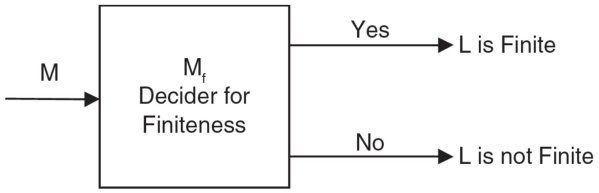
Put the output of T as the input to E. But, we know that the halting problem is undecidable, so the problem 'whether an arbitrary RE language is empty' is also undecidable.

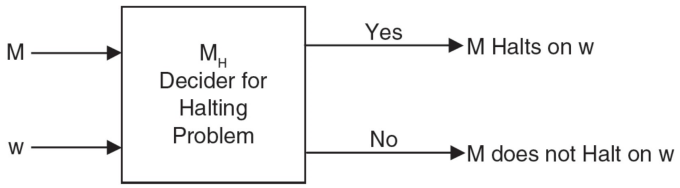
Example 10.19 Prove that the problem 'whether a recursively enumerable language L(M) is finite'is undecidable.

Solution: This can be proved by reducing the halting problem to it.

Let M be the TM accepting L. Suppose that Mf is a decider for the finite language problem which takes the TM M as input.



Suppose that there is decider for halting problem which takes TM M and a string w as input.



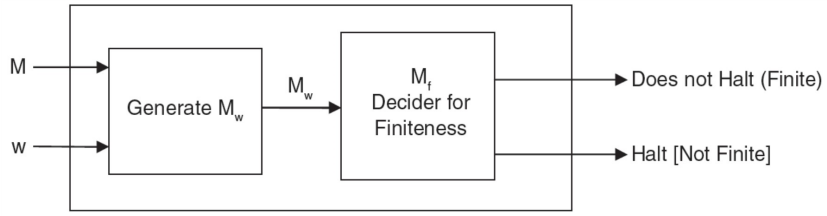
Now, reduce the halting problem to the finite language problem which constructs the machine M_w using the following process.

On input w to M,

i) Design the machine M_w such that when M enters into one of its halting states, M_w enters into its final state. M_w copies w on some special location of its input tape and then performs the same computation as M using the input w written on its input tape.
ii) If M enters into any one of its halting states, M_w halts reaching its final state.
iii) If M does not enter into its halting state, M_w does not halt and thus accepts nothing.

In other words, M_w accepts either the infinite language Σ^+ or the finite language ϕ.

The final decider is given in the following diagram.



Already,it is proved that the halting problem is undecidable.So, M_f is also undecidable