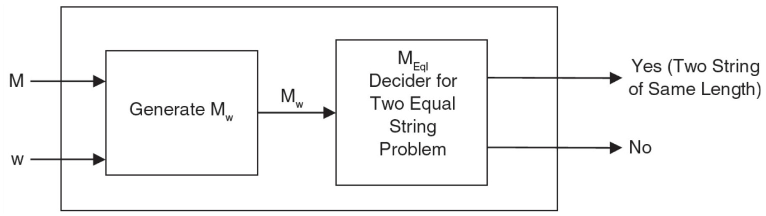
Example 10.20 Prove that the problem 'whether L(M) contains two strings of the same length' is undecidable.

Solution: This can be proved by reducing the halting problem to it. Reduce the halting problem to this problem by constructing a machine M_w using the following process.

On input w to M (the halting problem decider)

i) M_w copies w on some special location of its input tape and then performs the same computation as M using the input w written on its input tape.
ii) If w = a or w = b, M enters into one of its halting states and M_w reaches its final state. Thus,L(M_w) = a, b
iii) Otherwise, reject. Thus L(M_w) = ϕ.
The final decider is given in the following diagram.



Already,it is proved that the halting problem is undecidable.So,ME_{ql} is also undecidable.

Example 10.21 Prove that the virus detection is undecidable.

Solution: A computer virus is an executable program that can replicate itself and spread from one computer to another. Every file or program that becomes infected can act as a virus itself, allowing it to spread to other files and computers. But the virus detection problem is undecidable! This can be proved using the undecidability of halting problem.

Let us define a virus by an algorithm called the 'is-virus?'

```
For a program P,
define (halts for input P)
          if (is-virus?)          //Execute P assuming P is not a virus.
          {
           // If true
           Virus-code is evaluated. If it is a virus, and P must halt.
          }
           // If false
           Virus-code never executed. Hence, P must not halt.
```

Already, we have proved that the halting problem is undecidable. Hence, the virus detection problem is undecidable.