# RC Circuit

Lab #3

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# RC Circuit

## Objective

Infer the relationship between the time constant $\tau$, resistance $R$, and capacitance $C$ of an RC circuit.

## 1 Introduction

An RC circuit is a type of circuit made of a resistor and a capacitor connected in series, like so:

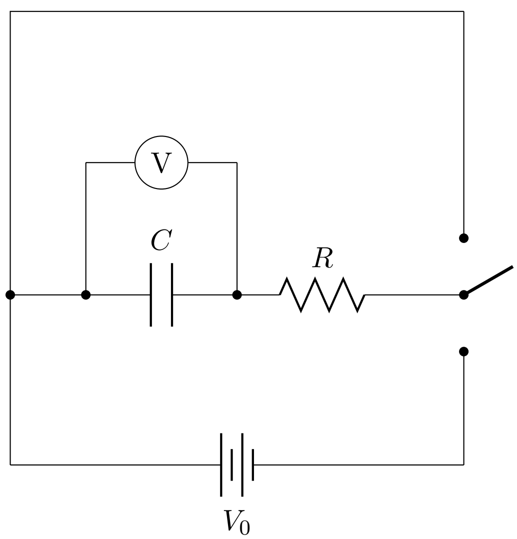


Figure 1: RC circuit

When a constant DC voltage $V_0$ is applied to the circuit, an electric field builds up inside the capacitor as it gradually charges. By Kirchhoff's voltage law, the circuit's behavior as a function of time is given by the first-order differential equation

$$V_0 - \frac{Q}{C} - R\dot{Q} = 0 \tag{1}$$

The solution to this equation is

$$V(t) = V_0\,(1 - e^{-t/RC}) \tag{2}$$

As $t$ approaches infinity, $V(t)$ approaches $V_0$.

The time constant $\tau$ of an RC circuit is defined such that

$$V(\tau) = V_0\,(1 - e^{-1}) \approx 0.63\,V_0. \tag{3}$$

Therefore,

$$\tau = RC. \tag{4}$$

Note that $\tau$ does not depend on $V_0$: the greater the applied voltage, the faster the capacitor charges.

When a capacitor is discharging into an RC circuit, it produces an exponentially decaying direct current. As a function of time, this is

$$V(t) = V_0\, e^{-t/\tau} \tag{5}$$

Substituting $t = \tau$,

$$V(\tau) = V_0\, e^{-1} \approx 0.37\, V_0. \tag{6}$$

## 2 Procedures and Results

We set up the circuit shown in Figure 1. We set $V_0$ to 4.5 V, $R$ to 1.6 k$\Omega$, and $C$ to 27 mF. We turned on the power and took voltage measurements every 5 seconds for 45 seconds.

Table 1: Charging from a 4.5-V DC power source.

| Time (s) | Voltage (V) | % of $V_0$ |
|---|---|---|
| 0 | 0 | 0.00% |
| 5 | 1.01 | 22.44% |
| 10 | 2.04 | 45.33% |
| 15 | 2.93 | 65.11% |
| 20 | 3.27 | 72.67% |
| 25 | 3.31 | 73.56% |
| 30 | 3.26 | 72.44% |
| 35 | 3.22 | 71.56% |
| 40 | 3.2 | 71.11% |
| 45 | 3.18 | 70.67% |

Next, we turned off the power and quickly disconnected the DC power source, so that the capacitor began to discharge. We took voltage measurements every 5 seconds for 60 seconds.

Table 2: Discharging after a 45-s charge.

| Time (s) | Voltage (V) | % of $V_0$ |
|---|---|---|
| 0 | 3.18 | 100.00% |
| 5 | 2.39 | 75.16% |
| 10 | 2.11 | 66.35% |
| 15 | 1.88 | 59.12% |
| 20 | 1.69 | 53.14% |
| 25 | 1.53 | 48.11% |
| 30 | 1.38 | 43.40% |
| 35 | 1.24 | 38.99% |
| 40 | 1.12 | 35.22% |
| 45 | 1.02 | 32.08% |
| 50 | 0.92 | 28.93% |
| 55 | 0.84 | 26.42% |
| 60 | 0.72 | 22.64% |

Then, we repeated these steps with a $200\,\Omega$ resistor and $V_0 = 12\,\mathrm{V}$.

Table 3: Charging with a $200\,\Omega$ resistor for 60 s.

| Time (s) | Voltage (V) | % of $V_0$ |
|---|---|---|
| 0 | 0 | 0.00% |
| 5 | 6.14 | 51.17% |
| 10 | 8.92 | 74.33% |
| 15 | 10.33 | 86.08% |
| 20 | 10.98 | 91.50% |
| 25 | 11.34 | 94.50% |
| 30 | 11.53 | 96.08% |
| 35 | 11.62 | 96.83% |
| 40 | 11.67 | 97.25% |
| 45 | 11.71 | 97.58% |
| 50 | 11.73 | 97.75% |
| 55 | 11.74 | 97.83% |
| 60 | 11.75 | 97.92% |

Table 4: Discharging for 70 s.

| Time (s) | Voltage (V) | % of $V_0$ |
|---|---|---|
| 0 | 11.76 | 100.00% |
| 5 | 5.05 | 42.94% |
| 10 | 2.36 | 20.07% |
| 15 | 1.24 | 10.54% |
| 20 | 0.67 | 5.70% |
| 25 | 0.35 | 2.98% |
| 30 | 0.2 | 1.70% |
| 35 | 0.13 | 1.11% |
| 40 | 0.08 | 0.68% |
| 45 | 0.06 | 0.51% |
| 50 | 0.04 | 0.34% |
| 55 | 0.03 | 0.26% |
| 60 | 0.02 | 0.17% |
| 65 | 0.02 | 0.17% |
| 70 | 0.02 | 0.17% |

## 3 Discussion

We know from Eq. 3 that during charging, the voltage rises to about 63% of its maximum level after one time constant. In Table 1, this occurs at roughly 15 s. This implies that $\tau \approx 15\,\mathrm{s}$. Using Eq. 4, we obtained an accepted value of $\tau = RC = (1600\,\Omega)(27\,\mathrm{mF}) = 43.2\,\mathrm{s}$. The percent error was 65%.

Similarly, by Eq. 6, during discharging, the voltage drops to about 37% of its original level after one time constant. In Table 2, this occurs between 10 s and 15 s, yielding $\tau \approx 12.5\,\mathrm{s}$. Again, the accepted value of $\tau$ is 43.2 s. The percent error was 71%.

In Table 3, the time constant observed during charging was between 5 s and 10 s, or approximately 7.5 s. In Table 4, the time constant observed during discharging was between 5 s and 10 s as well. The accepted value was $\tau = RC = (200\,\Omega)(27\,\mathrm{mF}) = 5.4\,\mathrm{s}$. The percent error was 39%.

## 4 Conclusion

The voltage in an RC circuit decays at a rate proportional to the product of the resistance and the capacitance, called the time constant, $\tau$.