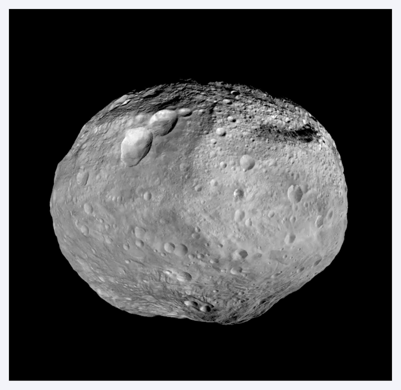
Asteroids in the Solar System

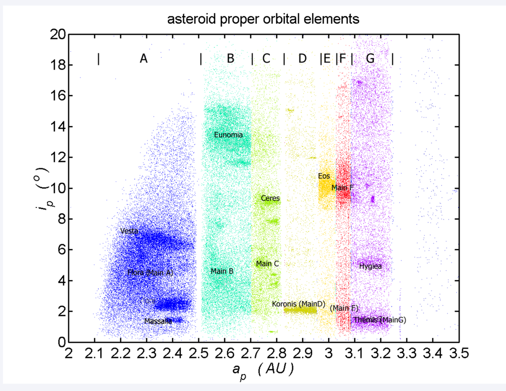
Asteroids is the most numerous and also most interesting group of bodies in **the Solar System**. The first asteroid was discovered in 1801 and more than half a million asteroids are known today.

In **the main asteroid belt** between *Mars* and *Jupiter*, asteroids form **families** — groupings created by a initial **breakup** of the same parent body, caused by a collision with another body. In our work, we focus on a large family called *Eunomia*, located in the middle main belt.

By studying collisional families, we can find out more about the creation of the Solar System and its **dynamical structure** [**nesvorny15**], for example we can support the **Late Heavy Bombardment** theory) [**broz13**].



(a) Asteroid *(4) Vesta* — second largest and most massive body of the main belt.



(b) Main asteroid belt in the space of **proper orbital elements** — proper semi-major axis a_{p} and proper inclination. $\sin I_{\mathrm{p}}$.

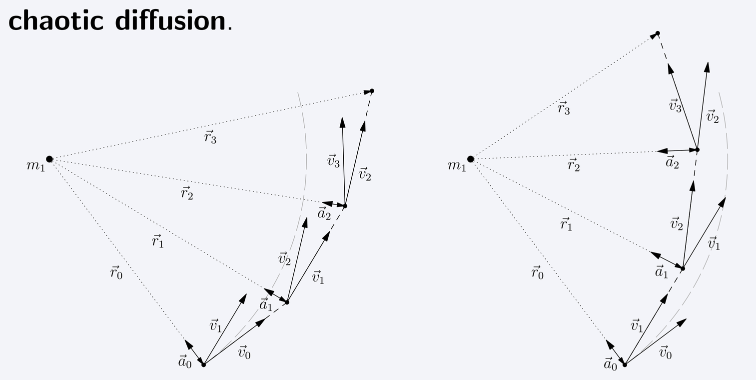
Methods of celestial mechanics

The fundamental problem of celestial mechanics is the **N-body problem** — calculate the position of bodies, that are gravitationally bound together according Newton's law of universal gravitation, at any time.

$$\vec{F}_i = m_i \vec{a}_i = -\sum_{\substack{j=1 \\ j \neq i}}^{N} G \frac{m_i m_j}{|\vec{r}_i - \vec{r}_j|^3} (\vec{r}_i - \vec{r}_j) , \qquad \text{pro } i \in \{1, 2, \dots, N\}$$

For simulation of orbital evolution, we use a **numerical integrator SWIFT**, which counts with

Yarkovsky effect,
YORP effect,
random collisions,
chaotic diffusion.



Obrázek: Illustration of a simpler integration method — **Euler's method** — which is in principle similar to ours.

The orbit around the Sun of an asteroid can be described with **orbital elements**:

semi-major axis a
eccentricity e
inclination I (or also $\sin I$)

They are subject to change by **perturbations** (e.g. gravitational forces of other planets); we can thus „average" them over long periods to **mean** and to **proper orbital elements**, where the latter are not subject to any periodical forces.

Obrázek: Comparison of **osculating** (actual) and **mean** semi-major axis (left), and **mean** and **proper** semi-major axis (right) for one particle simulated for 3.76 million years.

For identification of members of a family, we use the **hierarchical clustering method** (HCM): in the phase space (a_{p}, e_{p}, $\sin I_{\mathrm{p}}$) we choose a cut-off „distance" of bodies v_{cutoff} (with units of velocity), according to which, beginning with the parent body *(15) Eunomia*, we then determine the members.

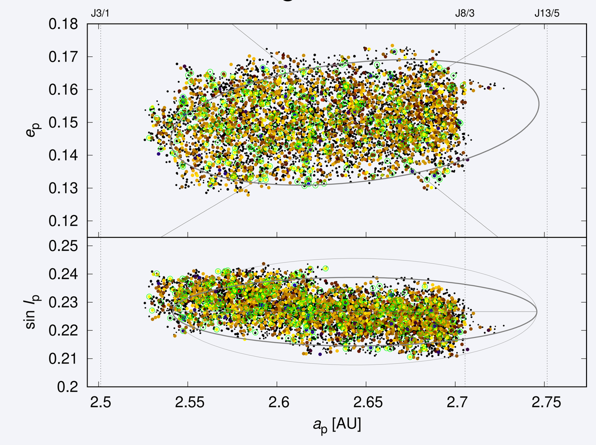
../obr/Nv_edit_en.png

Obrázek: Dependence of the number of the members of the family *Eunomia* on the chosen cut-off velocity v_{cutoff} while using the HCM.

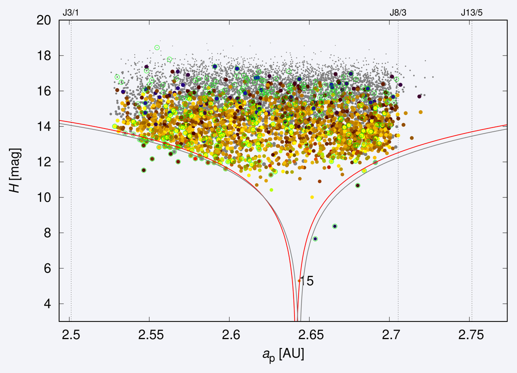
$$v_{\mathrm{cuttoff}} = n a_{\mathrm{p}} \sqrt{C_a \left(\frac{\Delta a_{\mathrm{p}}}{a_{\mathrm{p}}}\right)^2 + C_e (\Delta e_{\mathrm{p}})^2 + C_i (\Delta \sin i_{\mathrm{p}})^2}$$

Identification of members of the Eunomia family

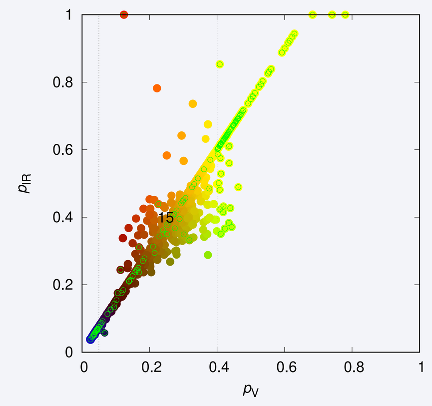
For determining the *Eunomia* family, we used the clustering algorithm. Then, we removed **interlopers** using the relationship between **semi-major axis drift** Δa_{p} and **absolute magnitude** H, and using two spectroscopic methods — the relationship of **albedoes** p_{V} a p_{IR} and the relationship of **color indexes** a^* a $i - z$. Before the removal, the member count was 6503; after using all the mentioned methods it was 6184.



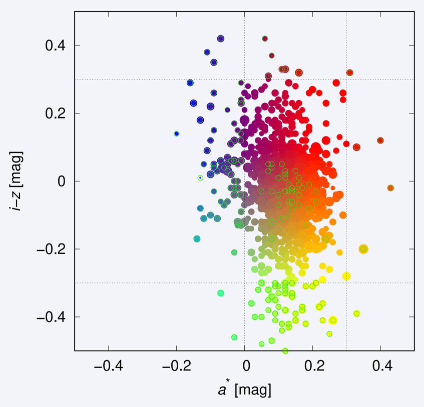
(a) Observed *Eunomia* family identified by HCM with $v_{\mathrm{cutoff}} = 44\,\mathrm{m/s}$ in space of **proper semi-major axis** a_{p} and **proper eccentricity** e_{p} (top) and in space of **proper semi-major axis** a_{p} and **proper inclination** $\sin I_{\mathrm{p}}$ (bottom). The color code is adapted from the **albedoes** p_{V} and p_{IR} from the WISE catalogue[**nugent15**].



(b) **Proper semi-major axis** a_{p} versus **absolute magnitude** H. We can see a typical „V"-shape, which is caused by an initial **velocity field** and the **Yarkovsky effect**, which is even magnified by the **YORP effect**, which leads to an increased concentration of small asteroids at the edges of the family.



(c) **Albedoes** p_{V} (in the visible spectrum) and p_{IR} (in infrared) from the WISE catalogue. The colors don't resemble real color. For identification of **interlopers**, the following values were chosen $0{,}05 \le p_{\mathrm{V}} \le 0{,}4$.



(d) **Color indexes** a^* and $i - z$ from the Sloan catalogue[**ivezic01**]. The colors don't resemble real color. For identification of **interlopers**, the following values were chosen $0 \le a^* \le 0{,}3$ a $-0{,}3 \le i - z \le 0{,}3$.

Simulation of orbital evolution

When creating the **synthetic** population of asteroids, we assigned the following properties to the particles

diameters (from observed data — we took the **size-frequency distribution** into account),
albedoes (from observed data),
rotational axis orientations (randomly; influence on the **Yarkovsky effect**),
initial velocities (simulating an **isotropic breakup** at the location on the orbit with values $f = 90°$ and $\omega + f = 50°$).

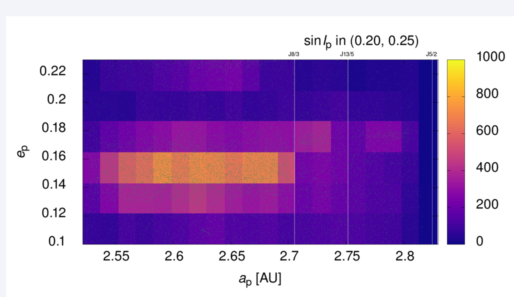
Obrázek: **Size-frequency distribution** of the *Eunomian* asteroids.

We simulated a population of **6210 particles** for 1,3 **billion years**. The computation was run on a **server of the Astronomical Institute of Charles University**; it took around **50000 CPU hours** and and the total amount of **binary data** was $3\,\mathrm{GB}$.

../obr/pngB_en/ae_5.png	../obr/pngB_en/ae_205.png	../obr/pngB_en/ae_605.png	../obr/pngB_en/ae_1105.png
../obr/pngB_en/ai_5.png	../obr/pngB_en/ai_205.png	../obr/pngB_en/ai_605.png	../obr/pngB_en/ai_1105.png
../obr/pngB_en/ei_5.png	../obr/pngB_en/ei_205.png	../obr/pngB_en/ei_605.png	../obr/pngB_en/ei_1105.png

Obrázek: Results of the simulation in space of (a_{p}, e_{p}), (a_{p}, $\sin I_{\mathrm{p}}$) and (e_{p}, $\sin I_{\mathrm{p}}$) at times $t = 5, 205, 605, 1105$ million year. The labels J3/1, J8/3, J13/5 a J5/2 indicate the most significant **resonances** with *Jupiter*. The black line at the top indicates the edge of the region, where the asteroid's orbit crosses *Mars'*. A similar border exists for **Jupiter** as well, but it is located outside these graphs (at $e = 0{,}65$). The purple rectangle labels the region chosen for a sample for the **background** population.

Due to the specific **proper elements** calculation process from initial velocities, at $t = 5$ million years, we can see a slightly unsymmetrical shape of the simulated family.
The mechanism, through which the asteroids **leave** the family is the following: due to the **Yarkovsky effect**, the asteroid gets close to a **resonance**, the eccentricity of its orbit **increases** until it starts to **cross the orbit** of *Mars* or *Jupiter*, whereat due to a **close encounter** it gets swung out of its orbit.



Obrázek: Graph (a_{p}, e_{p}) for the observed *Eunomia* family. The color code indicates the number of particles in the given **box**.

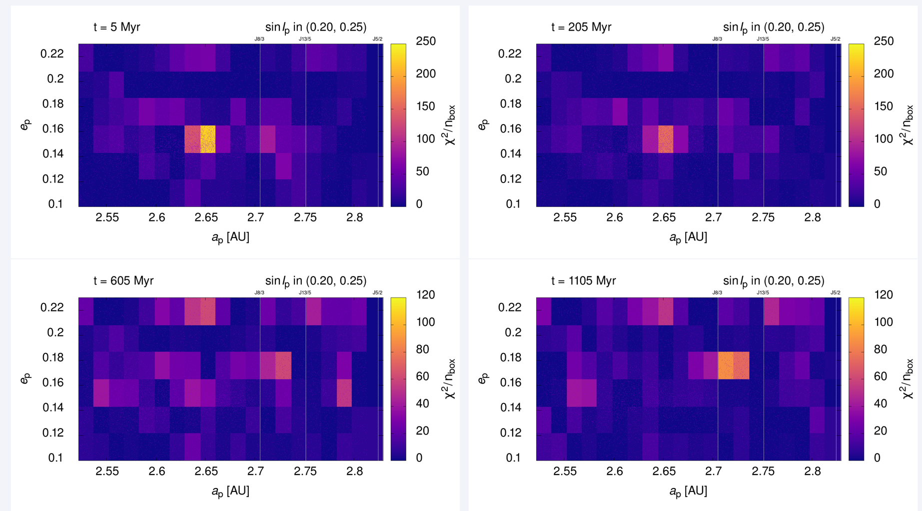
The asteroids initially located near the J5/2 resonance, were very quickly diffused, thus they are not present at the $t = 5\,\mathrm{My}$ graph.
Resonances J8/3 and J13/5 clearly divide the family into three parts, which have different widths, and thus the asteroids in them get diffused at different rates.
It is confirmed, that the J8/3 **resonance** is stronger than the J13/5 **resonance** (asteroids near the J8/3 resonance at $t = 205\,\mathrm{My}$ got diffused into a region of width $0{,}05 < e_{\mathrm{p}} < 0{,}5$, while near the J13/5 resonance, they reached only $0{,}1 < e_{\mathrm{p}} < 0{,}23$)
At the (a_{p}, $\sin I_{\mathrm{p}}$) graph, we can observe a slight „**tilt**" of the observer family (the part under $a \approx 2{,}62\,\mathrm{AU}$ has a higher inclination I_{p}), which we can unfortunately not yet spot on the simulated family.
With time, the concentration of asteroids in space **decreases**, which is caused by **all** the present **resonances**.

Age of the Eunomia family

Black-box method [broz19]

We divide asteroids of the observed and the simulated family into „boxes" in space (a_{p}, e_{p}, $\sin I_{\mathrm{p}}$) and we compare the number of asteroids in individual **boxes**. Additionally, we „mix" the simulated population with a sample of **background**, while keeping the **size-frequency distribution**. After this simple procedure, we calculate the chi-squared distribution (χ^2) of the data — for every **box**, we compute its contribution to the χ^2 value as

$$\frac{(N_{\mathrm{sim}} - N_{\mathrm{obs}})^2}{N_{\mathrm{sim}} + N_{\mathrm{obs}}} .$$

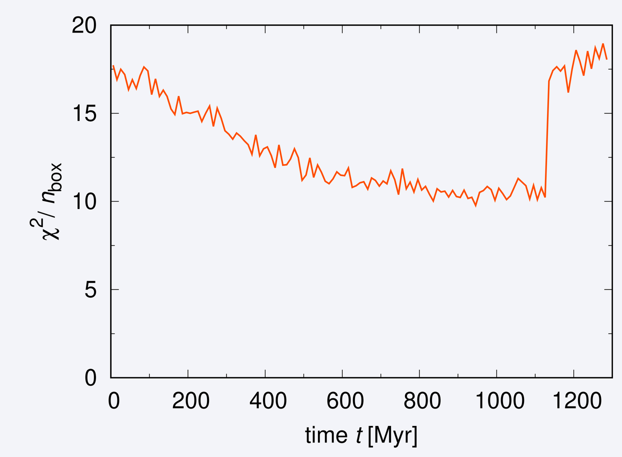


Obrázek: The χ^2 value for every **box** in space (a_{p}, e_{p}) for $t = 5$, 205, 605, 1105 million years. The dots show the **synthetic** population with the added **background**.

We can see, that at the beginning, the core around $2{,}65\,\mathrm{AU}$ differs the most (too many **synthetic** particles).

Due to a strong **contamination** from the *Adeona* family in the region $0.16 < e < 0.18$, we were forced to manually remove the observed members of this family.

We successfully described the **structure** of the *Eunomia* family, that can be seen on the graphs (a_{p}, e_{p}), (a_{p}, $\sin I_{\mathrm{p}}$) and (e_{p}, $\sin I_{\mathrm{p}}$). Some Unfortunately, we have to attribute some phenomena (e.g. compactness of the core) to the **insufficient length of the simulated period**. With almost complete probability, we can say, the the *Eunomia* family **is not younger** than 500 million years, but we can not yet estimate an upper limit (due to the flat dependency of the χ^2 value on time).



Obrázek: **Reduced** χ^2 value versus time. The jump at $t = 1125$ million years is due to a **loss of particles** in the simulation.

In future, we plan to simulate the *Eunomia* family for a **longer period** (4 billion years). Probably, we will get a minimal (statistically significant) value of the χ^2, from which we will be able to accurately estimate an upper limit for the age of the *Eunomia* family.

Another option is an analysis of the **surrounding families**, especially the *Adeona* family.
We can also focus on specific **taxonomic types** of asteroids (the *Eunomia* family is **S-type**) or try an **anisotropic initial velocity field** — simulate different types of breakup (**cratering**, **reaccumulation**, **catastrophic breakup**). Furthermore, we can try **different background samples** for different regions (between the J8/3 and J13/5 resonances, the concentration of asteroids is smaller than between the J3/1 and J8/3 resonances).

After **finishing the long-term simulation**, we plan to **publish** the results in a scientific journal (*Icarus*).

References