

Solution: Yes, it is possible. The sequence is 2 1 4 3 2.

1	01	010	10	1
101	0	1	0	101
2	1	4	3	2

But, for all types of cards, the solution is not possible. The game will continue for an infinite time. There is a condition which must be fulfilled to make the game halt. The condition is

1. At least one card must contain same starting character the top and bottom strings. Though if the condition is fulfilled, it cannot be said confirm that the game will halt.

The Post Correspondence Problem: Given two sequences of strings w_1, w_2, \dots, w_n and x_1, x_2, \dots, x_n over Σ . The solution of the problem is to find a non-empty sequence of integers i_1, i_2, \dots, i_k such that $w_{i_1} w_{i_2} \dots w_{i_k} = x_{i_1} x_{i_2} \dots x_{i_k}$.

The solution of PCP hides in the condition $w_{i_1} w_{i_2} \dots w_{i_k} = x_{i_1} x_{i_2} \dots x_{i_k}$.

If there exists a solution for a given instance of PCP, there may exist multiple solutions for that instance of PCP.

As an example, consider the previous example.

$\Sigma = \{0, 1\}$ and the sequence of string w_i, x_i are as follows.

i	w_i	x_i
1	01	0
2	1	101
3	10	0
4	010	1

The problem has a solution $i_1 = 2, i_2 = 1, i_3 = 4, i_4 = 3, i_5 = 2$ and the string is

101010101

Example 10.22

Find whether there is a solution for following PCP problem.

i	w_i	x_i
1	01	0
2	1	101
3	10	0
4	010	1

Solution: To get a solution, the top and bottom strings of at least one card must start with the same character. But here, the top and bottom strings of no cards start with same character. Thus, this PCP problem does not have any solution.