Question ID e8a6c1fc

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: e8a6c1fc

What is the value of $\tan \frac{92\pi}{3}$?

A. $-\sqrt{3}$

B. $-\frac{\sqrt{3}}{3}$

C. $\frac{\sqrt{3}}{3}$

D. $\sqrt{3}$

ID: e8a6c1fc Answer

Correct Answer: A

Rationale

Choice A is correct. A trigonometric ratio can be found using the unit circle, that is, a circle with radius 1 unit. If a central angle of a unit circle in the *xy*-plane centered at the origin has its starting side on the positive *x*-axis and its terminal side intersects the circle at a point (x, y), then the value of the tangent of the central angle is equal to the *y*-coordinate divided by the *x*-coordinate. There are 2π radians in a circle. Dividing $\frac{92\pi}{3}$ by 2π yields $\frac{92}{6}$, which is equivalent to $15 + \frac{2}{3}$. It follows that on the unit circle centered at the origin in the *xy*-plane, the angle $\frac{92\pi}{3}$ is the result of 15 revolutions from its starting side on the positive *x*-axis followed by a rotation through $\frac{2\pi}{3}$ radians. Therefore, the angles $\frac{92\pi}{3}$ and $\frac{2\pi}{3}$ are coterminal angles and $\tan\left(\frac{92\pi}{3}\right)$ is equal to $\tan\left(\frac{2\pi}{3}\right)$. Since $\frac{2\pi}{3}$ is greater than $\frac{\pi}{2}$ and less than π, it follows that the terminal side of the angle is in quadrant II and forms an angle of $\frac{\pi}{3}$, or 60°, with the negative *x*-axis. Therefore, the terminal side of the angle intersects the unit circle at the point $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. It follows that the value of $\tan\left(\frac{2\pi}{3}\right)$ is $\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$, which is equivalent to $-\sqrt{3}$. Therefore, the value of $\tan\left(\frac{92\pi}{3}\right)$ is $-\sqrt{3}$.

Choice B is incorrect. This is the value of $\frac{1}{\tan\left(\frac{92\pi}{3}\right)}$, not $\tan\left(\frac{92\pi}{3}\right)$.

Choice C is incorrect. This is the value of $\frac{1}{\tan\left(\frac{\pi}{3}\right)}$, not $\tan\left(\frac{92\pi}{3}\right)$.

Choice D is incorrect. This is the value of $\tan\left(\frac{\pi}{3}\right)$, not $\tan\left(\frac{92\pi}{3}\right)$.

Question Difficulty: Hard

Question ID 9f2728be

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 9f2728be

The graph of $x^2 + x + y^2 + y = \frac{199}{2}$ in the *xy*-plane is a circle. What is the length of the circle's radius?

ID: 9f2728be Answer

Correct Answer: 10

Rationale

The correct answer is 10. It's given that the graph of $x^2 + x + y^2 + y = \frac{199}{2}$ in the *xy*-plane is a circle. The equation of a circle in the *xy*-plane can be written in the form $(x-h)^2 + (y-k)^2 = r^2$, where the coordinates of the center of the circle are (h, k) and the length of the radius of the circle is r. The term $(x-h)^2$ in this equation can be obtained by adding the square of half the coefficient of x to both sides of the given equation to complete the square. The coefficient of x is 1. Half the coefficient of x is $\frac{1}{2}$. The square of half the coefficient of x is $\frac{1}{4}$. Adding $\frac{1}{4}$ to each side of $\left(x^2 + x\right) + \left(y^2 + y\right) = \frac{199}{2}$ yields $\left(x^2 + x + \frac{1}{4}\right) + \left(y^2 + y\right) = \frac{199}{2} + \frac{1}{4}$, or $\left(x + \frac{1}{2}\right)^2 + \left(y^2 + y\right) = \frac{199}{2} + \frac{1}{4}$. Similarly, the term $(y-k)^2$ can be obtained by adding the square of half the coefficient of y to both sides of this equation, which yields $\left(x + \frac{1}{2}\right)^2 + \left(y^2 + y + \frac{1}{4}\right) = \frac{199}{2} + \frac{1}{4} + \frac{1}{4}$, or $\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{199}{2} + \frac{1}{4} + \frac{1}{4}$. This equation is equivalent to $\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 100$, or $\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 10^2$. Therefore, the length of the circle's radius is 10.

Question Difficulty: Hard

Question ID fde10025

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: fde10025

A circle in the *xy*-plane has its center at $(-1, 1)$. Line t is tangent to this circle at the point $(5, -4)$. Which of the following points also lies on line t?

A. $(0, \frac{6}{5})$

B. $(4, 7)$

C. $(10, 2)$

D. $(11, 1)$

ID: fde10025 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the circle has its center at $(-1, 1)$ and that line t is tangent to this circle at the point $(5, -4)$. Therefore, the points $(-1, 1)$ and $(5, -4)$ are the endpoints of the radius of the circle at the point of tangency. The slope of a line or line segment that contains the points (a, b) and (c, d) can be calculated as $\frac{d-b}{c-a}$. Substituting $(-1, 1)$ for (a, b) and $(5, -4)$ for (c, d) in the expression $\frac{d-b}{c-a}$ yields $\frac{-4-1}{5-(-1)}$, or $-\frac{5}{6}$. Thus, the slope of this radius is $-\frac{5}{6}$. A line that's tangent to a circle is perpendicular to the radius of the circle at the point of tangency. It follows that line t is perpendicular to the radius at the point $(5, -4)$, so the slope of line t is the negative reciprocal of the slope of this radius. The negative reciprocal of $-\frac{5}{6}$ is $\frac{6}{5}$. Therefore, the slope of line t is $\frac{6}{5}$. Since the slope of line t is the same between any two points on line t, a point lies on line t if the slope of the line segment connecting the point and $(5, -4)$ is $\frac{6}{5}$. Substituting choice C, $(10, 2)$, for (a, b) and $(5, -4)$ for (c, d) in the expression $\frac{d-b}{c-a}$ yields $\frac{-4-2}{5-10}$, or $\frac{6}{5}$. Therefore, the point $(10, 2)$ lies on line t.

Choice A is incorrect. The slope of the line segment connecting $(0, \frac{6}{5})$ and $(5, -4)$ is $\frac{-4-\frac{6}{5}}{5-0}$, or $-\frac{26}{25}$, not $\frac{6}{5}$.

Choice B is incorrect. The slope of the line segment connecting $(4, 7)$ and $(5, -4)$ is $\frac{-4-7}{5-4}$, or -11, not $\frac{6}{5}$.

Choice D is incorrect. The slope of the line segment connecting $(11, 1)$ and $(5, -4)$ is $\frac{-4-1}{5-11}$, or $\frac{5}{6}$, not $\frac{6}{5}$.

Question Difficulty: Hard

Question ID 32f6a450

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 32f6a450

What is the diameter of the circle in the *xy*-plane with equation $(x-5)^2+(y-3)^2=16$?

A. 4

B. 8

C. 16

D. 32

ID: 32f6a450 Answer

Correct Answer: B

Rationale

Choice B is correct. The standard form of an equation of a circle in the *xy*-plane is $(x-h)^2+(y-k)^2=r^2$, where the coordinates of the center of the circle are (h,k) and the length of the radius of the circle is r. For the circle in the *xy*-plane with equation $(x-5)^2+(y-3)^2=16$, it follows that $r^2=16$. Taking the square root of both sides of this equation yields $r=4$ or $r=-4$. Because r represents the length of the radius of the circle and this length must be positive, $r=4$. Therefore, the radius of the circle is 4. The diameter of a circle is twice the length of the radius of the circle. Thus, $2(4)$ yields 8. Therefore, the diameter of the circle is 8.

Choice A is incorrect. This is the radius of the circle.

Choice C is incorrect. This is the square of the radius of the circle.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 82372955

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 82372955

In the *xy*-plane, a circle has center C with coordinates (h, k). Points A and B lie on the circle. Point A has coordinates $\left(h+1, k+\sqrt{102}\right)$, and $\angle ACB$ is a right angle. What is the length of $\overline{AB}$?

A. $\sqrt{206}$

B. $2\sqrt{102}$

C. $103\sqrt{2}$

D. $103\sqrt{3}$

ID: 82372955 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that points A and B lie on the circle with center C. Therefore, $\overline{AC}$ and $\overline{BC}$ are both radii of the circle. Since all radii of a circle are congruent, $\overline{AC}$ is congruent to $\overline{BC}$. The length of $\overline{AC}$, or the distance from point A to point C, can be found using the distance formula, which gives the distance between two points, (x_1, y_1) and (x_2, y_2), as $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. Substituting the given coordinates of point A, $\left(h+1, k+\sqrt{102}\right)$, for (x_1, y_1) and the given coordinates of point C, (h, k), for (x_2, y_2) in the distance formula yields $\sqrt{(h+1-h)^2+\left(k+\sqrt{102}-k\right)^2}$, or $\sqrt{1^2+\left(\sqrt{102}\right)^2}$, which is equivalent to $\sqrt{1+102}$, or $\sqrt{103}$. Therefore, the length of $\overline{AC}$ is $\sqrt{103}$ and the length of $\overline{BC}$ is $\sqrt{103}$. It's given that angle ACB is a right angle. Therefore, triangle ACB is a right triangle with legs $\overline{AC}$ and $\overline{BC}$ and hypotenuse $\overline{AB}$. By the Pythagorean theorem, if a right triangle has a hypotenuse with length c and legs with lengths a and b, then $a^2+b^2=c^2$. Substituting $\sqrt{103}$ for a and b in this equation yields $\left(\sqrt{103}\right)^2+\left(\sqrt{103}\right)^2=c^2$, or $103+103=c^2$, which is equivalent to $206=c^2$. Taking the positive square root of both sides of this equation yields $\sqrt{206}=c$. Therefore, the length of $\overline{AB}$ is $\sqrt{206}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This would be the length of $\overline{AB}$ if the length of $\overline{AC}$ were 103, not $\sqrt{103}$.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 40789a56

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 40789a56

Circle A in the *xy*-plane has the equation $(x+5)^2+(y-5)^2=4$. Circle B has the same center as circle A. The radius of circle B is two times the radius of circle A. The equation defining circle B in the *xy*-plane is $(x+5)^2+(y-5)^2=k$, where k is a constant. What is the value of k?

ID: 40789a56 Answer

Correct Answer: 16

Rationale

The correct answer is 16. An equation of a circle in the *xy*-plane can be written as $(x-t)^2+(y-u)^2=r^2$, where the center of the circle is (t,u) , the radius of the circle is r, and where t, u, and r are constants. It's given that the equation of circle A is $(x+5)^2+(y-5)^2=4$, which is equivalent to $(x+5)^2+(y-5)^2=2^2$. Therefore, the center of circle A is $(-5,5)$ and the radius of circle A is 2. It's given that circle B has the same center as circle A and that the radius of circle B is two times the radius of circle A. Therefore, the center of circle B is $(-5,5)$ and the radius of circle B is $2(2)$, or 4. Substituting -5 for t, 5 for u, and 4 for r into the equation $(x-t)^2+(y-u)^2=r^2$ yields $(x+5)^2+(y-5)^2=4^2$, which is equivalent to $(x+5)^2+(y-5)^2=16$. It follows that the equation of circle B in the *xy*-plane is $(x+5)^2+(y-5)^2=16$. Therefore, the value of k is 16.

Question Difficulty: Hard

Question ID 33be0f76

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 33be0f76

The equation $x^2+(y-2)^2=36$ represents circle A. Circle B is obtained by shifting circle A down 4 units in the *xy*-plane. Which of the following equations represents circle B?

A. x^2+ msup $=36$

B. x^2+ msup $=36$

C. msup $+(y-2)^2=36$

D. msup $+(y-2)^2=36$

ID: 33be0f76 Answer

Correct Answer: A

Rationale

Choice A is correct. The standard form of an equation of a circle in the *xy*-plane is $(x-h)^2+(y-k)^2=r^2$, where the coordinates of the center of the circle are (h, k) and the length of the radius of the circle is r. The equation of circle A, $x^2+(y-2)^2=36$, can be rewritten as $(x-0)^2+(y-2)^2=6^2$. Therefore, the center of circle A is at $(0, 2)$ and the length of the radius of circle A is 6. If circle A is shifted down 4 units, the *y*-coordinate of its center will decrease by 4; the radius of the circle and the *x*-coordinate of its center will not change. Therefore, the center of circle B is at $(0, 2-4)$, or $(0, -2)$, and its radius is 6. Substituting 0 for h, -2 for k, and 6 for r in the equation $(x-h)^2+(y-k)^2=r^2$ yields $(x-0)^2+(y-(-2))^2=(6)^2$, or $x^2+(y+2)^2=36$. Therefore, the equation $x^2+(y+2)^2=36$ represents circle B.

Choice B is incorrect. This equation represents a circle obtained by shifting circle A up, rather than down, 4 units.

Choice C is incorrect. This equation represents a circle obtained by shifting circle A right, rather than down, 4 units.

Choice D is incorrect. This equation represents a circle obtained by shifting circle A left, rather than down, 4 units.

Question Difficulty: Hard

Question ID fd77ca64

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: fd77ca64

$$x^2 + 14x + y^2 = 6y + 109$$

In the *xy*-plane, the graph of the given equation is a circle. What is the length of the circle's radius?

A. $\sqrt{109}$

B. $\sqrt{149}$

C. $\sqrt{167}$

D. $\sqrt{341}$

ID: fd77ca64 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that in the *xy*-plane, the graph of the given equation is a circle. The equation of a circle in the *xy*-plane can be written in the form $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the center of the circle and r is the length of the circle's radius. Subtracting $6y$ from both sides of the equation $x^2 + 14x + y^2 = 6y + 109$ yields $x^2 + 14x + y^2 - 6y = 109$. By completing the square, this equation can be rewritten as $\left(x^2 + 14x + \left(\frac{14}{2}\right)^2\right) + \left(y^2 - 6y + \left(\frac{-6}{2}\right)^2\right) = 109 + \left(\frac{14}{2}\right)^2 + \left(\frac{-6}{2}\right)^2$. This equation can be rewritten as $\left(x^2 + 14x + 49\right) + \left(y^2 - 6y + 9\right) = 109 + 49 + 9$, or $(x+7)^2 + (y-3)^2 = 167$. Therefore, $r^2 = 167$. Taking the square root of both sides of this equation yields $r = \sqrt{167}$ and $r = -\sqrt{167}$. Since r is the length of the circle's radius, r must be positive. Therefore, the length of the circle's radius is $\sqrt{167}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID ea7fd37b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: ea7fd37b

$$(x+4)^2+(y-19)^2=121$$

The graph of the given equation is a circle in the *xy*-plane. The point (a, b) lies on the circle. Which of the following is a possible value for a?

A. -16

B. -14

C. 11

D. 19

ID: ea7fd37b Answer

Correct Answer: B

Rationale

Choice B is correct. An equation of the form $(x-h)^2+(y-k)^2=r^2$, where h, k, and r are constants, represents a circle in the *xy*-plane with center (h, k) and radius r. Therefore, the circle represented by the given equation has center $(-4, 19)$ and radius 11. Since the center of the circle has an *x*-coordinate of -4 and the radius of the circle is 11, the least possible *x*-coordinate for any point on the circle is $-4-11$, or -15. Similarly, the greatest possible *x*-coordinate for any point on the circle is $-4+11$, or 7. Therefore, if the point (a, b) lies on the circle, it must be true that $-15 \leq a \leq 7$. Of the given choices, only -14 satisfies this inequality.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID fb2d9203

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: fb2d9203

Points Q and R lie on a circle with center P. The radius of this circle is 9 inches. Triangle PQR has a perimeter of 31 inches. What is the length, in inches, of $\overline{QR}$?

A. $13\sqrt{2}$

B. 13

C. $9\sqrt{2}$

D. 9

ID: fb2d9203 Answer

Correct Answer: B

Rationale

Choice B is correct. Since it's given that P is the center of a circle with a radius of 9 inches, and that points Q and R lie on that circle, it follows that $\overline{PQ}$ and $\overline{RP}$ of triangle PQR each have a length of 9 inches. Let the length of $\overline{QR}$ be x inches. It follows that the perimeter of triangle PQR is $9 + 9 + x$ inches. Since it's given that the perimeter of triangle PQR is 31 inches, it follows that $9 + 9 + x = 31$, or $18 + x = 31$. Subtracting 18 from both sides of this equation gives $x = 13$. Therefore, the length, in inches, of $\overline{QR}$ is 13.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID b2aa5d73

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: b2aa5d73

A circle in the *xy*-plane has its center at $(-5, 2)$ and has a radius of 9. An equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

ID: b2aa5d73 Answer

Correct Answer: -52

Rationale

The correct answer is -52. The equation of a circle in the *xy*-plane with its center at (h, k) and a radius of r can be written in the form $(x-h)^2 + (y-k)^2 = r^2$. It's given that a circle in the *xy*-plane has its center at $(-5, 2)$ and has a radius of 9. Substituting -5 for h, 2 for k, and 9 for r in the equation $(x-h)^2 + (y-k)^2 = r^2$ yields $(x-(-5))^2 + (y-2)^2 = 9^2$, or $(x+5)^2 + (y-2)^2 = 81$. It's also given that an equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. Therefore, $(x+5)^2 + (y-2)^2 = 81$ can be rewritten in the form $x^2 + y^2 + ax + by + c = 0$. The equation $(x+5)^2 + (y-2)^2 = 81$, or $(x+5)(x+5) + (y-2)(y-2) = 81$, can be rewritten as $x^2 + 5x + 5x + 25 + y^2 - 2y - 2y + 4 = 81$. Combining like terms on the left-hand side of this equation yields $x^2 + y^2 + 10x - 4y + 29 = 81$. Subtracting 81 from both sides of this equation yields $x^2 + y^2 + 10x - 4y - 52 = 0$, which is equivalent to $x^2 + y^2 + 10x + (-4)y + (-52) = 0$. This equation is in the form $x^2 + y^2 + ax + by + c = 0$. Therefore, the value of c is -52.

Question Difficulty: Hard

Question ID 8d56e2be

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 8d56e2be

A circle has center O, and points R and S lie on the circle. In triangle ORS, the measure of $\angle ROS$ is 88°. What is the measure of $\angle RSO$, in degrees? (Disregard the degree symbol when entering your answer.)

ID: 8d56e2be Answer

Correct Answer: 46

Rationale

The correct answer is 46. It's given that O is the center of a circle and that points R and S lie on the circle. Therefore, $\overline{OR}$ and $\overline{OS}$ are radii of the circle. It follows that $OR = OS$. If two sides of a triangle are congruent, then the angles opposite them are congruent. It follows that the angles $\angle RSO$ and $\angle ORS$, which are across from the sides of equal length, are congruent. Let x° represent the measure of $\angle RSO$. It follows that the measure of $\angle ORS$ is also x°. It's given that the measure of $\angle ROS$ is 88°. Because the sum of the measures of the interior angles of a triangle is 180°, the equation $x^\circ + x^\circ + 88^\circ = 180^\circ$, or $2x + 88 = 180$, can be used to find the measure of $\angle RSO$. Subtracting 88 from both sides of this equation yields $2x = 92$. Dividing both sides of this equation by 2 yields $x = 46$. Therefore, the measure of $\angle RSO$, in degrees, is 46.

Question Difficulty: Hard

Question ID 34eb7da8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 34eb7da8

The equation $x^2 + (y-1)^2 = 49$ represents circle A. Circle B is obtained by shifting circle A down 2 units in the *xy*-plane. Which of the following equations represents circle B?

A. msup $+ (y-1)^2 = 49$

B. $x^2 +$ msup $= 49$

C. msup $+ (y-1)^2 = 49$

D. $x^2 +$ msup $= 49$

ID: 34eb7da8 Answer

Correct Answer: D

Rationale

Choice D is correct. The graph in the *xy*-plane of an equation of the form $(x-h)^2 + (y-k)^2 = r^2$ is a circle with center (h, k) and a radius of length r. It's given that circle A is represented by $x^2 + (y-1)^2 = 49$, which can be rewritten as $x^2 + (y-1)^2 = 7^2$. Therefore, circle A has center $(0, 1)$ and a radius of length 7. Shifting circle A down two units is a rigid vertical translation of circle A that does not change its size or shape. Since circle B is obtained by shifting circle A down two units, it follows that circle B has the same radius as circle A, and for each point (x, y) on circle A, the point $(x, y-2)$ lies on circle B. Moreover, if (h, k) is the center of circle A, then $(h, k-2)$ is the center of circle B. Therefore, circle B has a radius of 7 and the center of circle B is $(0, 1-2)$, or $(0, -1)$. Thus, circle B can be represented by the equation $x^2 + (y+1)^2 = 7^2$, or $x^2 + (y+1)^2 = 49$.

Choice A is incorrect. This is the equation of a circle obtained by shifting circle A right 2 units.

Choice B is incorrect. This is the equation of a circle obtained by shifting circle A up 2 units.

Choice C is incorrect. This is the equation of a circle obtained by shifting circle A left 2 units.

Question Difficulty: Hard

Question ID ff2ced2c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: ff2ced2c

Which of the following equations represents a circle in the *xy*-plane that intersects the *y*-axis at exactly one point?

A. msup $+ (y-8)^2 = 16$

B. msup $+ (y-4)^2 = 16$

C. msup $+ (y-9)^2 = 16$

D. $x^2 +$ msup $= 16$

ID: ff2ced2c Answer

Correct Answer: C

Rationale

Choice C is correct. The graph of the equation $(x-h)^2 + (y-k)^2 = r^2$ in the *xy*-plane is a circle with center (h, k) and a radius of length r. The radius of a circle is the distance from the center of the circle to any point on the circle. If a circle in the *xy*-plane intersects the *y*-axis at exactly one point, then the perpendicular distance from the center of the circle to this point on the *y*-axis must be equal to the length of the circle's radius. It follows that the *x*-coordinate of the circle's center must be equivalent to the length of the circle's radius. In other words, if the graph of $(x-h)^2 + (y-k)^2 = r^2$ is a circle that intersects the *y*-axis at exactly one point, then $r = |h|$ must be true. The equation in choice C is $(x-4)^2 + (y-9)^2 = 16$, or $(x-4)^2 + (y-9)^2 = 4^2$. This equation is in the form $(x-h)^2 + (y-k)^2 = r^2$, where $h = 4$, $k = 9$, and $r = 4$, and represents a circle in the *xy*-plane with center $(4, 9)$ and radius of length 4. Substituting 4 for r and 4 for h in the equation $r = |h|$ yields $4 = |4|$, or $4 = 4$, which is true. Therefore, the equation in choice C represents a circle in the *xy*-plane that intersects the *y*-axis at exactly one point.

Choice A is incorrect. This is the equation of a circle that does not intersect the *y*-axis at any point.

Choice B is incorrect. This is an equation of a circle that intersects the *x*-axis, not the *y*-axis, at exactly one point.

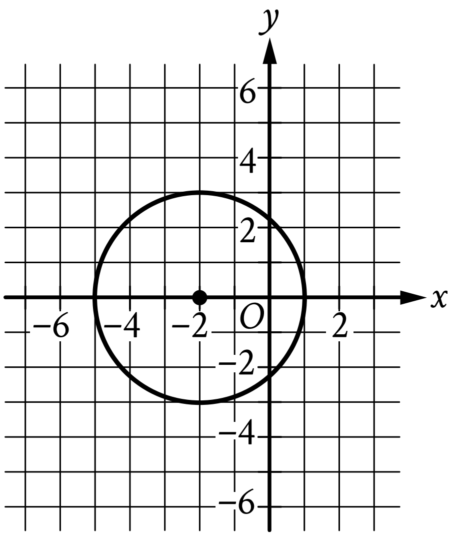
Choice D is incorrect. This is the equation of a circle with the center located on the *y*-axis and thus intersects the *y*-axis at exactly two points, not exactly one point.

Question Difficulty: Hard

Question ID 834ac03f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 834ac03f



Circle *A* (shown) is defined by the equation $(x+2)^2 + y^2 = 9$. Circle *B* (not shown) is the result of shifting circle *A* down 6 units and increasing the radius so that the radius of circle *B* is 2 times the radius of circle *A*. Which equation defines circle *B*?

A. msup $+ (y+6)^2 = (4)(9)$

B. 2msup $+ 2(y+6)^2 = 9$

C. msup $+ (y-6)^2 = (4)(9)$

D. 2msup $+ 2(y-6)^2 = 9$

ID: 834ac03f Answer

Correct Answer: A

Rationale

Choice A is correct. According to the graph, the center of circle *A* has coordinates $(-2, 0)$, and the radius of circle *A* is 3. It's given that circle *B* is the result of shifting circle *A* down 6 units and increasing the radius so that the radius of circle *B* is 2 times the radius of circle *A*. It follows that the center of circle *B* is 6 units below the center of circle *A*. The point that's 6 units below $(-2, 0)$ has the same *x*-coordinate as $(-2, 0)$ and has a *y*-coordinate that is 6 less than the *y*-coordinate of $(-2, 0)$. Therefore, the coordinates of the center of circle *B* are $(-2, 0-6)$, or $(-2, -6)$. Since the radius of circle *B* is 2 times the radius of circle *A*, the radius of circle *B* is $(2)(3)$. A circle in the *xy*-plane can be defined by an equation of the form $(x-h)^2 + (y-k)^2 = r^2$, where the coordinates of the center of the circle are (h, k) and the radius of the circle is r. Substituting -2 for h, -6 for k, and $(2)(3)$ for r in this equation yields $(x-(-2))^2 + (y-(-6))^2 = ((2)(3))^2$, which is equivalent to $(x+2)^2+(y+6)^2=(2)^2(3)^2$, or $(x+2)^2+(y+6)^2=(4)(9)$. Therefore, the equation $(x+2)^2+(y+6)^2=(4)(9)$ defines circle *B*.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This equation defines a circle that's the result of shifting circle *A* up, not down, by 6 units and increasing the radius.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 3e05efb1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 3e05efb1

A circle has center G, and points M and N lie on the circle. Line segments MH and NH are tangent to the circle at points M and N, respectively. If the radius of the circle is 168 millimeters and the perimeter of quadrilateral $GMHN$ is $3{,}856$ millimeters, what is the distance, in millimeters, between points G and H?

A. 168

B. $1{,}752$

C. $1{,}760$

D. $1{,}768$

ID: 3e05efb1 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the radius of the circle is 168 millimeters. Since points M and N both lie on the circle, segments GM and GN are both radii. Therefore, segments GM and GN each have length 168 millimeters. Two segments that are tangent to a circle and have a common exterior endpoint have equal length. Therefore, segment MH and segment NH have equal length. Let x represent the length of segment MH. Then x also represents the length of segment NH. It's given that the perimeter of quadrilateral $GMHN$ is $3{,}856$ millimeters. Since the perimeter of a quadrilateral is equal to the sum of the lengths of the sides of the quadrilateral, $3{,}856 = 168 + 168 + x + x$, or $3{,}856 = 336 + 2x$. Subtracting 336 from both sides of this equation yields $3{,}520 = 2x$, and dividing both sides of this equation by 2 yields $1{,}760 = x$. Therefore, the length of segment MH is $1{,}760$ millimeters. A line segment that's tangent to a circle is perpendicular to the radius of the circle at the point of tangency. Therefore, segment GM is perpendicular to segment MH. Since perpendicular segments form right angles, angle GMH is a right angle. Therefore, triangle GMH is a right triangle with legs of length $1{,}760$ millimeters and 168 millimeters, and hypotenuse GH. By the Pythagorean theorem, if a right triangle has a hypotenuse with length c and legs with lengths a and b, then $a^2 + b^2 = c^2$. Substituting $1{,}760$ for a and 168 for b in this equation yields $1{,}760^2 + 168^2 = c^2$, or $3{,}125{,}824 = c^2$. Taking the square root of both sides of this equation yields $\pm 1{,}768 = c$. Since c represents a length, which must be positive, the value of c is $1{,}768$. Therefore, the length of segment GH is $1{,}768$ millimeters, so the distance between points G and H is $1{,}768$ millimeters.

Choice A is incorrect. This is the distance between points G and M and between points G and N, not the distance between points G and H.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the distance between points M and H and between points N and H, not the distance between points G and H.

Question Difficulty: Hard

Question ID db84ccdc

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: db84ccdc

A circle in the *xy*-plane has a diameter with endpoints $(2, 4)$ and $(2, 14)$. An equation of this circle is $(x-2)^2 + (y-9)^2 = r^2$, where r is a positive constant. What is the value of r?

ID: db84ccdc Answer

Correct Answer: 5

Rationale

The correct answer is 5. The standard form of an equation of a circle in the *xy*-plane is $(x-h)^2 + (y-k)^2 = r^2$, where h, k, and r are constants, the coordinates of the center of the circle are (h, k), and the length of the radius of the circle is r. It's given that an equation of the circle is $(x-2)^2 + (y-9)^2 = r^2$. Therefore, the center of this circle is $(2, 9)$. It's given that the endpoints of a diameter of the circle are $(2, 4)$ and $(2, 14)$. The length of the radius is the distance from the center of the circle to an endpoint of a diameter of the circle, which can be found using the distance formula, $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Substituting the center of the circle $(2, 9)$ and one endpoint of the diameter $(2, 4)$ in this formula gives a distance of $\sqrt{(2-2)^2 + (9-4)^2}$, or $\sqrt{0^2 + 5^2}$, which is equivalent to 5. Since the distance from the center of the circle to an endpoint of a diameter is 5, the value of r is 5.

Question Difficulty: Hard

Question ID 9ca6e7b4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 9ca6e7b4

A circle has center O, and points A and B lie on the circle. The measure of arc AB is 45° and the length of arc AB is 3 inches. What is the circumference, in inches, of the circle?

A. 3

B. 6

C. 9

D. 24

ID: 9ca6e7b4 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the measure of arc AB is 45° and the length of arc AB is 3 inches. The arc measure of the full circle is 360°. If x represents the circumference, in inches, of the circle, it follows that $\frac{45^\circ}{360^\circ} = \frac{3 \text{ inches}}{x \text{ inches}}$. This equation is equivalent to $\frac{45}{360} = \frac{3}{x}$, or $\frac{1}{8} = \frac{3}{x}$. Multiplying both sides of this equation by $8x$ yields $1(x) = 3(8)$, or $x = 24$. Therefore, the circumference of the circle is 24 inches.

Choice A is incorrect. This is the length of arc AB.

Choice B is incorrect and may result from multiplying the length of arc AB by 2.

Choice C is incorrect and may result from squaring the length of arc AB.

Question Difficulty: Hard

Question ID 3a95868c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 3a95868c

Point O is the center of a circle. The measure of arc RS on this circle is 100°. What is the measure, in degrees, of its associated angle ROS?

ID: 3a95868c Answer

Correct Answer: 100

Rationale

The correct answer is 100. It's given that point O is the center of a circle and the measure of arc RS on the circle is 100°. It follows that points R and S lie on the circle. Therefore, $\overline{OR}$ and $\overline{OS}$ are radii of the circle. A central angle is an angle formed by two radii of a circle, with its vertex at the center of the circle. Therefore, $\angle ROS$ is a central angle. Because the degree measure of an arc is equal to the measure of its associated central angle, it follows that the measure, in degrees, of $\angle ROS$ is 100.

Question Difficulty: Hard

Question ID 9bf2678d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 9bf2678d

What is the value of $\sin 42\pi$?

A. 0

B. $\frac{1}{2}$

C. $\frac{\sqrt{2}}{2}$

D. 1

ID: 9bf2678d Answer

Correct Answer: A

Rationale

Choice A is correct. The sine of a number t is the *y*-coordinate of the point arrived at by traveling a distance of t units counterclockwise around the unit circle from the starting point $(1, 0)$. Since the unit circle has a circumference of 2π units, it follows that one full rotation around the circle is equal to a distance of 2π units. Therefore, a distance of 42π units around the circle from the starting point $(1, 0)$ would result in exactly 21 full rotations, arriving back at the point $(1, 0)$. So, $\sin 42\pi$ is equal to the *y*-coordinate of the point $(1, 0)$, which is 0.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This is the value of $\cos 42\pi$, not $\sin 42\pi$.

Question Difficulty: Hard

Question ID 55b41004

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 55b41004

A circle in the *xy*-plane has its center at $(-4, -6)$. Line k is tangent to this circle at the point $(-7, -7)$. What is the slope of line k?

A. -3

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

ID: 55b41004 Answer

Correct Answer: A

Rationale

Choice A is correct. A line that's tangent to a circle is perpendicular to the radius of the circle at the point of tangency. It's given that the circle has its center at $(-4, -6)$ and line k is tangent to the circle at the point $(-7, -7)$. The slope of a radius defined by the points (q, r) and (s, t) can be calculated as $\frac{t-r}{s-q}$. The points $(-7, -7)$ and $(-4, -6)$ define the radius of the circle at the point of tangency. Therefore, the slope of this radius can be calculated as $\frac{(-6)-(-7)}{(-4)-(-7)}$, or $\frac{1}{3}$. If a line and a radius are perpendicular, the slope of the line must be the negative reciprocal of the slope of the radius. The negative reciprocal of $\frac{1}{3}$ is -3. Thus, the slope of line k is -3.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the slope of the radius of the circle at the point of tangency, not the slope of line k.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard