Ideal Gas Fluid

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1 Introduction

Simple background to a 2D fluid simulation.

2 Equations

From my formalism Many Particle to Fluid, I derive a computational form to the fluid equations.

$$
\begin{aligned}
s &= \frac{k_B}{m} \log m\rho + \frac{3}{2m} k_B \log \frac{4\pi m^2 U}{2h^2} + \frac{5k_B}{2m} \\
U &= c\rho^{\gamma-1} e^{\alpha s} = \left[\frac{h^2 e^{-\frac{5}{3}}}{2\pi m^{\frac{8}{3}}} \right] \rho^{\frac{1}{3}-1} e^{\frac{2m}{3k} s} \\
S &= \int dt \; d^3a \; \rho_0 \left(\frac{1}{2} \dot{q}^2 - U \right)
\end{aligned}
$$

With [uniform density][5] (ie. $\nabla \rho_0 = 0$) and the Jacobian $\mathcal{J} = \frac{d^3q}{d^3a}$,

$$
\begin{aligned}
\delta S &= \int dt \; d^3a \; \left[\rho_0 \dot{q}_i \delta \dot{q}_i - \rho_0 \left(\frac{\partial U}{\partial \rho} \delta\rho + \frac{\partial U}{\partial s} \delta s \right) \right] \\
&= \int dt \; d^3a \; \left[-\rho_0 \ddot{q}_i \delta q_i - \rho_0^2 \frac{\partial U}{\partial \rho} \frac{\partial \frac{1}{\mathcal{J}}}{\partial \frac{\partial q^k}{\partial a^j}} \delta \frac{\partial q^k}{\partial a^j} \right] \\
&= \int dt \; d^3a \; \left[-\rho_0 \ddot{q}_k - \frac{\partial}{\partial a^j} \left(\rho^2 \frac{\partial U}{\partial \rho} \frac{\partial \mathcal{J}}{\partial \frac{\partial q^k}{\partial a^j}} \right) \right] \delta q_k
\end{aligned}
$$

which reproduces Morrison. More details shown [here][6].

The Jacobian can be written out,

$$
\begin{aligned}
\mathcal{J}(a,t) &= \frac{1}{2} \sum_{i,j,k,\ell} \epsilon_{k,\ell} \epsilon_{i,j} \frac{\partial q_k}{\partial a_i} \frac{\partial q_\ell}{\partial a_j} \\
&= \frac{\partial q_1}{\partial a_1} \frac{\partial q_2}{\partial a_2} - \frac{\partial q_1}{\partial a_2} \frac{\partial q_2}{\partial a_1} \\
\frac{\partial \mathcal{J}}{\partial \frac{\partial q^k}{\partial a^j}} &= \begin{bmatrix} \frac{\partial q_2}{\partial a_2} & -\frac{\partial q_2}{\partial a_1} \\ -\frac{\partial q_1}{\partial a_2} & \frac{\partial q_1}{\partial a_1} \end{bmatrix}_{kj}
\end{aligned}
$$