CS2030 Lecture 9

The Art of Being Lazy: 1

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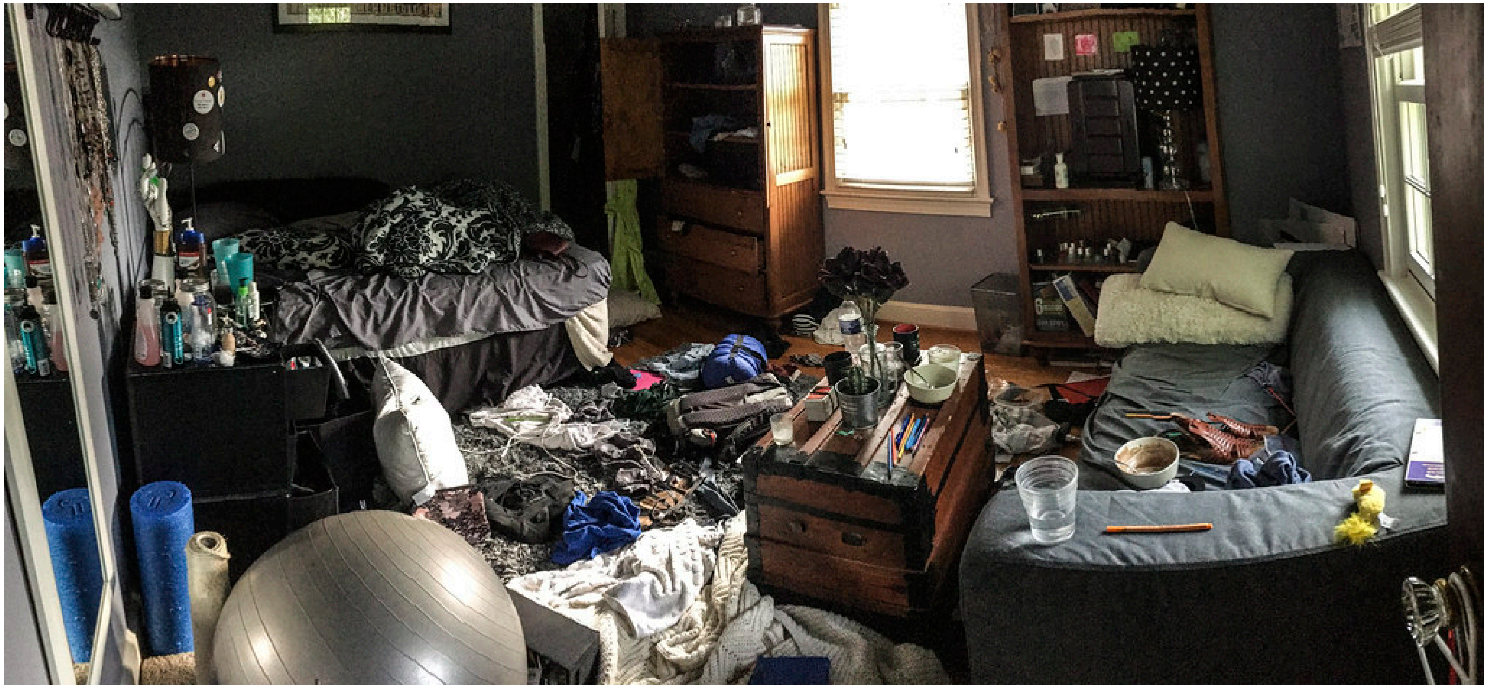
Eager vs Lazy Evaluation

Eager evaluation: computation is immediately carried out when the expression is encountered.

Lazy evaluation: computation is postponed, and carried out only upon demand.



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Benefits of Lazy Evaluation

Avoids unnecessary computation

Amortizes time complexity (when used with memoization (aka caching))

Allows for infinite data structures

New abstraction

`freeze(<expr>) => <expr>` is not evaluated now, but postponed indefinitely. `<expr>` is wrapped in a *thunk.*

`thaw(thunk) =>` forces the evaluation of the `<expr>` inside the thunk.

`thaw(freeze(<expr>)) ≡ <expr>`

Lazy Lists

`public class LazyList<T>`

<u>Constructors</u>:

`LLmake(a, b) ≡ new LazyList(a, freeze(b))`

Note that a is eagerly evaluated, but b is frozen.

`makeEmpty()`: returns an empty `LazyList`

<u>Methods</u>:

`head()`: returns `a`

`tail()`: returns `thaw(freeze(b))`

`isEmpty()`: returns `true` if `LazyList` is empty, otherwise `false`

Lazy Lists: example

Demo

```
LazyList<Integer> ll =
      LLmake(1, LLmake(2, LazyList.makeEmpty()));

System.out.println(ll);
=> head: 1
tail: thunk$1/0x0000000800060840@2f410acf

System.out.println(ll.tail());
=> head: 2
tail: thunk$16/0x0000000800066440@2a18f23c
```

Start with empty `LL` and add to the front (the head).

cf `ArrayList`, which starts with empty `AL` and adds to the back.

Integers from a (inclusive) to b (exclusive)

Demo

```
LazyList<Integer> intRange(int a, int b) {
    if (a >= b)
        return LazyList.makeEmpty();
    return LLmake(a, intRange(a+1, b)); }

var l2 = intRange(4, 12);
System.out.println(l2);
System.out.println(l2.tail());
System.out.println(l2.tail().tail());
=> head: 4
tail: thunk$17/0x0000000800065840@16c0663d
head: 5
tail: thunk$17/0x0000000800065840@23223dd8
head: 6
tail: thunk$17/0x0000000800065840@4ec6a292
```

map: apply function to every element

Demo

```
public <R> LazyList<R> map(Function<T,R> f) {
    if (this.isEmpty())
        return LazyList.makeEmpty();
    else
        return LLmake(f.apply(this.head()),
                            this.tail().map(f));
}
```

Note the use of recursion: we recursively apply `map` to the `tail` of the list.

But this is frozen because it is the 2nd argument of `LLmake`.

Compute the squares of 1,2,3 .. 10

Demo

```
intRange(1,11).map(x -> x*x).print();
=> (* 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, *)
```

Generate a LazyList of integers from 1 to 10

Square every element

Print it

Note the declarative style: no loops!

`filter`: keep elements that satisfy predicate

Demo

```
public LazyList<T> filter(Predicate<T> pred) {
    if (this.isEmpty())
        return this;
    else if (pred.test(this.head()))
        return LLmake(this.head(),
                      this.tail().filter(pred));
    else
        return this.tail().filter(pred);  }

intRange(1,21).filter(x-> x%2==0).print();
=> (* 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, *)
```

Fermat’s puzzle

26 is an integer such that one less is a perfect square, and one more is a perfect cube.

Is there any other such number?

```
 intRange(1, 50000)
            .filter(x-> isSquare(x-1))
            .filter(x-> isCube(x+1))
            .print();


=> (* 26, *)
```

But is it really lazy?

Instrumentation refers to the measure of a product's performance, to diagnose errors, and to write trace information. https://en.wikipedia.org/wiki/Instrumentation_(computer_programming)

```
<T> Predicate<T> predInstrument(Predicate<T> p,
                                String msg) {
      return x-> { System.out.println(msg + x);
                   return p.test(x); };
}
```

Let’s wrap our predicate with a print statement to see what is going on.

Instrumenting map

Do the same for our map

```
<T,U> Function<T,U> funcInstrument(Function<T,U> f
                                   String msg) {
        return x-> { System.out.println(msg + x);
                     return f.apply(x);};
}
```

Now use them to investigate

Demo

```
var ll =
  intRange(1, 21)
    .filter(predInstrument(x->x%2==0, "filter: "))
    .map(funcInstrument(x->x*x, "map: "));

=> filter: 1
filter: 2
map: 2
```

Predicate is called twice, and Function called once. Can you understand why?

Compare with ArrayList

Demo

```
List<Integer> list = new ArrayList<>();
for (int i=1; i<21; i++)
    list.add(i);


var result =
  listMap(funcInstrument(x->x*x, "map: "),
          listFilter(predInstrument(x->x%2==0,
                                    "filter: "),
                     list));
```

output

Demo

```
filter: 1
filter: 2
filter: 3
filter: 4
filter: 5
filter: 6
filter: 7
filter: 8
filter: 9
filter: 10
filter: 11
filter: 12
filter: 13
filter: 14
filter: 15
filter: 16
```

```
filter: 17
filter: 18
filter: 19
filter: 20
map: 2
map: 4
map: 6
map: 8
map: 10
map: 12
map: 14
map: 16
map: 18
map: 20
```

Let’s print our LazyList

Demo

```
intRange(1, 21)
    .filter(predInstrument(x->x%2==0, "filter: "))
    .map(funcInstrument(x->x*x, "map: ")).print();
```

```
filter: 1
filter: 2
map: 2
(* 4, filter: 3
filter: 4
map: 4
16, filter: 5
filter: 6
map: 6
36, filter: 7
```

```
  .  .  .
map: 16
256, filter: 17
filter: 18
map: 18
324, filter: 19
filter: 20
map: 20
400, *)
```

Notice how printing thaws the frozen computation!

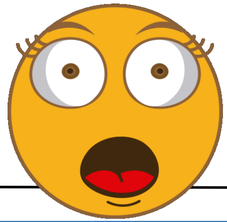
add: adding 2 LazyLists element-wise

Demo

```
public LazyList<T> add(LazyList<T> other,
                       BinaryOperator<T> add) {
  if (this.isEmpty() || other.isEmpty())
    return LazyList.makeEmpty();
  else
    return LLmake(add.apply(this.head(),
                            other.head()),
                  this.tail().add(other.tail(),
                                  add)); }


var ll = intRange(1, 11);
var twos = ll.map(x->2);
ll.add(twos, (x,y)->x+y).print();
=> (* 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, *)
```

Infinite LazyLists!



Demo

```
LazyList<Integer> integersFrom(int n) {
    return LLmake(n, integersFrom(n+1)); }
integersFrom(1).limit(50).print();


=> (* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,
15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40,
41, 42, 43, 44, 45, 46, 47, 48, 49, 50, *)
```

```
LazyList<T> limit(long maxSize) {
   if (maxSize==0)
         return LazyList.makeEmpty();
   return LLmake(this.head(),
                    this.tail().limit(maxSize - 1));   }
```

Prime numbers

Of course, one way to check if a number n is prime is to see if n can be found in a list, L, of primes. To generate L, an ancient method called the Sieve of Eratosthenes may be used:

1. Start by listing all integers greater than 1. Call this list C. Also, let L be an empty list.
2. Take the first number $p = 2$ in C, and add it to L. This is the first prime.
3. In C, cross out all multiples of p.
4. Let p be the next uncrossed number in C. This is the next prime. Add it to L, and repeat from Step 3.

	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~

Prime sieve

Demo

```
LazyList<Integer> sieve(LazyList<Integer> s) {
   return LLmake(s.head(),
                 sieve(s.tail()
                        .filter(x->
                               x%s.head()!=0)));
}

var primes = sieve(integersFrom(2));
```

Another way to get integers

Demo

```
LazyList<Integer> integers;

integers = LLmake(1, integers.map(x->x+1));
```

Fibonacci numbers

Demo

The infinite sequence:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

The next number is the sum of the previous two.

```
LazyList<Integer> fib;
fib = LLmake(0,
             LLmake(1,
                    fib.add(fib.tail(),
                            (x,y)->x+y)));
```

0	1	1	2	3	5			← fib
		0	1	1	2	3	5	← fib
		1	1	2	3	5		← fib.tail()

To be Continued