1 Pauli Matrices

$$\sigma_a = \begin{pmatrix} \delta_{a3} & \delta_{a1} - i\delta_{a2} \\ \delta_{a1} + i\delta_{a2} & -\delta_{a3} \end{pmatrix}$$

$$\begin{aligned}
[\sigma_a, \sigma_b] &= 2i\varepsilon_{abc}\,\sigma_c\,, \\
\{\sigma_a, \sigma_b\} &= 2\delta_{ab}\,I. \\
\sigma_a\sigma_b &= i\varepsilon_{abc}\,\sigma_c + \delta_{ab}I \\
(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) &= (\vec{a}\cdot\vec{b})\,I + i(\vec{a}\times\vec{b})\cdot\vec{\sigma} \\
e^{ia(\hat{n}\cdot\vec{\sigma})} &= I\cos a + i(\hat{n}\cdot\vec{\sigma})\sin a
\end{aligned}$$

$$\begin{aligned}
\hat{n}\cdot\vec{\sigma}|\pm_n\rangle &= \pm|\pm_n\rangle \\
\hat{n}\cdot\vec{\sigma}e^{i\theta\hat{m}\cdot\vec{\sigma}}|\pm_z\rangle &= \pm e^{i\theta\hat{m}\cdot\vec{\sigma}}|\pm_z\rangle \\
e^{-i\theta\hat{m}\cdot\vec{\sigma}}\hat{n}\cdot\vec{\sigma}e^{i\theta\hat{m}\cdot\vec{\sigma}} &= \sigma_z \\
\hat{m} &= \hat{z}\times\hat{n}/\sin\theta \\
\cos\theta &= \hat{n}\cdot\hat{z}
\end{aligned}$$

2 Harmonic Oscillator

$$\begin{aligned}
E\psi(x) &= \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2x^2\right)\psi(x) \\
x &= \sqrt{\frac{\hbar}{m\omega}}q \\
E\psi(q) &= \frac{\hbar\omega}{2}\left(-\frac{d^2}{dq^2} + q^2\right)\psi(q) \\
p &= -i\frac{d}{dq} \\
[q,p] &= i \\
a &= \frac{1}{\sqrt{2}}(q+ip) = \frac{1}{\sqrt{2}}\left(q + \frac{d}{dq}\right) \\
a^\dagger &= \frac{1}{\sqrt{2}}(q-ip) = \frac{1}{\sqrt{2}}\left(q - \frac{d}{dq}\right) \\
[a, a^\dagger] &= \frac{1}{2}[q+ip, q-ip] = 1 \\
[\hat{H}, a] &= -\hbar\omega a. \\
[\hat{H}, a^\dagger] &= \hbar\omega\, a^\dagger. \\
a \mid n\rangle &= \sqrt{n}\ |n-1\rangle \\
a^\dagger \mid n\rangle &= \sqrt{n+1}\mid n+1\rangle
\end{aligned}$$

3 Angular Momentum

$$[L_x, L_y] = i\hbar L_z,\ \ [L_y, L_z] = i\hbar L_x,\ \ [L_z, L_x] = i\hbar L_y,$$

$$\begin{aligned}
[L_l, L_m] &= i\hbar\sum_{n=1}^{3}\varepsilon_{lmn}L_n, \\
[L^2, L_x] &= [L^2, L_y] = [L^2, L_z] = 0\ .
\end{aligned}$$

$$\begin{aligned}
J_+ &= J_x + iJ_y, \\
J_- &= J_x - iJ_y, \\
[J_z, J_\pm] &= \pm\hbar J_\pm. \\
[J_+, J_-] &= 2\hbar J_z. \\
J_zJ_\pm|j\,m\rangle &= (J_\pm J_z + [J_z, J_\pm])\,|j\,m\rangle \\
&= (J_\pm J_z \pm \hbar J_\pm)\,|j\,m\rangle \\
&= \hbar\,(m\pm 1)\,J_\pm|j\,m\rangle
\end{aligned}$$

4 Integrals

$$\int_{-\infty}^{\infty} e^{-ax^2+bx+c}\,dx = \sqrt{\frac{\pi}{a}}\,e^{\frac{b^2}{4a}+c},$$

$$ax^2+bx+c \ =\ a(x-h)^2+k, \quad \text{where}$$
$$h = -\frac{b}{2a} \quad \text{and} \quad k = c - ah^2 = c - \frac{b^2}{4a}.$$