STEVENS INSTITUTE OF TECHNOLOGY
FE 535 Financial Risk Management
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Lab 1

FRM Team 8
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1 Task 1

Team members:

Aidana Bekboeva
*Program:*MS in Financial Analytics
Academic background: Bachelor's in Applied Mathematics and Informatics (American University of Central Asia, Bishkek, Kyrgyzstan)
Personal information: Within the framework of my bachelor's program, I have gained experience in multiple programming languages, however, both R and Python will be new to me. I am pursuing graduate school immediately following my undergraduate degree.

Jayesh Kartik
*Program:*MS in Financial Analytics
Academic background: Bachelor's in engineering from the branch of computer science
Personal information: My name is Jayesh, I am from India. I have worked in Amazon and NTT Data Services as associates, having an experience of 14 months in the corporate world. I have 8/10 exposure to programming languages though I am new to these mathematical concepts.

Anthony Laino
Program: MS in Business Intelligence and Analytics
Academic background: Studied communications and business at Villanova University in his undergraduate, and now is enrolled in the MBA program with a BIA concentration
Personal information: Novice with programming but prefers to use R.

2 Task 2

Programming languages and statistical software preferences:

Programming language	Aidana Bekboeva	Jayesh Kartik	Anthony Laino
Preferred:			
R	10/10	9/10	5/10
Excel	9/10	10/10	9/10
Other:			
Java	5/10	7/10	2/10
C/C++	4/10	9/10	2/10
Python	4/10	8/10	2/10

3 Task 3

3.1 Using expectation and variance operations, show analytically that

$$E[R_A] = 252 \cdot E[R_d]$$
$$\sqrt{V[R_A]} = \sqrt{252 \cdot \; V[R_d]}$$
$$\text{with } R = \sum\,_{d=1}^{252} R_d$$

Expectation:

Let's substitute R_A with $R = \sum\,_{d=1}^{252} R_d$ in the following equation:
$E[R_A] = 252 \cdot E[R_d]$

Hence,

$E\left[\sum_{d=1}^{252} R_d\right] = \sum_{d=1}^{252} \cdot E[R_d] = 252 \cdot E[R_d]$

Q.E.D.

Variance:

Let's substitute R_A with $R = \sum\,_{d=1}^{252} R_d$ in the following equation:
$\sqrt{V[R_A]} = \sqrt{252\, V[R_d]}$

Hence,

$\sqrt{V\left[\sum_{d=1}^{252} R_d\right]} = \sqrt{V[252\, R_d]} = \sqrt{252\, V[R_d]}$

Q.E.D.

3.2 What is the assumption behind this to hold true?

The number of trading days per year is an essential parameter when computing expectation E and volatility V. And although the number of trading days can be quite different in other markets(in other countries), in this task we assume the US Market, that has 252 trading days per year.

4 Task 4

Go to Yahoo Finance and download historical data for SPY and IEF ETFs, dating from 2004 to 2021. Compute the daily log returns using the adjusted close price column. This should result in two time series (two columns). Based on this, report the annual mean return and volatility for each ETF.

4.1 The first approach.

We followed the following steps to solve this problem using the 'daily-return' approach.

1. First, we downloaded the historical data for SPY and IEF ETFs, dating from 2004 to 2021 with a daily frequency. Then, in the same spreadsheet [Figure 1: Task 4 Part 1] we computed the daily log returns using the adjusted close prices, utilizing the following function: $log10(FV/PV)$

	A	B	C	D	E	F	G	H	I	J	K
1	Date	SPY	IEF	Daily_log return SPY	Daily-log return IEF						
2	1/2/2004	77.69033	51.73874								
3	1/5/2004	78.53548	51.69566	0.004698953	-0.000361722						
4	1/6/2004	78.61231	52.08362	0.000424606	0.003247061		Return				
5	1/7/2004	78.87772	52.2006	0.001463796	0.000974375		Mu1(Reward of SPY)		0.000172825	0.04451	
6	1/8/2004	79.19205	52.2006	0.001727242	0		Mu2(Reward of IEF)		7.55515E-05	0.019221	
7	1/9/2004	78.50054	52.81636	-0.003808914	0.005092923		Volatility				
8	1/12/2004	79.08028	52.81636	0.003195521	0		Sigma1(Volatility of SPY)			0.005186	
9	1/13/2004	78.6193	53.09963	-0.002538998	0.002323069		Sigma2(Volatility of IEF)			0.001753	
10	1/14/2004	79.27584	53.20432	0.00361168	0.000855377						
11	1/15/2004	79.47141	53.23516	0.001070077	0.000251683						
12	1/16/2004	79.78572	53.01349	0.001714239	-0.001812184						
13	1/20/2004	79.76477	52.92109	-0.000114051	-0.000757582						
14	1/21/2004	80.39337	53.07505	0.003409122	0.001261639						
15	1/22/2004	80.18383	53.30285	-0.001133448	0.00186001						
16	1/23/2004	79.92543	52.90258	-0.001401815	-0.003273635						
17	1/26/2004	80.93122	52.68091	0.005431111	-0.001823568						
18	1/27/2004	80.10005	52.89026	-0.004483295	0.001722409						
19	1/28/2004	79.18506	52.50849	-0.004989553	-0.003146148						
20	1/29/2004	79.2619	52.44688	0.000421245	-0.00050988						
21	1/30/2004	79.2619	52.68709	0	0.001984563						
22	2/2/2004	79.60413	52.6346	0.001871132	-0.000432886						
23	2/3/2004	79.47141	52.83839	-0.000724687	0.001678241						
24	2/4/2004	78.82189	52.7519	-0.00356408	-0.000711412						
25	2/5/2004	79.05233	52.55429	0.00126782	-0.001629958						
26	2/6/2004	79.93938	52.87545	0.004846084	0.002645895						
27	2/9/2004	79.96033	52.999	0.000113802	0.001013607						

Figure 1: Task 4 Part 1

2. Based off the results (two columns of time series), we worked on calculating the annual mean return and volatility.

Annual mean return:

1. Column I [Figure 1] returns mean μ for daily log returns for both ETF's.
Cell I5 contains function =AVERAGE(D3:D4533) and returns the value of $\mu_1 = 0.000172825$
Cell I6 contains function =AVERAGE(E3:E4533) and returns $\mu_2 = 7.55515E - 05$

2. Since we need annual returns, we can use daily mean return to calculate it using the function: $= (\mu + 1)^{252} - 1$. It returns annual mean returns for both ETF's and stores them in cells J5 and J6 [Figure 1]: $\mu_{a1} = 0.04451$, $\mu_{a2} = 0.019221$

Volatility:

In order to calculate volatility, we do the following:
Cell J8 =STDEV(D3:D4533) $\sigma_1 = 0.005186$
Cell J9 =STDEV(E3:E4533) $\sigma_2 = 0.001753$

4.2 The second approach.

This second approach suggests that we use annual returns rather then daily returns in the beginning. For that, we refer to the website [https://quantstats.shinyapps.io/yahoo/] once again, now downloading the historical data with a monthly frequency, easily convertible to annual values. Now, having a different set of data, we computed the same functions on that columns (Figure 2: Task 4 Part 2):

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Date	SPY	IEF	R1-SPY	R2_IEF											
2	1/30/2004	79.2619	52.687				Return									
3	2/27/2004	80.3375	53.5363	0.00585	0.00694		Mu1	0.00360174	0.04322086							
4	3/31/2004	79.2734	54.2307	-0.0058	0.0056		Mu2	0.00155552	0.0186662							
5	4/30/2004	77.7734	51.8737	-0.0083	-0.0193											
6	5/28/2004	79.1052	51.5982	0.00737	-0.0023		Volatality									
7	6/30/2004	80.5687	51.9353	0.00796	0.00283		Sigma 1	0.01804387								
8	7/30/2004	77.9729	52.5646	-0.0142	0.00523		Sigma 2	0.00756043								
9	8/31/2004	78.1628	54.052	0.00106	0.01212											
10	9/30/2004	78.9474	54.2263	0.00434	0.0014		Correlation									
11	10/29/2004	79.9646	54.7627	0.00556	0.00427		Rho12	-0.29352439								
12	11/30/2004	83.5244	53.6796	0.01892	-0.0087		Sigma12	-4.0042E-05								
13	12/31/2004	86.0402	54.3062	0.01289	0.00504											
14	1/31/2005	84.1112	54.7793	-0.0098	0.00377		Covariance Matrix(sigma		-4.0042E-05	0.00032558		Sigma Inverse		2354.49	19144.1	
15	2/28/2005	85.8694	54.0297	0.00898	-0.006				5.716E-05	-4.0042E-05				3361	2354.49	
16	3/31/2005	84.2986	53.8116	-0.008	-0.0018											
17	4/29/2005	82.7193	55.1658	-0.0082	0.01079							Wo		21498.6		0.78998
18	5/31/2005	85.3849	56.18	0.01377	0.00791									5715.49		0.21002
19	6/30/2005	85.5142	56.4486	0.00066	0.00207											
20	7/29/2005	88.7861	55.2142	0.01631	-0.0096											
21	8/31/2005	87.9538	56.3187	-0.0041	0.0086											
22	9/30/2005	88.6597	55.3319	0.00347	-0.0077											

Figure 2: Task 4 Part 2

Columns D and E [Figure 2] return two time series of monthly log returns, computed using the formula $log10(FV/PV)$. Using two new times series, we can again solve for annual mean return and volatility.

Annual mean return:

1. We will refer to mean monthly log returns as μ_{m1} and μ_{m2}.
Cell H3 contains function =AVERAGE(D3:D217) and returns the value of $\mu_{m1} = 0.003601738$
Cell H4 contains function =AVERAGE(E3:E217) and returns the value of $\mu_{m2} = 0.001555516$

2. In need for annual mean returns, we just multiply values mu_{m1} and mu_{m2} by 12, resulting in $\mu_{a1} = 0.043220856$, $\mu_{a2} = 0.018666195$

Volatility

To calculate volatility, we refer to STDEV Excel function, see [Figure 2].
Cell H7 =STDEV(D3:D217) $\sigma_1 = 0.01804387$
Cell H8 =STDEV(E3:E217) $\sigma_2 = 0.007560432$

4.3 How do both results compare?

We refer back to both sets of results of our annual mean returns:

Approach 1: $\mu_{a1} = 0.04451$, $\mu_{a2} = 0.019221$
Approach 2: $\mu_{a1} = 0.043220856$, $\mu_{a2} = 0.018666195$

We can see that these results are approximately identical for both approaches. Naturally, the annual returns for the same period of time of the same ETFs should not be unequal.

5 Task 5

We will assign ρ to correlation, and compute its value by simply utilizing the Excel function CORREL [Figure 2]:

Cell H11 is RHO1,2 = =CORREL(D3:D217,E3:E217)
Where result ρ = -0.293524393

Using correlation, we now can calculate covariance:
Covariance = Correlation * Standard deviation (σ_1, σ_2)
Covariance = -4.00424E-05 [Sigma 12 at cell G12]

The covariance matrix, then, is the following:

-4.00424E-05	0.000325581
5.71601E-05	-4.00424E-05

Figure 3: covariance matrix

6 Task 6

We have two ETFs SPY and IEF. We calculated the daily return of both the ETFs using $LOG10 = (FV/PV)$, and then took the average of the daily which is μ_1 and μ_2 respectively.

Similarly we applied STDEV function on both the IEFs and got σ_1 and σ_2.

Now we assigned a weight of 0.1(w1) to SPY in our portfolio which implied to change the weight of IEF to 1-w1=w2. This is basically our portfolio.

Again, we calculated return of the portfolio by doing the calculation with the formula weight1SPY daily return+weight2IEF daily returns.

Similarly, following the above steps, we calculated the Average return of the portfolio which is μ_p and volatility of the portfolio that is σ_p. A is our risk aversion value. And we kept on randomly changing the value of A from 1 to 100 and got the different composition of our weights in the portfolio.

We finally plotted a graph and our results are the following:

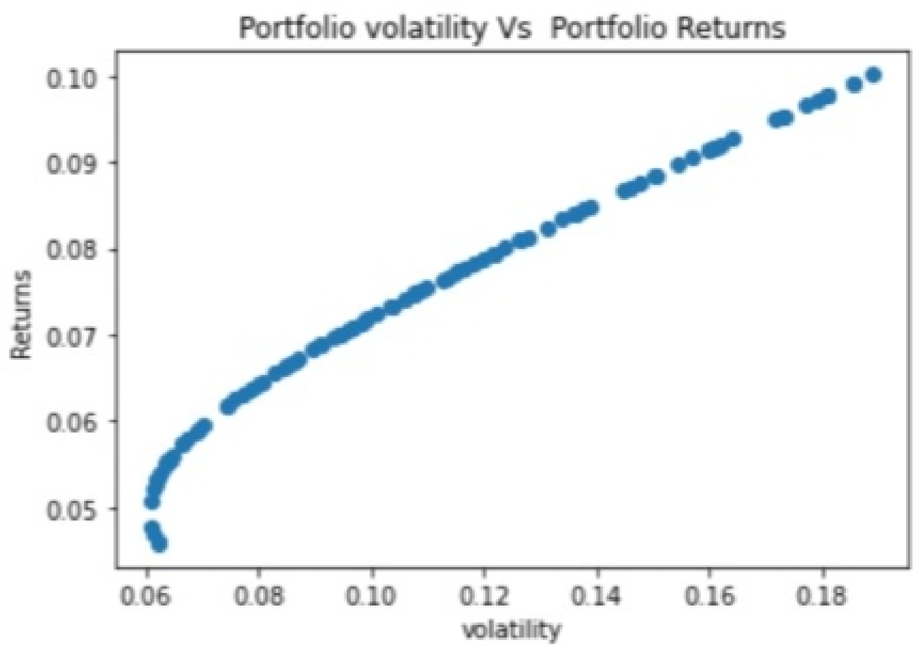


Figure 4: Task 6

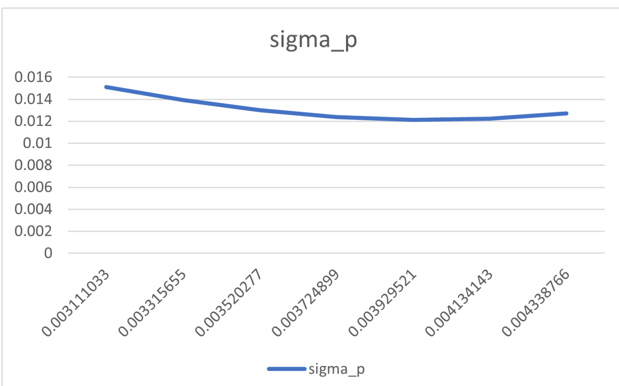
7 Task 7

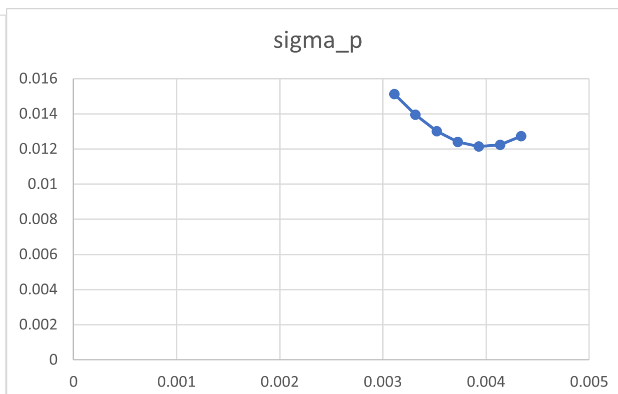
In this task, we calculate the average return and volatility for the portfolio.

mu_p	sigma_p	w
0.00311103	0.01512086	0,2
0.00331565	0.0139423	.1,1.9
0.00352028	0.01301505	.2,1.8
0.0037249	0.01239564	0.3,1.7
0.00392952	0.01213131	.4,1.6
0.00413414	0.01224508	.5,1.5
0.00433877	0.01272681	.6,1.4

Figure 5: Task 6

For a sequence of $\omega \in (0, 2)$, we computed the portfolio mean return and volatility and plotted them on the y-axis and x-axis:





What does it tell us in terms of the capital market line (CML)?

The sharpe Ratio is above the CML and thus this tells us that the asset should be sold

What does a negative (respectively positive) weight in the risk-free asset imply?

A negative weight means that the asset should be shorted, while the positive weight means investing in the asset.