Path Integral Computation

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Abstract

An "efficient" means of computing the path integral as in Quantum Mechanics. Written because I couldn't find something that immediately made sense to me, so I write my own.

1 Introduction

I work with an approximation of the path integral equation in Chapter 8 of "Principles of Quantum Mechanics" by R. Shankar [1]. I don't explain it, I just show how to compute it.

2 Formulation

The original integral is of the form,

$$\lim_{\substack{N\to+\infty \\ \epsilon\to 0}} \int\int ... \int \mathbf{D}\left(x^N\right) \exp\left[\frac{im}{\hbar m}\sum_{i=0}^{N-1}\frac{(x_{i+1}-x_i)^2}{\epsilon}\right]$$

If we limit the space of the x variables to a N-dimensional cube, and discretize the values they take on into M distinct values, we end up with,

$$\sum_{x_0}\sum_{x_1}...\sum_{x_{N-1}} \mathbf{D}\left(x^N\right) \exp\left[\frac{im}{\hbar m}\sum_{i=0}^{N-1}\frac{(x_{i+1}-x_i)^2}{\epsilon}\right]$$

$$\sum\sum...\sum \mathbf{D}\left(x^N\right) \prod_{i=0}^{N-1} \exp\left[\frac{im}{\hbar m}\frac{(x_{i+1}-x_i)^2}{\epsilon}\right]$$

$$\sum\sum...\sum \Delta x^N \prod_{i=0}^{N-1} E(x_{i+1}, x_i)$$

$$\Delta x^N \sum\sum...\sum E(x_{N-1}, x_{N-2})E(x_{N-2}, x_{N-3})...E(x_1, x_0)$$

where the summations sum over the possible values of x_i.

In the given notation, we can see that each summation resembles a matrix multiplication.

References

[1] Ramamurti Shankar. *Principles of quantum mechanics*. Springer Science & Business Media, 2012.