

# Homework #2

FE-570

October 10, 2022

## Problem 2.1.

For this problem we will load the dataset `sampleTQdata.RData` provided in Canvas. This is a generic data set in TAQ format for one trading day 09:30am - 4:00pm. Load the dataset using the R file provided.

We will use these data sets to get some practice working with tick level data.

1. Load the TAQ dataset. Explore the data, and answer the following questions:

- i) how many trades are in the dataset?
- ii) plot the trade prices  $p_t$  and the best-bid  $b_t$  and best-ask prices  $a_t$  for the entire dataset.
- iii) same as in ii) but only for trades with counts 100:200

2. Count how many trades take place within the spread ( $p_t \in (b_t, a_t)$ ), and how many at the touch ( $p_t = b_t$  or  $p_t = a_t$ ). Give separately the three numbers, and test if their sum reproduces the total trade count from 1.i).

3. Determine the “trade direction”  $d_t$  of each trade, which shows if it is a buy ( $d_t = +1$ ) or a sell ( $d_t = -1$ ).

There are two ways to do this, implement each of them in R.

i) **Tick test.** This test uses only the trade prices  $p_t$ , but not the quotes  $a_t, b_t$ . Under this test the trade is classified as buy/sell according to:

$$\begin{aligned} d_t &= +1 \text{ (buy) if } p_t > p_{t-1} \text{ (uptick) or if } p_t = p_{t-1} > p_{t-2} \text{ (zero-uptick)} \\ d_t &= -1 \text{ (sell) if } p_t < p_{t-1} \text{ (downtick) or if } p_t = p_{t-1} < p_{t-2} \text{ (zero-downtick)} \end{aligned}$$

Note that zero-uptick/downtick results apply also if there are multiple (more than 2) trades with the same price. For example if the trade prices are  $p_t = (20.0, 20.0, 20.0, 19.9)$ , then the trade signs are  $(+, +, +, ?)$ .

ii) **Lee-Ready rule.** This test uses both trade prices  $p_t$  and quotes  $a_t, b_t$ . The Lee-Ready rule decides if a trade is a buy or sell by comparing the trade price  $p_t$  with the mid-price  $m_t = \frac{1}{2}(a_t + b_t)$  (the half-point between best-bid  $b_t$  and best-ask  $a_t$ ). If the trade price is exactly equal to the mid-price  $p_t = m_t$  then use the tick rule in point (i) above.

$$(1) \quad d_t = \begin{cases} +1 \text{ (buy)} & \text{if } p_t > m_t \\ -1 & \text{if } p_t < m_t \\ \text{tick rule} & \text{if } p_t = m_t \end{cases}$$

The Lee-Ready rule is implemented as the function `getTradeDirection` in *highfrequency*. You have to implement it independently. Then test that your implementation gives the same result as the `getTradeDirection` implementation.

Compare the results of the two methods (tick rule and Lee-Ready rule) for determining the trade direction. What fraction of trades are classified the same way under the two tests?