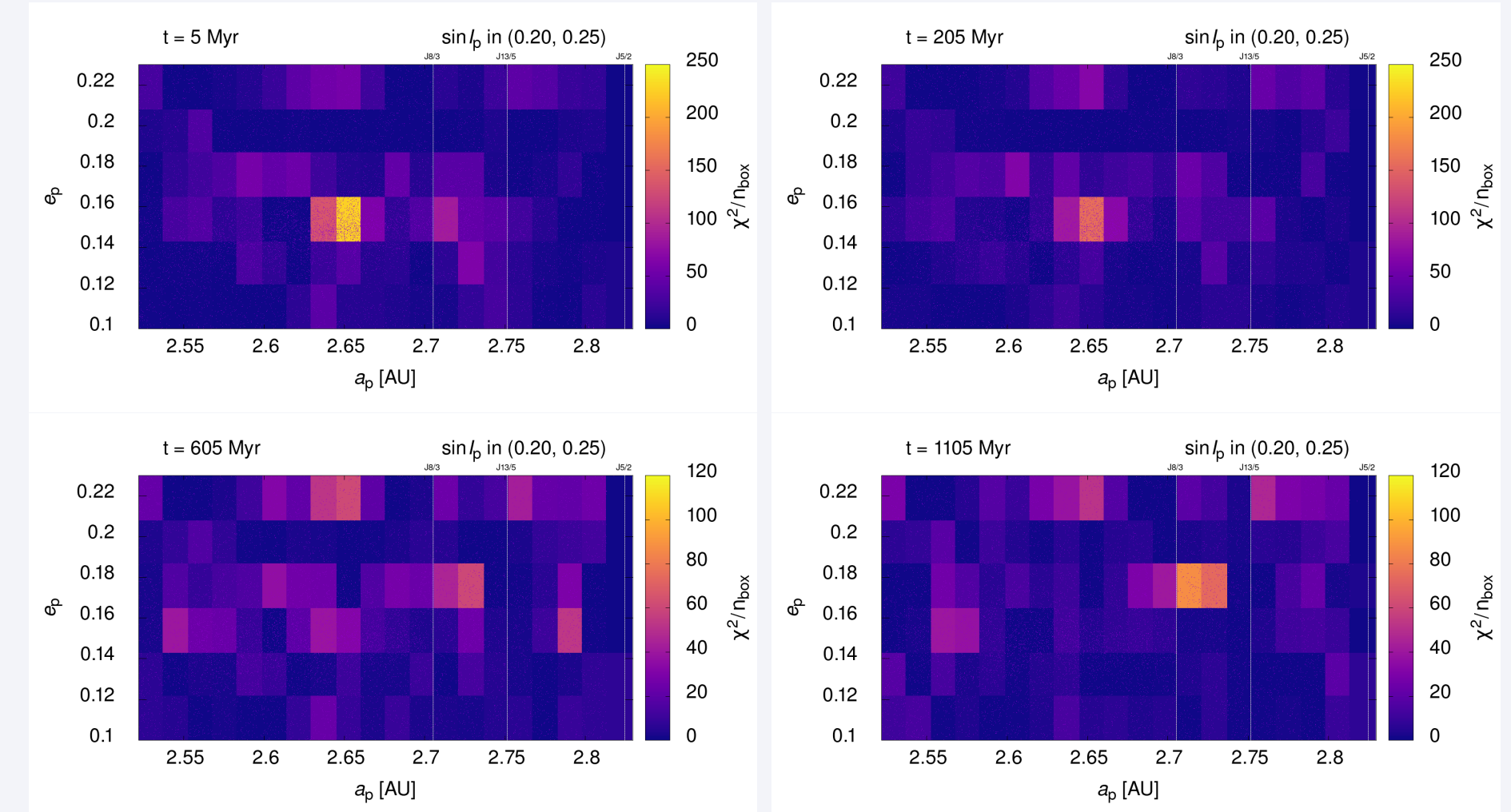
Obrázek: ( $a_p, e_p$ ) graph of the simulated *Eunomia* family for  $t = 5, 205, 605, 1105$  million years. Notice the change in the color code.

## Age of the Eunomia family

### Black-box method [5]

We divide asteroids of the observed and the simulated family into „boxes“ in space ( $a_p, e_p, \sin i_p$ ) and we compare the number of asteroids in individual **boxes**. Additionally, we „mix“ the simulated population with a sample of **background**, while keeping the **size-frequency distribution**. After this simple procedure, we calculate the chi-squared distribution ( $\chi^2$ ) of the data — for every **box**, we compute its contribution to the  $\chi^2$  value as

$$\frac{(N_{\text{sim}} - N_{\text{obs}})^2}{N_{\text{sim}} + N_{\text{obs}}}.$$

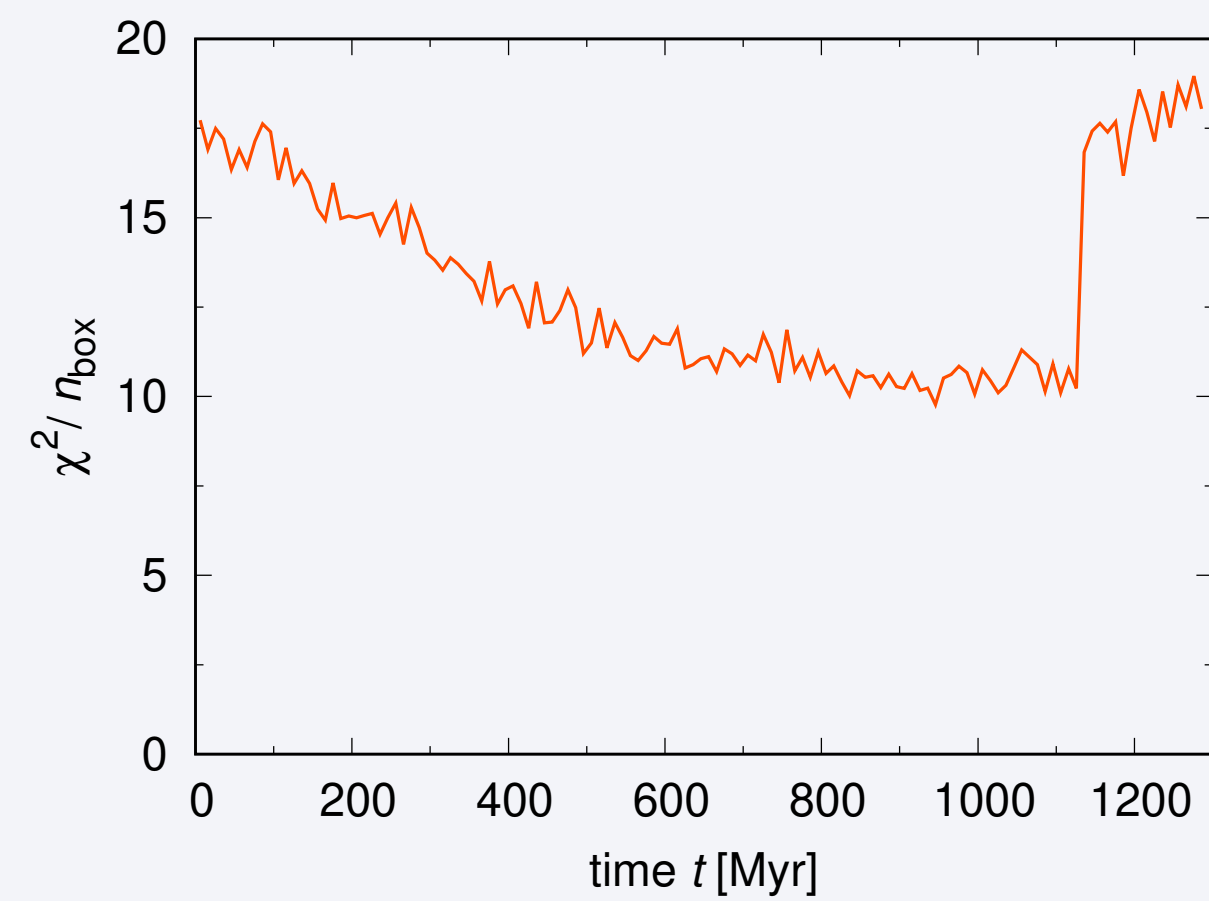


Obrázek: The  $\chi^2$  value for every **box** in space ( $a_p, e_p$ ) for  $t = 5, 205, 605, 1105$  million years. The dots show the **synthetic** population with the added **background**.

We can see, that at the beginning, the core around  $2.65 \text{ AU}$  differs the most (too many **synthetic** particles).

Due to a strong **contamination** from the *Adeona* family in the region  $0.16 < e < 0.18$ , we were forced to manually remove the observed members of this family.

We successfully described the **structure** of the *Eunomia* family, that can be seen on the graphs ( $a_p, e_p$ ), ( $a_p, \sin i_p$ ) and ( $e_p, \sin i_p$ ). Some Unfortunately, we have to attribute some phenomena (e.g. compactness of the core) to the **insufficient length of the simulated period**. With almost complete probability, we can say, the the *Eunomia* family is **not younger** than 500 million years, but we can not yet estimate an upper limit (due to the flat dependency of the  $\chi^2$  value on time).



Obrázek: **Reduced**  $\chi^2$  value versus time. The jump at  $t = 1125$  million years is due to a **loss of particles** in the simulation.

In future, we plan to simulate the *Eunomia* family for a **longer period** (4 billion years). Probably, we will get a minimal (statistically significant) value of the  $\chi^2$ , from which we will be able to accurately estimate an upper limit for the age of the *Eunomia* family.

Another option is an analysis of the **surrounding families**, especially the *Adeona* family.

We can also focus on specific **taxonomic types** of asteroids (the *Eunomia* family is **S-type**) or try an **anisotropic initial velocity field** — simulate different types of breakup (**cratering**, **reaccumulation**, **catastrophic breakup**). Furthermore, we can try **different background samples** for different regions (between the J8/3 and J13/5 resonances, the concentration of asteroids is smaller than between the J3/1 and J8/3 resonances).

After **finishing the long-term simulation**, we plan to **publish** the results in a scientific journal (*Icarus*).

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