Question ID 005e9982

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 005e9982

$$f(x) = 9{,}000(0.66)^x$$

The given function f models the number of advertisements a company sent to its clients each year, where x represents the number of years since 1997, and $0 \leq x \leq 5$. If $y = f(x)$ is graphed in the *xy*-plane, which of the following is the best interpretation of the *y*-intercept of the graph in this context?

A. The minimum estimated number of advertisements the company sent to its clients during the 5 years was $1{,}708$.

B. The minimum estimated number of advertisements the company sent to its clients during the 5 years was $9{,}000$.

C. The estimated number of advertisements the company sent to its clients in 1997 was $1{,}708$.

D. The estimated number of advertisements the company sent to its clients in 1997 was $9{,}000$.

ID: 005e9982 Answer

Correct Answer: D

Rationale

Choice D is correct. The *y*-intercept of a graph in the *xy*-plane is the point where $x = 0$. For the given function f, the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane can be found by substituting 0 for x in the equation $y = 9{,}000(0.66)^x$, which gives $y = 9{,}000(0.66)^0$. This is equivalent to $y = 9{,}000(1)$, or $y = 9{,}000$. Therefore, the *y*-intercept of the graph of $y = f(x)$ is $(0, 9{,}000)$. It's given that the function f models the number of advertisements a company sent to its clients each year. Therefore, $f(x)$ represents the estimated number of advertisements the company sent to its clients each year. It's also given that x represents the number of years since 1997. Therefore, $x = 0$ represents 0 years since 1997, or 1997. Thus, the best interpretation of the *y*-intercept of the graph of $y = f(x)$ is that the estimated number of advertisements the company sent to its clients in 1997 was $9{,}000$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID fe81a236

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: fe81a236

The function g is defined by $g(x) = x(x-2)(x+6)^2$. The value of $g(7-w)$ is 0, where w is a constant. What is the sum of all possible values of w?

ID: fe81a236 Answer

Correct Answer: 25

Rationale

The correct answer is 25. The value of $g(7-w)$ is the value of $g(x)$ when $x = 7 - w$, where w is a constant. Substituting $7 - w$ for x in the given equation yields $g(7-w) = (7-w)(7-w-2)(7-w+6)^2$, which is equivalent to $g(7-w) = (7-w)(5-w)(13-w)^2$. It's given that the value of $g(7-w)$ is 0. Substituting 0 for $g(7-w)$ in the equation $g(7-w) = (7-w)(5-w)(13-w)^2$ yields $0 = (7-w)(5-w)(13-w)^2$. Since the product of the three factors on the right-hand side of this equation is equal to 0, at least one of these three factors must be equal to 0. Therefore, the possible values of w can be found by setting each factor equal to 0. Setting the first factor equal to 0 yields $7 - w = 0$. Adding w to both sides of this equation yields $7 = w$. Therefore, 7 is one possible value of w. Setting the second factor equal to 0 yields $5 - w = 0$. Adding w to both sides of this equation yields $5 = w$. Therefore, 5 is a second possible value of w. Setting the third factor equal to 0 yields $(13-w)^2 = 0$. Taking the square root of both sides of this equation yields $13 - w = 0$. Adding w to both sides of this equation yields $13 = w$. Therefore, 13 is a third possible value of w. Adding the three possible values of w yields $7 + 5 + 13$, or 25. Therefore, the sum of all possible values of w is 25.

Question Difficulty: Hard

Question ID 54bf74ea

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 54bf74ea

$$p(t) = 90{,}000(1.06)^t$$

The given function p models the population of Lowell t years after a census. Which of the following functions best models the population of Lowell m months after the census?

A. $r(m) = \frac{90{,}000}{12}(1.06)^m$

B. $r(m) = 90{,}000\left(\frac{1.06}{12}\right)^m$

C. $r(m) = 90{,}000\left(\frac{1.06}{12}\right)^{\frac{m}{12}}$

D. $r(m) = 90{,}000(1.06)^{\frac{m}{12}}$

ID: 54bf74ea Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the function p models the population of Lowell t years after a census. Since there are 12 months in a year, m months after the census is equivalent to $\frac{m}{12}$ years after the census. Substituting $\frac{m}{12}$ for t in the equation $p(t) = 90{,}000(1.06)^t$ yields $p\left(\frac{m}{12}\right) = 90{,}000(1.06)^{\frac{m}{12}}$. Therefore, the function r that best models the population of Lowell m months after the census is $r(m) = 90{,}000(1.06)^{\frac{m}{12}}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID a51ff0d6

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: a51ff0d6

$$f(x) = (x+7)^2 + 4$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

ID: a51ff0d6 Answer

Correct Answer: -7

Rationale

The correct answer is -7. For a quadratic function defined by an equation of the form $f(x) = a(x-h)^2 + k$, where a, h, and k are constants and $a > 0$, the function reaches its minimum when $x = h$. In the given function, $a = 1$, $h = -7$, and $k = 4$. Therefore, the value of x for which $f(x)$ reaches its minimum is -7.

Question Difficulty: Hard

Question ID 32dc74a4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 32dc74a4

A right rectangular prism has a height of 9 inches. The length of the prism's base is x inches, which is 7 inches more than the width of the prism's base. Which function V gives the volume of the prism, in cubic inches, in terms of the length of the prism's base?

A. $V(x) = x(x+9)(x+7)$

B. $V(x) = x(x+9)(x-7)$

C. $V(x) = 9x(x+7)$

D. $V(x) = 9x(x-7)$

ID: 32dc74a4 Answer

Correct Answer: D

Rationale

Choice D is correct. The volume of a right rectangular prism can be represented by a function V that gives the volume of the prism, in cubic inches, in terms of the length of the prism's base. The volume of a right rectangular prism is equal to the area of its base times its height. It's given that the length of the prism's base is x inches, which is 7 inches more than the width of the prism's base. This means that the width of the prism's base is $x - 7$ inches. It follows that the area of the prism's base, in square inches, is $x(x-7)$ and the volume, in cubic inches, of the prism is $x(x-7)(9)$. Thus, the function V that gives the volume of this right rectangular prism, in cubic inches, in terms of the length of the prism's base, x, is $V(x) = 9x(x-7)$.

Choice A is incorrect. This function would give the volume of the prism if the height were 9 inches more than the length of its base and the width of the base were 7 inches more than its length.

Choice B is incorrect. This function would give the volume of the prism if the height were 9 inches more than the length of its base.

Choice C is incorrect. This function would give the volume of the prism if the width of the base were 7 inches more than its length, rather than the length of the base being 7 inches more than its width.

Question Difficulty: Hard

Question ID 1d670e5f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 1d670e5f

$$f(x) = x^2 - 48x + 2{,}304$$

What is the minimum value of the given function?

ID: 1d670e5f Answer

Correct Answer: 1728

Rationale

The correct answer is $1{,}728$. The given function can be rewritten in the form $f(x) = a(x-h)^2 + k$, where a is a positive constant and the minimum value, k, of the function occurs when the value of x is h. By completing the square, $f(x) = x^2 - 48x + 2{,}304$ can be written as $f(x) = x^2 - 48x + \left(\frac{48}{2}\right)^2 + 2{,}304 - \left(\frac{48}{2}\right)^2$, or $f(x) = (x-24)^2 + 1{,}728$. This equation is in the form $f(x) = a(x-h)^2 + k$, where $a = 1$, $h = 24$, and $k = 1{,}728$. Therefore, the minimum value of the given function is $1{,}728$.

Question Difficulty: Hard

Question ID 78eae128

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 78eae128

$$y = 2(x-d)(x+d)(x+g)(x-d)$$

In the given equation, d and g are unique positive constants. When the equation is graphed in the *xy*-plane, how many distinct *x*-intercepts does the graph have?

A. 4

B. 3

C. 2

D. 1

ID: 78eae128 Answer

Correct Answer: B

Rationale

Choice B is correct. An *x*-intercept of a graph in the *xy*-plane is a point on the graph where the value of y is 0. Substituting 0 for y in the given equation yields $0 = 2(x-d)(x+d)(x+g)(x-d)$. By the zero product property, the solutions to this equation are $x = d$, $x = -d$, $x = -g$, and $x = d$. However, $x = d$ and $x = d$ are identical. It's given that d and g are unique positive constants. It follows that the equation $0 = 2(x-d)(x+d)(x+g)(x-d)$ has 3 unique solutions: $x = d$, $x = -d$, and $x = -g$. Thus, the graph of the given equation has 3 distinct *x*-intercepts.

Choice A is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 80803aed

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 80803aed

The function g is defined by $g(x) = (x + 14)(t - x)$, where t is a constant. In the *xy*-plane, the graph of $y = g(x)$ passes through the point $(24, 0)$. What is the value of $g(0)$?

ID: 80803aed Answer

Correct Answer: 336

Rationale

The correct answer is 336. By the zero product property, if $(x + 14)(t - x) = 0$, then $x + 14 = 0$, which gives $x = -14$, or $(t - x) = 0$, which gives $x = t$. Therefore, $g(x) = 0$ when $x = -14$ and when $x = t$. Since the graph of $y = g(x)$ passes through the point $(24, 0)$, it follows that $g(24) = 0$, so $t = 24$. Substituting 24 for t in the equation $g(x) = (x + 14)(t - x)$ yields $g(x) = (x + 14)(24 - x)$. The value of $g(0)$ can be calculated by substituting 0 for x in this equation, which yields $g(0) = (0 + 14)(24 - 0)$, or $g(0) = 336$.

Question Difficulty: Hard

Question ID e24f00be

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: e24f00be

The product of two positive integers is 462. If the first integer is 5 greater than twice the second integer, what is the smaller of the two integers?

ID: e24f00be Answer

Correct Answer: 14

Rationale

The correct answer is 14. Let x represent the first integer and y represent the second integer. If the first integer is 5 greater than twice the second integer, then $x = 2y + 5$. It's given that the product of the two integers is 462; therefore $xy = 462$. Substituting $2y + 5$ for x in this equation yields $(2y + 5)(y) = 462$, which can be written as $2y^2 + 5y = 462$. Subtracting 462 from each side of this equation yields $2y^2 + 5y - 462 = 0$. The left-hand side of this equation can be factored by finding two values whose product is $2(-462)$, or -924, and whose sum is 5. The two values whose product is -924 and whose sum is 5 are 33 and -28. Thus, the equation $2y^2 + 5y - 462 = 0$ can be rewritten as $2y^2 - 28y + 33y - 462 = 0$, which is equivalent to $2y(y - 14) + 33(y - 14) = 0$, or $(2y + 33)(y - 14) = 0$. By the zero product property, it follows that $2y + 33 = 0$ or $y - 14 = 0$. Subtracting 33 from both sides of the equation $2y + 33 = 0$ yields $2y = -33$. Dividing both sides of this equation by 2 yields $y = -\frac{33}{2}$. Since y is a positive integer, the value of y isn't $-\frac{33}{2}$. Adding 14 to both sides of the equation $y - 14 = 0$ yields $y = 14$. Substituting 14 for y in the equation $xy = 462$ yields $x(14) = 462$. Dividing both sides of this equation by 14 yields $x = 33$. Therefore, the two integers are 14 and 33, so the smaller of the two integers is 14.

Question Difficulty: Hard

Question ID a2d5ec41

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: a2d5ec41

Two variables, x and y, are related such that for each increase of 1 in the value of x, the value of y increases by a factor of 4. When $x = 0$, $y = 200$. Which equation represents this relationship?

A. $y = 4$msup

B. $y = 4$msup

C. $y = 200$msup

D. $y = 200$msup

ID: a2d5ec41 Answer

Correct Answer: D

Rationale

Choice D is correct. Since the value of y increases by a constant factor, 4, for each increase of 1 in the value of x, the relationship between x and y is exponential. An exponential relationship between x and y can be represented by an equation of the form $y = a(b)^x$, where a is the value of x when $y = 0$ and y increases by a factor of b for each increase of 1 in the value of x. Since $y = 200$ when $x = 0$, $a = 200$. Since y increases by a factor of 4 for each increase of 1 in the value of x, $b = 4$. Substituting 200 for a and 4 for b in the equation $y = a(b)^x$ yields $y = 200(4)^x$. Thus, the equation $y = 200(4)^x$ represents the relationship between x and y.

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect. This equation represents a relationship where for each increase of 1 in the value of x, the value of y increases by a factor of 200, not 4, and when $x = 0$, y is equal to 4, not 200.

Choice C is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 5ec80061

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 5ec80061

$$f(t) = 8{,}000(0.65)^t$$

The given function f models the number of coupons a company sent to their customers at the end of each year, where t represents the number of years since the end of 1998, and $0 \leq t \leq 5$. If $y = f(t)$ is graphed in the *ty*-plane, which of the following is the best interpretation of the *y*-intercept of the graph in this context?

A. The minimum estimated number of coupons the company sent to their customers during the 5 years was $1{,}428$.

B. The minimum estimated number of coupons the company sent to their customers during the 5 years was $8{,}000$.

C. The estimated number of coupons the company sent to their customers at the end of 1998 was $1{,}428$.

D. The estimated number of coupons the company sent to their customers at the end of 1998 was $8{,}000$.

ID: 5ec80061 Answer

Correct Answer: D

Rationale

Choice D is correct. The *y*-intercept of a graph in the *ty*-plane is the point where $t = 0$. For the given function f, the *y*-intercept of the graph of $y = f(t)$ in the *ty*-plane can be found by substituting 0 for t in the equation $y = 8{,}000(0.65)^t$, which gives $y = 8{,}000(0.65)^0$. This is equivalent to $y = 8{,}000(1)$, or $y = 8{,}000$. Therefore, the *y*-intercept of the graph of $y = f(t)$ is $(0, 8{,}000)$. It's given that the function f models the number of coupons a company sent to their customers at the end of each year. Therefore, $f(t)$ represents the estimated number of coupons the company sent to their customers at the end of each year. It's also given that t represents the number of years since the end of 1998. Therefore, $t = 0$ represents 0 years since the end of 1998, or the end of 1998. Thus, the best interpretation of the *y*-intercept of the graph of $y = f(t)$ is that the estimated number of coupons the company sent to their customers at the end of 1998 was $8{,}000$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID a6c47b9c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: a6c47b9c

The function f is defined by $f(x)=(-8)(2)^x+22$. What is the *y*-intercept of the graph of $y=f(x)$ in the *xy*-plane?

A. $(0,14)$

B. $(0,2)$

C. $(0,22)$

D. $(0,-8)$

ID: a6c47b9c Answer

Correct Answer: A

Rationale

Choice A is correct. The *y*-intercept of the graph of $y=f(x)$ in the *xy*-plane occurs at the point on the graph where $x=0$. In other words, when $x=0$, the corresponding value of $f(x)$ is the *y*-coordinate of the *y*-intercept. Substituting 0 for x in the given equation yields $f(0)=(-8)(2)^0+22$, which is equivalent to $f(0)=(-8)(1)+22$, or $f(0)=14$. Thus, when $x=0$, the corresponding value of $f(x)$ is 14. Therefore, the *y*-intercept of the graph of $y=f(x)$ in the *xy*-plane is $(0,14)$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This could be the *y*-intercept for $f(x)=(-8)(2)^x$, not $f(x)=(-8)(2)^x+22$.

Question Difficulty: Hard

Question ID 1145bfcc

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 1145bfcc

$$y = x^2 - 14x + 22$$

The given equation relates the variables x and y. For what value of x does the value of y reach its minimum?

ID: 1145bfcc Answer

Correct Answer: 7

Rationale

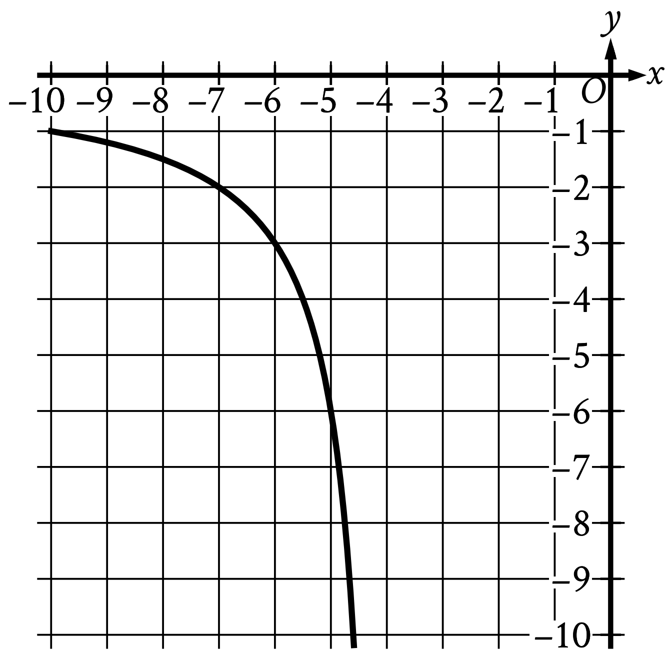
The correct answer is 7. When an equation is of the form $y = ax^2 + bx + c$, where a, b, and c are constants, the value of y reaches its minimum when $x = -\frac{b}{2a}$. Since the given equation is of the form $y = ax^2 + bx + c$, it follows that $a = 1$, $b = -14$, and $c = 22$. Therefore, the value of y reaches its minimum when $x = -\frac{(-14)}{2(1)}$, or $x = 7$.

Question Difficulty: Hard

Question ID c7f7ccdd

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: c7f7ccdd



The rational function f is defined by an equation in the form $f(x)=\frac{a}{x+b}$, where a and b are constants. The partial graph of $y=f(x)$ is shown. If $g(x)=f(x+4)$, which equation could define function g?

A. $g(x)=\frac{6}{x}$

B. $g(x)=\frac{6}{x+4}$

C. $g(x)=\frac{6}{x+8}$

D. $g(x)=\frac{6(x+4)}{x+4}$

ID: c7f7ccdd Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that $f(x)=\frac{a}{x+b}$ and that the graph shown is a partial graph of $y=f(x)$. Substituting y for $f(x)$ in the equation $f(x)=\frac{a}{x+b}$ yields $y=\frac{a}{x+b}$. The graph passes through the point $(-7,-2)$. Substituting -7 for x and -2 for y in the equation $y=\frac{a}{x+b}$ yields $-2=\frac{a}{-7+b}$. Multiplying each side of this equation by $-7+b$ yields $-2(-7+b)=a$, or $14-2b=a$. The graph also passes through the point $(-5,-6)$. Substituting -5 for x and -6 for y in the equation $y=\frac{a}{x+b}$ yields $-6=\frac{a}{-5+b}$. Multiplying each side of this equation by $-5+b$ yields $-6(-5+b)=a$, or $30-6b=a$. Substituting $14-2b$ for a in this equation yields $30-6b=14-2b$. Adding $6b$ to each side of this equation yields $30 = 14 + 4b$. Subtracting 14 from each side of this equation yields $16 = 4b$. Dividing each side of this equation by 4 yields $4 = b$. Substituting 4 for b in the equation $14 - 2b = a$ yields $14 - 2(4) = a$, or $6 = a$. Substituting 6 for a and 4 for b in the equation $f(x) = \frac{a}{x+b}$ yields $f(x) = \frac{6}{x+4}$. It's given that $g(x) = f(x+4)$. Substituting $x + 4$ for x in the equation $f(x) = \frac{6}{x+4}$ yields $f(x+4) = \frac{6}{x+4+4}$, which is equivalent to $f(x+4) = \frac{6}{x+8}$. It follows that $g(x) = \frac{6}{x+8}$.

Choice A is incorrect. This could define function g if $g(x) = f(x-4)$.

Choice B is incorrect. This could define function g if $g(x) = f(x)$.

Choice D is incorrect. This could define function g if $g(x) = f(x) \cdot (x+4)$.

Question Difficulty: Hard

Question ID 2833ad7d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 2833ad7d

A model estimates that at the end of each year from 2015 to 2020, the number of squirrels in a population was 150% more than the number of squirrels in the population at the end of the previous year. The model estimates that at the end of 2016, there were 180 squirrels in the population. Which of the following equations represents this model, where n is the estimated number of squirrels in the population t years after the end of 2015 and $t \leq 5$?

A. $n = 72$msup

B. $n = 72$msup

C. $n = 180$msup

D. $n = 180$msup

ID: 2833ad7d Answer

Correct Answer: B

Rationale

Choice B is correct. Since the model estimates that the number of squirrels in the population increased by a fixed percentage, 150%, each year, the model can be represented by an exponential equation of the form $n = a\left(1 + \frac{p}{100}\right)^t$, where a is the estimated number of squirrels in the population at the end of 2015, and the model estimates that at the end of each year, the number is $p\%$ more than the number at the end of the previous year. Since the model estimates that at the end of each year, the number was 150% more than the number at the end of the previous year, $p = 150$. Substituting 150 for p in the equation $n = a\left(1 + \frac{p}{100}\right)^t$ yields $n = a\left(1 + \frac{150}{100}\right)^t$, which is equivalent to $n = a(1 + 1.5)^t$, or $n = a(2.5)^t$. It's given that the estimated number of squirrels at the end of 2016 was 180. This means that when $t = 1$, $n = 180$. Substituting 1 for t and 180 for n in the equation $n = a(2.5)^t$ yields $180 = a(2.5)^1$, or $180 = 2.5a$. Dividing each side of this equation by 2.5 yields $72 = a$. Substituting 72 for a in the equation $n = a(2.5)^t$ yields $n = 72(2.5)^t$.

Choice A is incorrect. This equation represents a model where at the end of each year, the estimated number of squirrels was 150% of, not 150% more than, the estimated number at the end of the previous year.

Choice C is incorrect. This equation represents a model where at the end of each year, the estimated number of squirrels was 150% of, not 150% more than, the estimated number at the end of the previous year, and the estimated number of squirrels at the end of 2015, not the end of 2016, was 180.

Choice D is incorrect. This equation represents a model where the estimated number of squirrels at the end of 2015, not the end of 2016, was 180.

Question Difficulty: Hard

Question ID 317d165b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 317d165b

$$f(x) = 9(4)^x$$

The function f is defined by the given equation. If $g(x) = f(x+2)$, which of the following equations defines the function g?

A. $g(x) = 18(4)^x$

B. $g(x) = 144(4)^x$

C. $g(x) = 18(8)^x$

D. $g(x) = 81(16)^x$

ID: 317d165b Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that $f(x) = 9(4)^x$ and $g(x) = f(x+2)$. Substituting $x+2$ for x in $f(x) = 9(4)^x$ gives $f(x+2) = 9(4)^{x+2}$. Rewriting this equation using properties of exponents gives $f(x+2) = 9(4)^x(4)^2$, which is equivalent to $f(x+2) = 9(4)^x(16)$. Multiplying 9 and 16 in this equation gives $f(x+2) = 144(4)^x$. Since $g(x) = f(x+2)$, $g(x) = 144(4)^x$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2a093c45

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 2a093c45

$$f(x) = (x - 10)(x + 13)$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

A. -130

B. -13

C. $-\frac{23}{2}$

D. $-\frac{3}{2}$

ID: 2a093c45 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $f(x) = (x - 10)(x + 13)$, which can be rewritten as $f(x) = x^2 + 3x - 130$. Since the coefficient of the x^2-term is positive, the graph of $y = f(x)$ in the *xy*-plane opens upward and reaches its minimum value at its vertex. The *x*-coordinate of the vertex is the value of x such that $f(x)$ reaches its minimum. For an equation in the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants, the *x*-coordinate of the vertex is $-\frac{b}{2a}$. For the equation $f(x) = x^2 + 3x - 130$, $a = 1$, $b = 3$, and $c = -130$. It follows that the *x*-coordinate of the vertex is $-\frac{3}{2(1)}$, or $-\frac{3}{2}$. Therefore, $f(x)$ reaches its minimum when the value of x is $-\frac{3}{2}$.

Alternate approach: The value of x for the vertex of a parabola is the *x*-value of the midpoint between the two *x*-intercepts of the parabola. Since it's given that $f(x) = (x - 10)(x + 13)$, it follows that the two *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane occur when $x = 10$ and $x = -13$, or at the points $(10, 0)$ and $(-13, 0)$. The midpoint between two points, (x_1, y_1) and (x_2, y_2), is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Therefore, the midpoint between $(10, 0)$ and $(-13, 0)$ is $\left(\frac{10+(-13)}{2}, \frac{0+0}{2}\right)$, or $\left(-\frac{3}{2}, 0\right)$. It follows that $f(x)$ reaches its minimum when the value of x is $-\frac{3}{2}$.

Choice A is incorrect. This is the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane.

Choice B is incorrect. This is one of the *x*-coordinates of the *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID a4a86ebb

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: a4a86ebb

In the *xy*-plane, a parabola has vertex $(9, -14)$ and intersects the *x*-axis at two points. If the equation of the parabola is written in the form $y = ax^2 + bx + c$, where a, b, and c are constants, which of the following could be the value of $a + b + c$?

A. -23

B. -19

C. -14

D. -12

ID: a4a86ebb Answer

Correct Answer: D

Rationale

Choice D is correct. The equation of a parabola in the *xy*-plane can be written in the form $y = a(x - h)^2 + k$, where a is a constant and (h, k) is the vertex of the parabola. If a is positive, the parabola will open upward, and if a is negative, the parabola will open downward. It's given that the parabola has vertex $(9, -14)$. Substituting 9 for h and -14 for k in the equation $y = a(x - h)^2 + k$ gives $y = a(x - 9)^2 - 14$, which can be rewritten as $y = a(x - 9)(x - 9) - 14$, or $y = a\left(x^2 - 18x + 81\right) - 14$. Distributing the factor of a on the right-hand side of this equation yields $y = ax^2 - 18ax + 81a - 14$. Therefore, the equation of the parabola, $y = ax^2 - 18ax + 81a - 14$, can be written in the form $y = ax^2 + bx + c$, where $a = a$, $b = -18a$, and $c = 81a - 14$. Substituting $-18a$ for b and $81a - 14$ for c in the expression $a + b + c$ yields $(a) + (-18a) + (81a - 14)$, or $64a - 14$. Since the vertex of the parabola, $(9, -14)$, is below the *x*-axis, and it's given that the parabola intersects the *x*-axis at two points, the parabola must open upward. Therefore, the constant a must have a positive value. Setting the expression $64a - 14$ equal to the value in choice D yields $64a - 14 = -12$. Adding 14 to both sides of this equation yields $64a = 2$. Dividing both sides of this equation by 64 yields $a = \frac{2}{64}$, which is a positive value. Therefore, if the equation of the parabola is written in the form $y = ax^2 + bx + c$, where a, b, and c are constants, the value of $a + b + c$ could be -12.

Choice A is incorrect. If the equation of a parabola with a vertex at $(9, -14)$ is written in the form $y = ax^2 + bx + c$, where a, b, and c are constants and $a + b + c = -23$, then the value of a will be negative, which means the parabola will open downward, not upward, and will intersect the *x*-axis at zero points, not two points.

Choice B is incorrect. If the equation of a parabola with a vertex at $(9, -14)$ is written in the form $y = ax^2 + bx + c$, where a, b, and c are constants and $a + b + c = -19$, then the value of a will be negative, which means the parabola will open downward, not upward, and will intersect the *x*-axis at zero points, not two points.

Choice C is incorrect. If the equation of a parabola with a vertex at $(9, -14)$ is written in the form $y = ax^2 + bx + c$, where a, b, and c are constants and $a + b + c = -14$, then the value of a will be 0, which is inconsistent with the equation of a parabola.

Question Difficulty: Hard

Question ID b91b2899

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: b91b2899

$$f(x) = (1.84)^{\frac{x}{4}}$$

The function f is defined by the given equation. The equation can be rewritten as $f(x) = \left(1 + \frac{p}{100}\right)^x$, where p is a constant. Which of the following is closest to the value of p?

A. 16

B. 21

C. 46

D. 96

ID: b91b2899 Answer

Correct Answer: A

Rationale

Choice A is correct. The equation $f(x) = (1.84)^{\frac{x}{4}}$ can be rewritten as $f(x) = (1.84)^{\left(\frac{1}{4}\right)(x)}$, which is equivalent to $f(x) = \left(1.84^{\frac{1}{4}}\right)^x$, or approximately $f(x) = (1.16467)^x$. Since it's given that $f(x) = (1.84)^{\frac{x}{4}}$ can be rewritten as $f(x) = \left(1 + \frac{p}{100}\right)^x$, where p is a constant, it follows that $1 + \frac{p}{100}$ is approximately equal to 1.16467. Therefore, $\frac{p}{100}$ is approximately equal to 0.16467. It follows that the value of p is approximately equal to 16.467. Of the given choices, 16 is closest to the value of p.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID f5f840a0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: f5f840a0

For the function f, $f(0) = 86$, and for each increase in x by 1, the value of $f(x)$ decreases by 80%. What is the value of $f(2)$?

ID: f5f840a0 Answer

Correct Answer: 3.44, 86/25

Rationale

The correct answer is 3.44. It's given that $f(0) = 86$ and that for each increase in x by 1, the value of $f(x)$ decreases by 80%. Because the output of the function decreases by a constant percentage for each 1-unit increase in the value of x, this relationship can be represented by an exponential function of the form $f(x) = a(b)^x$, where a represents the initial value of the function and b represents the rate of decay,
expressed as a decimal. Because $f(0) = 86$, the value of a must be 86. Because the value of $f(x)$ decreases by 80% for each 1-unit increase in x, the value of b must be $(1 - 0.80)$, or 0.2. Therefore, the function f can be defined by $f(x) = 86(0.2)^x$. Substituting 2 for x in this function yields $f(2) = 86(0.2)^2$, which is equivalent to $f(2) = 86(0.04)$, or $f(2) = 3.44$. Either 3.44 or $86/25$ may be entered as the correct answer.

Alternate approach: It's given that $f(0) = 86$ and that for each increase in x by 1, the value of $f(x)$ decreases by 80%. Therefore, when $x = 1$, the value of $f(x)$ is $(100 - 80)\%$, or 20%, of 86, which can be expressed as $(0.20)(86)$. Since $(0.20)(86) = 17.2$, the value of $f(1)$ is 17.2. Similarly, when $x = 2$, the value of $f(x)$ is 20% of 17.2, which can be expressed as $(0.20)(17.2)$. Since $(0.20)(17.2) = 3.44$, the value of $f(2)$ is 3.44. Either 3.44 or $86/25$ may be entered as the correct answer.

Question Difficulty: Hard

Question ID 03d0309b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 03d0309b

$$f(x) = (x-1)(x+3)(x-2)$$

In the *xy*-plane, when the graph of the function f, where $y = f(x)$, is shifted up 6 units, the resulting graph is defined by the function g. If the graph of $y = g(x)$ crosses through the point $(4, b)$, where b is a constant, what is the value of b?

ID: 03d0309b Answer

Correct Answer: 48

Rationale

The correct answer is 48. It's given that in the *xy*-plane, when the graph of the function f, where $y = f(x)$, is shifted up 6 units, the resulting graph is defined by the function g. Therefore, function g can be defined by the equation $g(x) = f(x) + 6$. It's given that $f(x) = (x-1)(x+3)(x-2)$. Substituting $(x-1)(x+3)(x-2)$ for $f(x)$ in the equation $g(x) = f(x) + 6$ yields $g(x) = (x-1)(x+3)(x-2) + 6$. For the point $(4, b)$, the value of x is 4. Substituting 4 for x in the equation $g(x) = (x-1)(x+3)(x-2) + 6$ yields $g(4) = (4-1)(4+3)(4-2) + 6$, or $g(4) = 48$. It follows that the graph of $y = g(x)$ crosses through the point $(4, 48)$. Therefore, the value of b is 48.

Question Difficulty: Hard

Question ID 1caa83ee

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 1caa83ee

For the function q, the value of $q(x)$ decreases by 45% for every increase in the value of x by 1. If $q(0) = 14$, which equation defines q?

A. $q(x) = 0.55(14)^x$

B. $q(x) = 1.45(14)^x$

C. $q(x) = 14(0.55)^x$

D. $q(x) = 14(1.45)^x$

ID: 1caa83ee Answer

Correct Answer: C

Rationale

Choice C is correct. Since the value of $q(x)$ decreases by a fixed percentage, 45%, for every increase in the value of x by 1, the function q is a decreasing exponential function. A decreasing exponential function can be written in the form $q(x) = a\left(1 - \frac{p}{100}\right)^x$, where a is the value of $q(0)$ and the value of $q(x)$ decreases by $p\%$ for every increase in the value of x by 1. If $q(0) = 14$, then $a = 14$. Since the value of $q(x)$ decreases by 45% for every increase in the value of x by 1, $p = 45$. Substituting 14 for a and 45 for p in the equation $q(x) = a\left(1 - \frac{p}{100}\right)^x$ yields $q(x) = 14\left(1 - \frac{45}{100}\right)^x$, which is equivalent to $q(x) = 14(1 - 0.45)^x$, or $q(x) = 14(0.55)^x$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. For this function, the value of $q(x)$ increases, rather than decreases, by 45% for every increase in the value of x by 1.

Question Difficulty: Hard

Question ID ae4d719d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: ae4d719d

The function f is defined by $f(x) = a^x + b$, where a and b are constants and $a > 0$. In the *xy*-plane, the graph of $y = f(x)$ has a *y*-intercept at $(0, -25)$ and passes through the point $(2, 23)$. What is the value of $a + b$?

ID: ae4d719d Answer

Correct Answer: -19

Rationale

The correct answer is -19. It's given that function f is defined by $f(x) = a^x + b$, where a and b are constants and $a > 0$. It's also given that the graph of $y = f(x)$ in the *xy*-plane has a *y*-intercept at $(0, -25)$ and passes through the point $(2, 23)$. Since the graph has a *y*-intercept at $(0, -25)$, $f(0) = -25$. Substituting 0 for x in the given equation yields $f(0) = a^0 + b$, or $f(0) = 1 + b$, and substituting -25 for $f(0)$ in this equation yields $-25 = 1 + b$. Subtracting 1 from each side of this equation yields $-26 = b$. Substituting -26 for b in the equation $f(x) = a^x + b$ yields $f(x) = a^x - 26$. Since the graph also passes through the point $(2, 23)$, $f(2) = 23$. Substituting 2 for x in the equation $f(x) = a^x - 26$ yields $f(2) = a^2 - 26$, and substituting 23 for $f(2)$ yields $23 = a^2 - 26$. Adding 26 to each side of this equation yields $49 = a^2$. Taking the square root of both sides of this equation yields $\pm 7 = a$. Since it's given that $a > 0$, the value of a is 7. It follows that the value of $a + b$ is $7 - 26$, or -19.

Question Difficulty: Hard

Question ID 6a8a7fbd

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 6a8a7fbd

The function f is defined by $f(x) = a\sqrt{x+b}$, where a and b are constants. In the *xy*-plane, the graph of $y = f(x)$ passes through the point $(-24, 0)$, and $f(24) < 0$. Which of the following must be true?

A. $f(0) = 24$

B. $f(0) = -24$

C. $a > b$

D. $a < b$

ID: 6a8a7fbd Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $f(24) < 0$. Substituting 24 for $f(x)$ in the equation $f(x) = a\sqrt{x+b}$ yields $f(24) = a\sqrt{24+b}$. Therefore, $a\sqrt{24+b} < 0$. Since $\sqrt{24+b}$ can't be negative, it follows that $a < 0$. It's also given that the graph of $y = f(x)$ passes through the point $(-24, 0)$. It follows that when $x = -24$, $f(x) = 0$. Substituting -24 for x and 0 for $f(x)$ in the equation $f(x) = a\sqrt{x+b}$ yields $0 = a\sqrt{-24+b}$. By the zero product property, either $a = 0$ or $\sqrt{-24+b} = 0$. Since $a < 0$, it follows that $\sqrt{24+b} = 0$. Squaring both sides of this equation yields $-24 + b = 0$. Adding 24 to both sides of this equation yields $b = 24$. Since $a < 0$ and b is 24, it follows that $a < b$ must be true.

Choice A is incorrect. The value of $f(0)$ is $a\sqrt{b}$, which must be negative.

Choice B is incorrect. The value of $f(0)$ is $a\sqrt{b}$, which could be -24, but doesn't have to be.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 81aa6aa9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 81aa6aa9

When the quadratic function f is graphed in the *xy*-plane, where $y = f(x)$, its vertex is $(-3, 6)$. One of the *x*-intercepts of this graph is $\left(-\frac{17}{4}, 0\right)$. What is the other *x*-intercept of the graph?

A. $\left(-\frac{29}{4}, 0\right)$

B. $\left(-\frac{7}{4}, 0\right)$

C. $\left(\frac{5}{4}, 0\right)$

D. $\left(\frac{17}{4}, 0\right)$

ID: 81aa6aa9 Answer

Correct Answer: B

Rationale

Choice B is correct. Since the line of symmetry for the graph of a quadratic function contains the vertex of the graph, the *x*-coordinate of the vertex of the graph of $y = f(x)$ is the *x*-coordinate of the midpoint of its two *x*-intercepts. The midpoint of two points with *x*-coordinates x_1 and x_2 has *x*-coordinate x_m, where $x_m = \frac{x_1 + x_2}{2}$. It's given that the vertex is $(-3, 6)$ and one of the *x*-intercepts is $\left(-\frac{17}{4}, 0\right)$. Substituting -3 for x_m and $-\frac{17}{4}$ for x_1 in the equation $x_m = \frac{x_1 + x_2}{2}$ yields $-3 = \frac{-\frac{17}{4} + x_2}{2}$. Multiplying each side of this equation by 2 yields $-6 = -\frac{17}{4} + x_2$. Adding $\frac{17}{4}$ to each side of this equation yields $-\frac{7}{4} = x_2$. Therefore, the other *x*-intercept is $\left(-\frac{7}{4}, 0\right)$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID b017359f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: b017359f

A submersible device is used for ocean research. The function $g(x)=-\frac{1}{55}(x+19)(x-35)$ gives the depth below the surface of the ocean, in meters, of the submersible device x minutes after collecting a sample, where $x>0$. How many minutes after collecting the sample did it take for the submersible device to reach the surface of the ocean?

ID: b017359f Answer

Correct Answer: 35

Rationale

The correct answer is 35. It's given that the function $g(x)=-\frac{1}{55}(x+19)(x-35)$ gives the depth below the surface of the ocean, in meters, of the submersible device x minutes after collecting a sample, where $x>0$. It follows that when the submersible device is at the surface of the ocean, the value of $g(x)$ is 0. Substituting 0 for $g(x)$ in the equation $g(x)=-\frac{1}{55}(x+19)(x-35)$ yields $0=-\frac{1}{55}(x+19)(x-35)$. Multiplying both sides of this equation by -55 yields $0=(x+19)(x-35)$. Since a product of two factors is equal to 0 if and only if at least one of the factors is 0, either $x+19=0$ or $x-35=0$. Subtracting 19 from both sides of the equation $x+19=0$ yields $x=-19$. Adding 35 to both sides of the equation $x-35=0$ yields $x=35$. Since $x>0$, 35 minutes after collecting the sample the submersible device reached the surface of the ocean.

Question Difficulty: Hard

Question ID 3e4e3220

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 3e4e3220

x	y
21	-8
23	8
25	-8

The table shows three values of x and their corresponding values of y, where $y = f(x) + 4$ and f is a quadratic function. What is the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane?

ID: 3e4e3220 Answer

Correct Answer: -2112

Rationale

The correct answer is $-2{,}112$. It's given that f is a quadratic function. It follows that f can be defined by an equation of the form $f(x) = a(x-h)^2 + k$, where a, h, and k are constants. It's also given that the table shows three values of x and their corresponding values of y, where $y = f(x) + 4$. Substituting $a(x-h)^2 + k$ for $f(x)$ in this equation yields $y = a(x-h)^2 + k + 4$. This equation represents a quadratic relationship between x and y, where $k + 4$ is either the maximum or the minimum value of y, which occurs when $x = h$. For quadratic relationships between x and y, the maximum or minimum value of y occurs at the value of x halfway between any two values of x that have the same corresponding value of y. The table shows that *x*-values of 21 and 25 correspond to the same *y*-value, -8. Since 23 is halfway between 21 and 25, the maximum or minimum value of y occurs at an *x*-value of 23. The table shows that when $x = 23$, $y = 8$. It follows that $h = 23$ and $k + 4 = 8$. Subtracting 4 from both sides of the equation $k + 4 = 8$ yields $k = 4$. Substituting 23 for h and 4 for k in the equation $y = a(x-h)^2 + k + 4$ yields $y = a(x-23)^2 + 4 + 4$, or $y = a(x-23)^2 + 8$. The value of a can be found by substituting any *x*-value and its corresponding *y*-value for x and y, respectively, in this equation. Substituting 25 for x and -8 for y in this equation yields $-8 = a(25-23)^2 + 8$, or $-8 = a(2)^2 + 8$. Subtracting 8 from both sides of this equation yields $-16 = a(2)^2$, or $-16 = 4a$. Dividing both sides of this equation by 4 yields $-4 = a$. Substituting -4 for a, 23 for h, and 4 for k in the equation $f(x) = a(x-h)^2 + k$ yields $f(x) = -4(x-23)^2 + 4$. The *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane is the point on the graph where $x = 0$. Substituting 0 for x in the equation $f(x) = -4(x-23)^2 + 4$ yields $f(0) = -4(0-23)^2 + 4$, or $f(0) = -4(-23)^2 + 4$. This is equivalent to $f(0) = -2{,}112$, so the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane is $(0, -2{,}112)$. Thus, the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane is $-2{,}112$.

Question Difficulty: Hard

Question ID 16ff7151

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 16ff7151

$$f(x) = (x-2)(x+15)$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

ID: 16ff7151 Answer

Correct Answer: -13/2, -6.5

Rationale

The correct answer is $-\frac{13}{2}$. The value of x for which $f(x)$ reaches its minimum can be found by rewriting the given equation in the form $f(x) = (x-h)^2 + k$, where $f(x)$ reaches its minimum, k, when the value of x is h. The given equation, $f(x) = (x-2)(x+15)$, can be rewritten as $f(x) = x^2 + 13x - 30$. By completing the square, this equation can be rewritten as $f(x) = \left(x^2 + 13x + \left(\frac{13}{2}\right)^2\right) - 30 - \left(\frac{13}{2}\right)^2$, which is equivalent to $f(x) = \left(x + \frac{13}{2}\right)^2 - \frac{289}{4}$, or $f(x) = \left(x - \left(-\frac{13}{2}\right)\right)^2 - \frac{289}{4}$. Therefore, $f(x)$ reaches its minimum when the value of x is $-\frac{13}{2}$. Note that -13/2 and -6.5 are examples of ways to enter a correct answer.

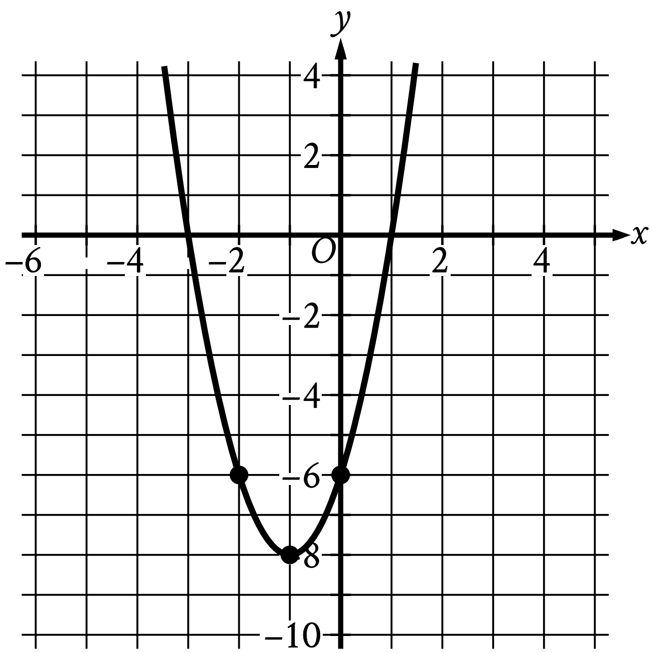
Alternate approach: The graph of $y = f(x)$ in the *xy*-plane is a parabola. The value of x for the vertex of a parabola is the *x*-value of the midpoint between the two *x*-intercepts of the parabola. Since it's given that $f(x) = (x-2)(x+15)$, it follows that the two *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane occur when $x = 2$ and $x = -15$, or at the points $(2, 0)$ and $(-15, 0)$. The midpoint between two points, (x_1, y_1) and (x_2, y_2), is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Therefore, the midpoint between $(2, 0)$ and $(-15, 0)$ is $\left(\frac{2-15}{2}, \frac{0+0}{2}\right)$, or $\left(-\frac{13}{2}, 0\right)$. It follows that $f(x)$ reaches its minimum when the value of x is $-\frac{13}{2}$. Note that -13/2 and -6.5 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID a44eb7d8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: a44eb7d8



The graph of $y = 2x^2 + bx + c$ is shown, where b and c are constants. What is the value of bc?

ID: a44eb7d8 Answer

Correct Answer: -24

Rationale

The correct answer is -24. Since the graph passes through the point $(0, -6)$, it follows that when the value of x is 0, the value of y is -6. Substituting 0 for x and -6 for y in the given equation yields $-6 = 2(0)^2 + b(0) + c$, or $-6 = c$. Therefore, the value of c is -6. Substituting -6 for c in the given equation yields $y = 2x^2 + bx - 6$. Since the graph passes through the point $(-1, -8)$, it follows that when the value of x is -1, the value of y is -8. Substituting -1 for x and -8 for y in the equation $y = 2x^2 + bx - 6$ yields $-8 = 2(-1)^2 + b(-1) - 6$, or $-8 = 2 - b - 6$, which is equivalent to $-8 = -4 - b$. Adding 4 to each side of this equation yields $-4 = -b$. Dividing each side of this equation by -1 yields $4 = b$. Since the value of b is 4 and the value of c is -6, it follows that the value of bc is $(4)(-6)$, or -24.

Alternate approach: The given equation represents a parabola in the *xy*-plane with a vertex at $(-1, -8)$. Therefore, the given equation, $y = 2x^2 + bx + c$, which is written in standard form, can be written in vertex form, $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola and a is the value of the coefficient on the x^2 term when the equation is written in standard form. It follows that $a = 2$. Substituting 2 for a, -1 for h, and -8 for k in this equation yields $y = 2(x - (-1))^2 + (-8)$, or $y = 2(x + 1)^2 - 8$. Squaring the binomial on the right-hand side of this equation yields $y = 2\left(x^2 + 2x + 1\right) - 8$. Multiplying each term inside the parentheses on the right-hand side of this equation by 2 yields

$y = 2x^2 + 4x + 2 - 8$, which is equivalent to $y = 2x^2 + 4x - 6$. From the given equation $y = 2x^2 + bx + c$, it follows that the value of b is 4 and the value of c is -6. Therefore, the value of bc is $(4)(-6)$, or -24.

Question Difficulty: Hard

Question ID 79eb41b9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 79eb41b9

The quadratic function g models the depth, in meters, below the surface of the water of a seal t minutes after the seal entered the water during a dive. The function estimates that the seal reached its maximum depth of 302.4 meters 6 minutes after it entered the water and then reached the surface of the water 12 minutes after it entered the water. Based on the function, what was the estimated depth, to the nearest meter, of the seal 10 minutes after it entered the water?

ID: 79eb41b9 Answer

Correct Answer: 168

Rationale

The correct answer is 168. The quadratic function g gives the estimated depth of the seal, $g(t)$, in meters, t minutes after the seal enters the water. It's given that function g estimates that the seal reached its maximum depth of 302.4 meters 6 minutes after it entered the water. Therefore, function g can be expressed in vertex form as $g(t) = a(t-6)^2 + 302.4$, where a is a constant. Since it's also given that the seal reached the surface of the water after 12 minutes, $g(12) = 0$. Substituting 12 for t and 0 for $g(t)$ in $g(t) = a(t-6)^2 + 302.4$ yields $0 = a(12-6)^2 + 302.4$, or $36a = -302.4$. Dividing both sides of this equation by 36 gives $a = -8.4$. Substituting -8.4 for a in $g(t) = a(t-6)^2 + 302.4$ gives $g(t) = -8.4(t-6)^2 + 302.4$. Substituting 10 for t in $g(t)$ gives $g(10) = -8.4(10-6)^2 + 302.4$, which is equivalent to $g(10) = -8.4(4)^2 + 302.4$, or $g(10) = 168$. Therefore, the estimated depth, to the nearest meter, of the seal 10 minutes after it entered the water was 168 meters.

Question Difficulty: Hard

Question ID f85886ff

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: f85886ff

A machine launches a softball from ground level. The softball reaches a maximum height of 51.84 meters above the ground at 1.8 seconds and hits the ground at 3.6 seconds. Which equation represents the height above ground h, in meters, of the softball t seconds after it is launched?

A. $h = -t^2 + 3.6$

B. $h = -t^2 + 51.84$

C. $h = -16$msup $- 3.6$

D. $h = -16$msup $+ 51.84$

ID: f85886ff Answer

Correct Answer: D

Rationale

Choice D is correct. An equation representing the height above ground h, in meters, of a softball t seconds after it is launched by a machine from ground level can be written in the form $h = -a(t-b)^2 + c$, where a, b, and c are positive constants. In this equation, b represents the time, in seconds, at which the softball reaches its maximum height of c meters above the ground. It's given that this softball reaches a maximum height of 51.84 meters above the ground at 1.8 seconds; therefore, $b = 1.8$ and $c = 51.84$. Substituting 1.8 for b and 51.84 for c in the equation $h = -a(t-b)^2 + c$ yields $h = -a(t-1.8)^2 + 51.84$. It's also given that this softball hits the ground at 3.6 seconds; therefore, $h = 0$ when $t = 3.6$. Substituting 0 for h and 3.6 for t in the equation $h = -a(t-1.8)^2 + 51.84$ yields $0 = -a(3.6-1.8)^2 + 51.84$, which is equivalent to $0 = -a(1.8)^2 + 51.84$, or $0 = -3.24a + 51.84$. Adding $3.24a$ to both sides of this equation yields $3.24a = 51.84$. Dividing both sides of this equation by 3.24 yields $a = 16$. Substituting 16 for a in the equation $h = -a(t-1.8)^2 + 51.84$ yields $h = -16(t-1.8)^2 + 51.84$. Therefore, $h = -16(t-1.8)^2 + 51.84$ represents the height above ground h, in meters, of this softball t seconds after it is launched.

Choice A is incorrect. This equation represents a situation where the maximum height is 3.6 meters above the ground at 0 seconds, not 51.84 meters above the ground at 1.8 seconds.

Choice B is incorrect. This equation represents a situation where the maximum height is 51.84 meters above the ground at 0 seconds, not 1.8 seconds.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID d5b08036

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: d5b08036

$$y = 576^{(2x+2)}$$

The graph of the given equation in the *xy*-plane has a *y*-intercept of (r, s). Which of the following equivalent equations displays the value of s as a constant, a coefficient, or the base?

A. $y =$ msup

B. $y =$ msup

C. $y = \frac{1}{24}$msup

D. $y = \frac{1}{576}$msup

ID: d5b08036 Answer

Correct Answer: A

Rationale

Choice A is correct. The *y*-intercept of a graph in the *xy*-plane is the point where $x = 0$. Substituting 0 for x in the given equation, $y = 576^{(2x+2)}$, yields $y = 576^{(2(0)+2)}$, which is equivalent to $y = 576^2$, or $y = 331{,}776$. Therefore, the graph of the given equation in the *xy*-plane has a *y*-intercept of $(0, 331{,}776)$. It follows that $r = 0$ and $s = 331{,}776$. Thus, the equivalent equation $y = 331{,}776^{(x+1)}$ displays the value of s as the base.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 95954b57

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 95954b57

The function f is defined by $f(x) = a^x + b$, where a and b are constants. In the xy-plane, the graph of $y = f(x)$ has an x-intercept at $(2, 0)$ and a y-intercept at $(0, -323)$. What is the value of b?

ID: 95954b57 Answer

Correct Answer: -324

Rationale

The correct answer is -324. It's given that the function f is defined by $f(x) = a^x + b$, where a and b are constants. It's also given that the graph of $y = f(x)$ has a y-intercept at $(0, -323)$. It follows that $f(0) = -323$. Substituting 0 for x and -323 for $f(x)$ in $f(x) = a^x + b$ yields $-323 = a^0 + b$, or $-323 = 1 + b$. Subtracting 1 from each side of this equation yields $-324 = b$. Therefore, the value of b is -324.

Question Difficulty: Hard

Question ID 1b30fd79

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 1b30fd79

x	$g(x)$
-27	3
-9	0
21	5

The table shows three values of x and their corresponding values of $g(x)$, where $g(x) = \frac{f(x)}{x+3}$ and f is a linear function. What is the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane?

A. $(0, 36)$

B. $(0, 12)$

C. $(0, 4)$

D. $(0, -9)$

ID: 1b30fd79 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the table shows values of x and their corresponding values of $g(x)$, where $g(x) = \frac{f(x)}{x+3}$. It's also given that f is a linear function. It follows that an equation that defines f can be written in the form $f(x) = mx + b$, where m represents the slope and b represents the *y*-coordinate of the *y*-intercept $(0, b)$ of the graph of $y = f(x)$ in the *xy*-plane. The slope of the graph of $y = f(x)$ can be found using two points, (x_1, y_1) and (x_2, y_2), that are on the graph of $y = f(x)$, and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Since the table shows values of x and their corresponding values of $g(x)$, substituting values of x and $g(x)$ in the equation $g(x) = \frac{f(x)}{x+3}$ can be used to define function f. Using the first pair of values from the table, $x = -27$ and $g(x) = 3$, yields $3 = \frac{f(-27)}{-27+3}$, or $3 = \frac{f(-27)}{-24}$. Multiplying each side of this equation by -24 yields $-72 = f(-27)$, so the point $(-27, -72)$ is on the graph of $y = f(x)$. Using the second pair of values from the table, $x = -9$ and $g(x) = 0$, yields $0 = \frac{f(-9)}{-9+3}$, or $0 = \frac{f(-9)}{-6}$. Multiplying each side of this equation by -6 yields $0 = f(-9)$, so the point $(-9, 0)$ is on the graph of $y = f(x)$. Substituting $(-27, -72)$ and $(-9, 0)$ for (x_1, y_1) and (x_2, y_2), respectively, in the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ yields $m = \frac{0-(-72)}{-9-(-27)}$, or $m = 4$. Substituting 4 for m in the equation $f(x) = mx + b$ yields $f(x) = 4x + b$. Since $0 = f(-9)$, substituting -9 for x and 0 for $f(x)$ in the equation $f(x) = 4x + b$ yields $0 = 4(-9) + b$, or $0 = -36 + b$. Adding 36 to both sides of this equation yields $36 = b$. It follows that 36 is the *y*-coordinate of the *y*-intercept $(0, b)$ of the graph of $y = f(x)$. Therefore, the *y*-intercept of the graph of $y = f(x)$ is $(0, 36)$.

Choice B is incorrect. 12 is the *y*-coordinate of the *y*-intercept of the graph of $y = g(x)$.

Choice C is incorrect. 4 is the slope of the graph of $y = f(x)$.

Choice D is incorrect. -9 is the *x*-coordinate of the *x*-intercept of the graph of $y = f(x)$.

Question Difficulty: Hard

Question ID 41ab1df4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 41ab1df4

$$f(x) = 272(2)^x$$

The function f is defined by the given equation. If $h(x) = f(x-4)$, which of the following equations defines function h?

A. $h(x) = 17(2)^x$

B. $h(x) = 68(2)^x$

C. $h(x) = 272(16)^x$

D. $h(x) = 272(8)^x$

ID: 41ab1df4 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $f(x) = 272(2)^x$ and $h(x) = f(x-4)$. Substituting $x-4$ for x in $f(x) = 272(2)^x$ yields $f(x-4) = 272(2)^{x-4}$. Substituting $h(x)$ for $f(x-4)$ in this equation yields $h(x) = 272(2)^{x-4}$. Using the properties of exponents, the function $h(x) = 272(2)^{x-4}$ can be rewritten as $h(x) = \frac{272(2)^x}{2^4}$, which is equivalent to $h(x) = \frac{272(2)^x}{16}$, or $h(x) = 17(2)^x$. Therefore, of the given choices, an equation that defines function h is $h(x) = 17(2)^x$.

Choice B is incorrect. This equation defines function h if $h(x) = f(x-2)$, not $h(x) = f(x-4)$.

Choice C is incorrect. This equation defines function h if $h(x) = f(4x)$, not $h(x) = f(x-4)$.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 7b54c7af

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 7b54c7af

The functions f and g are defined by the given equations, where $x \geq 0$. Which of the following equations displays, as a constant or coefficient, the maximum value of the function it defines, where $x \geq 0$?

I. $f(x) = 33(0.4)^{x+3}$
II. $g(x) = 33(0.16)(0.4)^{x-2}$

A. I only

B. II only

C. I and II

D. Neither I nor II

ID: 7b54c7af Answer

Correct Answer: B

Rationale

Choice B is correct. Functions f and g are both exponential functions with a base of 0.40. Since 0.40 is less than 1, functions f and g are both decreasing exponential functions. This means that $f(x)$ and $g(x)$ decrease as x increases. Since $f(x)$ and $g(x)$ decrease as x increases, the maximum value of each function occurs at the least value of x for which the function is defined. It's given that functions f and g are defined for $x \geq 0$. Therefore, the maximum value of each function occurs at $x = 0$. Substituting 0 for x in the equation defining f yields $f(0) = 33(0.4)^{0+3}$, which is equivalent to $f(0) = 33(0.4)^3$, or $f(0) = 2.112$. Therefore, the maximum value of f is 2.112. Since the equation $f(x) = 33(0.4)^{x+3}$ doesn't display the value 2.112, the equation defining f doesn't display the maximum value of f. Substituting 0 for x in the equation defining g yields $g(0) = 33(0.16)(0.4)^{0-2}$, which can be rewritten as $g(0) = 33(0.16)\left(\frac{1}{0.4^2}\right)$, or $g(0) = 33(0.16)\left(\frac{1}{0.16}\right)$, which is equivalent to $g(0) = 33$. Therefore, the maximum value of g is 33. Since the equation $g(x) = 33(0.16)(0.4)^{x-2}$ displays the value 33, the equation defining g displays the maximum value of g. Thus, only equation II displays, as a constant or coefficient, the maximum value of the function it defines.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID f9e32510

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: f9e32510

The function $f(x) = \frac{1}{9}(x-7)^2 + 3$ gives a metal ball's height above the ground $f(x)$, in inches, x seconds after it started moving on a track, where $0 \leq x \leq 10$. Which of the following is the best interpretation of the vertex of the graph of $y = f(x)$ in the *xy*-plane?

A. The metal ball's minimum height was 3 inches above the ground.

B. The metal ball's minimum height was 7 inches above the ground.

C. The metal ball's height was 3 inches above the ground when it started moving.

D. The metal ball's height was 7 inches above the ground when it started moving.

ID: f9e32510 Answer

Correct Answer: A

Rationale

Choice A is correct. The graph of a quadratic equation in the form $y = a(x-h)^2 + k$, where a, h, and k are positive constants, is a parabola that opens upward with vertex (h, k). The given function $f(x) = \frac{1}{9}(x-7)^2 + 3$ is in the form $y = a(x-h)^2 + k$, where $y = f(x)$, $a = \frac{1}{9}$, $h = 7$, and $k = 3$. Therefore, the graph of $y = f(x)$ is a parabola that opens upward with vertex $(7, 3)$. Since the parabola opens upward, the vertex is the lowest point on the graph. It follows that the *y*-coordinate of the vertex of the graph of $y = f(x)$ is the minimum value of $f(x)$. Therefore, the minimum value of $f(x)$ is 3. It's given that $f(x) = \frac{1}{9}(x-7)^2 + 3$ represents the metal ball's height above the ground, in inches, x seconds after it started moving on a track. Therefore, the best interpretation of the vertex of the graph of $y = f(x)$ is that the metal ball's minimum height was 3 inches above the ground.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 23923e5b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 23923e5b

A quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. The model indicates the object has an initial height of 10 feet above the ground and reaches its maximum height of $1{,}034$ feet above the ground 8 seconds after being launched. Based on the model, what is the height, in feet, of the object above the ground 10 seconds after being launched?

A. 234

B. 778

C. 970

D. $1{,}014$

ID: 23923e5b Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that a quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. This quadratic function can be defined by an equation of the form $f(x) = a(x-h)^2 + k$, where $f(x)$ is the height of the object x seconds after it was launched, and a, h, and k are constants such that the function reaches its maximum value, k, when $x = h$. Since the model indicates the object reaches its maximum height of $1{,}034$ feet above the ground 8 seconds after being launched, $f(x)$ reaches its maximum value, $1{,}034$, when $x = 8$. Therefore, $k = 1{,}034$ and $h = 8$. Substituting 8 for h and $1{,}034$ for k in the function $f(x) = a(x-h)^2 + k$ yields $f(x) = a(x-8)^2 + 1{,}034$. Since the model indicates the object has an initial height of 10 feet above the ground, the value of $f(x)$ is 10 when $x = 0$. Substituting 0 for x and 10 for $f(x)$ in the equation $f(x) = a(x-8)^2 + 1{,}034$ yields $10 = a(0-8)^2 + 1{,}034$, or $10 = 64a + 1{,}034$. Subtracting $1{,}034$ from both sides of this equation yields $64a = -1{,}024$. Dividing both sides of this equation by 64 yields $a = -16$. Therefore, the model can be represented by the equation $f(x) = -16(x-8)^2 + 1{,}034$. Substituting 10 for x in this equation yields $f(10) = -16(10-8)^2 + 1{,}034$, or $f(10) = 970$. Therefore, based on the model, 10 seconds after being launched, the height of the object above the ground is 970 feet.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 6ef1d0a7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 6ef1d0a7

$$f(x) = (x - 44)(x - 46)$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

A. 46

B. 45

C. 44

D. -1

ID: 6ef1d0a7 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that $f(x) = (x - 44)(x - 46)$, which can be rewritten as $f(x) = x^2 - 90x + 2{,}024$. Since the coefficient of the x^2-term is positive, the graph of $y = f(x)$ in the *xy*-plane opens upward and reaches its minimum value at its vertex. For an equation in the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants, the *x*-coordinate of the vertex is $-\frac{b}{2a}$. For the equation $f(x) = x^2 - 90x + 2{,}024$, $a = 1$, $b = -90$, and $c = 2{,}024$. It follows that the *x*-coordinate of the vertex is $-\frac{(-90)}{2(1)}$, or 45. Therefore, $f(x)$ reaches its minimum when the value of x is 45.

Choice A is incorrect. This is one of the *x*-coordinates of the *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane.

Choice C is incorrect. This is one of the *x*-coordinates of the *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane.

Choice D is incorrect. This is the *y*-coordinate of the vertex of the graph of $y = f(x)$ in the *xy*-plane.

Question Difficulty: Hard

Question ID 1ba110f2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 1ba110f2

$$P(t) = 260(1.04)^{\left(\frac{6}{4}\right)t}$$

The function P models the population, in thousands, of a certain city t years after 2003. According to the model, the population is predicted to increase by 4% every n months. What is the value of n?

A. 8

B. 12

C. 18

D. 72

ID: 1ba110f2 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the function P models the population, in thousands, of a certain city t years after 2003. The value of the base of the given exponential function, 1.04, corresponds to an increase of 4% for every increase of 1 in the exponent, $\left(\frac{6}{4}\right)t$. If the exponent is equal to 0, then $\left(\frac{6}{4}\right)t = 0$. Multiplying both sides of this equation by $\left(\frac{4}{6}\right)$ yields $t = 0$. If the exponent is equal to 1, then $\left(\frac{6}{4}\right)t = 1$. Multiplying both sides of this equation by $\left(\frac{4}{6}\right)$ yields $t = \frac{4}{6}$, or $t = \frac{2}{3}$. Therefore, the population is predicted to increase by 4% every $\frac{2}{3}$ of a year. It's given that the population is predicted to increase by 4% every n months. Since there are 12 months in a year, $\frac{2}{3}$ of a year is equivalent to $\left(\frac{2}{3}\right)(12)$, or 8, months. Therefore, the value of n is 8.

Choice B is incorrect. This is the number of months in which the population is predicted to increase by 4% according to the model $P(t) = 260(1.04)^t$, not $P(t) = 260(1.04)^{\left(\frac{6}{4}\right)t}$.

Choice C is incorrect. This is the number of months in which the population is predicted to increase by 4% according to the model $P(t) = 260(1.04)^{\left(\frac{4}{6}\right)t}$, not $P(t) = 260(1.04)^{\left(\frac{6}{4}\right)t}$.

Choice D is incorrect. This is the number of months in which the population is predicted to increase by 4% according to the model $P(t) = 260(1.04)^{\left(\frac{1}{6}\right)t}$, not $P(t) = 260(1.04)^{\left(\frac{6}{4}\right)t}$.

Question Difficulty: Hard

Question ID aefae524

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: aefae524

$$P(t) = 290(1.04)^{\left(\frac{4}{6}\right)t}$$

The function P models the population, in thousands, of a certain city t years after 2005. According to the model, the population is predicted to increase by $n\%$ every 18 months. What is the value of n?

A. 0.38

B. 1.04

C. 4

D. 6

ID: aefae524 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the function P models the population of the city t years after 2005. Since there are 12 months in a year, 18 months is equivalent to $\frac{18}{12}$ years. Therefore, the expression $\frac{18}{12}x$ can represent the number of years in x 18-month periods. Substituting $\frac{18}{12}x$ for t in the given equation yields $P\left(\frac{18}{12}x\right) = 290(1.04)^{\left(\frac{4}{6}\right)\left(\frac{18}{12}x\right)}$, which is equivalent to $P\left(\frac{18}{12}x\right) = 290(1.04)^x$. Therefore, for each 18-month period, the predicted population of the city is 1.04 times, or 104% of, the previous population. This means that the population is predicted to increase by 4% every 18 months.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. Each year, the predicted population of the city is 1.04 times the previous year's predicted population, which is not the same as an increase of 1.04%.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 49de5e98

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 49de5e98

$$f(x) = 4x^2 - 50x + 126$$

The given equation defines the function f. For what value of x does $f(x)$ reach its minimum?

ID: 49de5e98 Answer

Correct Answer: 25/4, 6.25

Rationale

The correct answer is $\frac{25}{4}$. The given equation can be rewritten in the form $f(x) = a(x-h)^2 + k$, where a, h, and k are constants. When $a > 0$, h is the value of x for which $f(x)$ reaches its minimum. The given equation can be rewritten as $f(x) = 4\left(x^2 - \frac{50}{4}x\right) + 126$, which is equivalent to $f(x) = 4\left(x^2 - \frac{50}{4}x + \left(\frac{50}{8}\right)^2 - \left(\frac{50}{8}\right)^2\right) + 126$. This equation can be rewritten as $f(x) = 4\left(\left(x - \frac{50}{8}\right)^2 - \left(\frac{50}{8}\right)^2\right) + 126$, or $f(x) = 4\left(x - \frac{50}{8}\right)^2 - 4\left(\frac{50}{8}\right)^2 + 126$, which is equivalent to $f(x) = 4\left(x - \frac{25}{4}\right)^2 - \frac{121}{4}$. Therefore, $h = \frac{25}{4}$, so the value of x for which $f(x)$ reaches its minimum is $\frac{25}{4}$. Note that 25/4 and 6.25 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 3de3402c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 3de3402c

The functions f and g are defined by the given equations, where $x \geq 0$. Which of the following equations displays, as a constant or coefficient, the maximum value of the function it defines, where $x \geq 0$?

I. $f(x) = 18(1.25)^x + 41$

II. $g(x) = 9(0.73)^x$

A. I only

B. II only

C. I and II

D. Neither I nor II

ID: 3de3402c Answer

Correct Answer: B

Rationale

Choice B is correct. For the function f, since the base of the exponent, 1.25, is greater than 1, the value of $(1.25)^x$ increases as x increases. Therefore, the value of $18(1.25)^x$ and the value of $18(1.25)^x + 41$ also increase as x increases. Since f is therefore an increasing function where $x \geq 0$, the function f has no maximum value. For the function g, since the base of the exponent, 0.73, is less than 1, the value of $(0.73)^x$ decreases as x increases. Therefore, the value of $9(0.73)^x$ also decreases as x increases. It follows that the maximum value of $g(x)$ for $x \geq 0$ occurs when $x = 0$. Substituting 0 for x in the function g yields $g(0) = 9(0.73)^0$, which is equivalent to $g(0) = 9(1)$, or $g(0) = 9$. Therefore, the maximum value of $g(x)$ for $x \geq 0$ is 9, which appears as a coefficient in equation II. So, of the two equations given, only II displays, as a constant or coefficient, the maximum value of the function it defines, where $x \geq 0$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 5e98384e

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 5e98384e

The function f is defined by $f(x) = (x - 6)(x - 2)(x + 6)$. In the *xy*-plane, the graph of $y = g(x)$ is the result of translating the graph of $y = f(x)$ up 4 units. What is the value of $g(0)$?

ID: 5e98384e Answer

Correct Answer: 76

Rationale

The correct answer is 76. It's given that the graph of $y = g(x)$ is the result of translating the graph of $y = f(x)$ up 4 units in the *xy*-plane. It follows that the graph of $y = g(x)$ is the same as the graph of $y = f(x) + 4$. Substituting $g(x)$ for y in the equation $y = f(x) + 4$ yields $g(x) = f(x) + 4$. It's given that $f(x) = (x - 6)(x - 2)(x + 6)$. Substituting $(x - 6)(x - 2)(x + 6)$ for $f(x)$ in the equation $g(x) = f(x) + 4$ yields $g(x) = (x - 6)(x - 2)(x + 6) + 4$. Substituting 0 for x in this equation yields $g(0) = (0 - 6)(0 - 2)(0 + 6) + 4$, or $g(0) = 76$. Thus, the value of $g(0)$ is 76.

Question Difficulty: Hard

Question ID c56a9f57

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: c56a9f57

The first term of a sequence is 9. Each term after the first is 4 times the preceding term. If w represents the nth term of the sequence, which equation gives w in terms of n?

A. $w = 4(9^n)$

B. $w = 4(9^{n-1})$

C. $w = 9(4^n)$

D. $w = 9(4^{n-1})$

ID: c56a9f57 Answer

Correct Answer: D

Rationale

Choice D is correct. Since w represents the nth term of the sequence and 9 is the first term of the sequence, the value of w is 9 when the value of n is 1. Since each term after the first is 4 times the preceding term, the value of w is $9(4)$ when the value of n is 2. Therefore, the value of w is $9(4)(4)$, or $9(4)^2$, when the value of n is 3. More generally, the value of w is $9\left(4^{n-1}\right)$ for a given value of n. Therefore, the equation $w = 9\left(4^{n-1}\right)$ gives w in terms of n.

Choice A is incorrect. This equation describes a sequence for which the first term is 36, rather than 9, and each term after the first is 9, rather than 4, times the preceding term.

Choice B is incorrect. This equation describes a sequence for which the first term is 4, rather than 9, and each term after the first is 9, rather than 4, times the preceding term.

Choice C is incorrect. This equation describes a sequence for which the first term is 36, rather than 9.

Question Difficulty: Hard

Question ID 2f88a547

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 2f88a547

At the time that an article was first featured on the home page of a news website, there were 40 comments on the article. An exponential model estimates that at the end of each hour after the article was first featured on the home page, the number of comments on the article had increased by 190% of the number of comments on the article at the end of the previous hour. Which of the following equations best represents this model, where C is the estimated number of comments on the article t hours after the article was first featured on the home page and $t \leq 4$?

A. $C = 40$msup

B. $C = 40$msup

C. $C = 40$msup

D. $C = 40$msup

ID: 2f88a547 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that an exponential model estimates that the number of comments on an article increased by a fixed percentage at the end of each hour. Therefore, the model can be represented by an exponential equation of the form $C = Ka^t$, where C is the estimated number of comments on the article t hours after the article was first featured on the home page and K and a are constants. It's also given that when the article was first featured on the home page of the news website, there were 40 comments on the article. This means that when $t = 0$, $C = 40$. Substituting 0 for t and 40 for C in the equation $C = Ka^t$ yields $40 = Ka^0$, or $40 = K$. It's also given that the number of comments on the article at the end of an hour had increased by 190% of the number of comments on the article at the end of the previous hour. Multiplying the percent increase by the number of comments on the article at the end of the previous hour yields the number of estimated additional comments the article has on its home page: $(40)\left(\frac{190}{100}\right)$, or 76 comments. Thus, the estimated number of comments for the following hour is the sum of the comments from the end of the previous hour and the number of additional comments, which is $40 + 76$, or 116. This means that when $t = 1$, $C = 116$. Substituting 1 for t, 116 for C, and 40 for K in the equation $C = Ka^t$ yields $116 = 40a^1$, or $116 = 40a$. Dividing both sides of this equation by 40 yields $2.9 = a$. Substituting 40 for K and 2.9 for a in the equation $C = Ka^t$ yields $C = 40(2.9)^t$. Thus, the equation that best represents this model is $C = 40(2.9)^t$.

Choice A is incorrect. This model represents a situation where the number of comments at the end of each hour increased by 19% of the number of comments at the end of the previous hour, rather than 190%.

Choice B is incorrect. This model represents a situation where the number of comments at the end of each hour increased by 90% of the number of comments at the end of the previous hour, rather than 190%.

Choice C is incorrect. This model represents a situation where the number of comments at the end of each hour was 19 times the number of comments at the end of the previous hour, rather than increasing by 190% of the number of comments at the end of the previous hour.

Question Difficulty: Hard

Question ID d93eac68

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: d93eac68

Function f is defined by $f(x) = -a^x + b$, where a and b are constants. In the *xy*-plane, the graph of $y = f(x) - 15$ has a *y*-intercept at $\left(0, -\frac{99}{7}\right)$. The product of a and b is $\frac{65}{7}$. What is the value of a?

ID: d93eac68 Answer

Correct Answer: 5

Rationale

The correct answer is 5. It's given that $f(x) = -a^x + b$. Substituting $-a^x + b$ for $f(x)$ in the equation $y = f(x) - 15$ yields $y = -a^x + b - 15$. It's given that the *y*-intercept of the graph of $y = f(x) - 15$ is $\left(0, -\frac{99}{7}\right)$. Substituting 0 for x and $-\frac{99}{7}$ for y in the equation $y = -a^x + b - 15$ yields $-\frac{99}{7} = -a^0 + b - 15$, which is equivalent to $-\frac{99}{7} = -1 + b - 15$, or $-\frac{99}{7} = b - 16$. Adding 16 to both sides of this equation yields $\frac{13}{7} = b$. It's given that the product of a and b is $\frac{65}{7}$, or $ab = \frac{65}{7}$. Substituting $\frac{13}{7}$ for b in this equation yields $(a)\left(\frac{13}{7}\right) = \frac{65}{7}$. Dividing both sides of this equation by $\frac{13}{7}$ yields $a = 5$.

Question Difficulty: Hard

Question ID ebf50998

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: ebf50998

Function f is defined by $f(x) = (x+6)(x+5)(x+1)$. Function g is defined by $g(x) = f(x-1)$. The graph of $y = g(x)$ in the *xy*-plane has *x*-intercepts at $(a, 0)$, $(b, 0)$, and $(c, 0)$, where a, b, and c are distinct constants. What is the value of $a + b + c$?

A. -15

B. -9

C. 11

D. 15

ID: ebf50998 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that $g(x) = f(x-1)$. Since $f(x) = (x+6)(x+5)(x+1)$, it follows that $f(x-1) = (x-1+6)(x-1+5)(x-1+1)$. Combining like terms yields $f(x-1) = (x+5)(x+4)(x)$. Therefore, $g(x) = x(x+5)(x+4)$. The *x*-intercepts of a graph in the *xy*-plane are the points where $y = 0$. The *x*-coordinates of the *x*-intercepts of the graph of $y = g(x)$ in the *xy*-plane can be found by solving the equation $0 = x(x+5)(x+4)$. Applying the zero product property to this equation yields three equations: $x = 0$, $x + 5 = 0$, and $x + 4 = 0$. Solving each of these equations for x yields $x = 0$, $x = -5$, and $x = -4$, respectively. Therefore, the *x*-intercepts of the graph of $y = g(x)$ are $(0, 0)$, $(-5, 0)$, and $(-4, 0)$. It follows that the values of a, b, and c are 0, -5, and -4. Thus, the value of $a + b + c$ is $0 + (-5) + (-4)$, which is equal to -9.

Choice A is incorrect. This is the value of $a + b + c$ if $g(x) = f(x+1)$.

Choice C is incorrect. This is the value of $a + b + c - 1$ if $g(x) = (x-6)(x-5)(x-1)$.

Choice D is incorrect. This is the value of $a + b + c$ if $f(x) = (x-6)(x-5)(x-1)$.

Question Difficulty: Hard

Question ID 7189ece4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 7189ece4

For the exponential function f, the value of $f(1)$ is k, where k is a constant. Which of the following equivalent forms of the function f shows the value of k as the coefficient or the base?

A. $f(x) = 50(2)^{x+1}$

B. $f(x) = 80(2)^{x}$

C. $f(x) = 128(2)^{x-1}$

D. $f(x) = 205(2)^{x-2}$

ID: 7189ece4 Answer

Correct Answer: C

Rationale

Choice C is correct. For the form of the function in choice C, $f(x) = 128(1.6)^{x-1}$, the value of $f(1)$ can be found as $128(1.6)^{1-1}$, which is equivalent to $128(1.6)^{0}$, or 128. Therefore, $k = 128$, which is shown in $f(x) = 128(1.6)^{x-1}$ as the coefficient.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID ca44c7ce

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: ca44c7ce

$$f(x) = |59 - 2x|$$

The function f is defined by the given equation. For which of the following values of k does $f(k) = 3k$?

A. $\frac{59}{5}$

B. $\frac{59}{2}$

C. $\frac{177}{5}$

D. 59

ID: ca44c7ce Answer

Correct Answer: A

Rationale

Choice A is correct. The value of k for which $f(k) = 3k$ can be found by substituting k for x and $3k$ for $f(x)$ in the given equation, $f(x) = |59 - 2x|$, which yields $3k = |59 - 2k|$. For this equation to be true, either $-3k = 59 - 2k$ or $3k = 59 - 2k$. Adding $2k$ to both sides of the equation $-3k = 59 - 2k$ yields $-k = 59$. Dividing both sides of this equation by -1 yields $k = -59$. To check whether -59 is the value of k, substituting -59 for k in the equation $3k = |59 - 2k|$ yields $3(-59) = |59 - 2(-59)|$, which is equivalent to $-177 = |177|$, or $-177 = 177$, which isn't a true statement. Therefore, -59 isn't the value of k. Adding $2k$ to both sides of the equation $3k = 59 - 2k$ yields $5k = 59$. Dividing both sides of this equation by 5 yields $k = \frac{59}{5}$. To check whether $\frac{59}{5}$ is the value of k, substituting $\frac{59}{5}$ for k in the equation $3k = |59 - 2k|$ yields $3\left(\frac{59}{5}\right) = \left|59 - 2\left(\frac{59}{5}\right)\right|$, which is equivalent to $\frac{177}{5} = \left|\frac{177}{5}\right|$, or $\frac{177}{5} = \frac{177}{5}$, which is a true statement. Therefore, the value of k for which $f(k) = 3k$ is $\frac{59}{5}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 5b955063

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 5b955063

A rectangle has an area of 155 square inches. The length of the rectangle is 4 inches less than 7 times the width of the rectangle. What is the width of the rectangle, in inches?

ID: 5b955063 Answer

Correct Answer: 5

Rationale

The correct answer is 5. Let x represent the width, in inches, of the rectangle. It's given that the length of the rectangle is 4 inches less than 7 times its width, or $7x - 4$ inches. The area of a rectangle is equal to its width multiplied by its length. Multiplying the width, x inches, by the length, $7x - 4$ inches, yields $x(7x - 4)$ square inches. It's given that the rectangle has an area of 155 square inches, so it follows that $x(7x - 4) = 155$, or $7x^2 - 4x = 155$. Subtracting 155 from both sides of this equation yields $7x^2 - 4x - 155 = 0$. Factoring the left-hand side of this equation yields $(7x + 31)(x - 5) = 0$. Applying the zero product property to this equation yields two solutions: $x = -\frac{31}{7}$ and $x = 5$. Since x is the rectangle's width, in inches, which must be positive, the value of x is 5. Therefore, the width of the rectangle, in inches, is 5.

Question Difficulty: Hard

Question ID 99b8a5c8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 99b8a5c8

$$f(x) = ax^2 + 4x + c$$

In the given quadratic function, a and c are constants. The graph of $y = f(x)$ in the *xy*-plane is a parabola that opens upward and has a vertex at the point (h, k), where h and k are constants. If $k < 0$ and $f(-9) = f(3)$, which of the following must be true?

I. $c < 0$

II. $a \geq 1$

A. I only

B. II only

C. I and II

D. Neither I nor II

ID: 99b8a5c8 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the graph of $y = f(x)$ in the *xy*-plane is a parabola with vertex (h, k). If $f(-9) = f(3)$, then for the graph of $y = f(x)$, the point with an *x*-coordinate of -9 and the point with an *x*-coordinate of 3 have the same *y*-coordinate. In the *xy*-plane, a parabola is a symmetric graph such that when two points have the same *y*-coordinate, these points are equidistant from the vertex, and the *x*-coordinate of the vertex is halfway between the *x*-coordinates of these two points. Therefore, for the graph of $y = f(x)$, the points with *x*-coordinates -9 and 3 are equidistant from the vertex, (h, k), and h is halfway between -9 and 3. The value that is halfway between -9 and 3 is $\frac{-9+3}{2}$, or -3. Therefore, $h = -3$. The equation defining f can also be written in vertex form, $f(x) = a(x - h)^2 + k$. Substituting -3 for h in this equation yields $f(x) = a(x - (-3))^2 + k$, or $f(x) = a(x + 3)^2 + k$. This equation is equivalent to $f(x) = a\left(x^2 + 6x + 9\right) + k$, or $f(x) = ax^2 + 6ax + 9a + k$. Since $f(x) = ax^2 + 4x + c$, it follows that $6a = 4$ and $9a + k = c$. Dividing both sides of the equation $6a = 4$ by 6 yields $a = \frac{4}{6}$, or $a = \frac{2}{3}$. Since $\frac{2}{3} < 1$, it's not true that $a \geq 1$. Therefore, statement II isn't true. Substituting $\frac{2}{3}$ for a in the equation $9a + k = c$ yields $9\left(\frac{2}{3}\right) + k = c$, or $6 + k = c$. Subtracting 6 from both sides of this equation yields $k = c - 6$. If $k < 0$, then $c - 6 < 0$, or $c < 6$. Since c could be any value less than 6, it's not necessarily true that $c < 0$. Therefore, statement I isn't necessarily true. Thus, neither I nor II must be true.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 98f7ab7a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 98f7ab7a

Function f is defined by $f(x) = -a^x + b$, where a and b are constants. In the *xy*-plane, the graph of $y = f(x) - 12$ has a *y*-intercept at $\left(0, -\frac{75}{7}\right)$. The product of a and b is $\frac{320}{7}$. What is the value of a?

ID: 98f7ab7a Answer

Correct Answer: 20

Rationale

The correct answer is 20. It's given that $f(x) = -a^x + b$. Substituting $-a^x + b$ for $f(x)$ in the equation $y = f(x) - 12$ yields $y = -a^x + b - 12$. It's given that the *y*-intercept of the graph of $y = f(x) - 12$ is $\left(0, -\frac{75}{7}\right)$. Substituting 0 for x and $-\frac{75}{7}$ for y in the equation $y = -a^x + b - 12$ yields $-\frac{75}{7} = -a^0 + b - 12$, which is equivalent to $-\frac{75}{7} = -1 + b - 12$, or $-\frac{75}{7} = b - 13$. Adding 13 to both sides of this equation yields $\frac{16}{7} = b$. It's given that the product of a and b is $\frac{320}{7}$, or $ab = \frac{320}{7}$. Substituting $\frac{16}{7}$ for b in this equation yields $(a)\left(\frac{16}{7}\right) = \frac{320}{7}$. Dividing both sides of this equation by $\frac{16}{7}$ yields $a = 20$.

Question Difficulty: Hard

Question ID 06fac60b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 06fac60b

$$f(x) = 5{,}470(0.64)^{\frac{x}{12}}$$

The function f gives the value, in dollars, of a certain piece of equipment after x months of use. If the value of the equipment decreases each year by $p\%$ of its value the preceding year, what is the value of p?

A. 4

B. 5

C. 36

D. 64

ID: 06fac60b Answer

Correct Answer: C

Rationale

Choice C is correct. For a function of the form $f(x) = a(r)^{\frac{x}{k}}$, where a, r, and k are constants and $r < 1$, the value of $f(x)$ decreases by $100(1 - r)\%$ for every increase of x by k. In the given function, $a = 5{,}470$, $r = 0.64$, and $k = 12$. Therefore, for the given function, the value of $f(x)$ decreases by $100(1 - 0.64)\%$, or 36%, for every increase of x by 12. Since $f(x)$ represents the value, in dollars, of the equipment after x months of use, it follows that the value of the equipment decreases every 12 months by 36% of its value the preceding 12 months. Since there are 12 months in a year, the value of the equipment decreases each year by 36% of its value the preceding year. Thus, the value of p is 36.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID da9efa2f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: da9efa2f

$$f(x) = (x - 14)(x + 19)$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

A. -266

B. -19

C. $-\frac{33}{2}$

D. $-\frac{5}{2}$

ID: da9efa2f Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $f(x) = (x - 14)(x + 19)$, which can be rewritten as $f(x) = x^2 + 5x - 266$. Since the coefficient of the x^2-term is positive, the graph of $y = f(x)$ in the *xy*-plane opens upward and reaches its minimum value at its vertex. The *x*-coordinate of the vertex is the value of x such that $f(x)$ reaches its minimum. For an equation in the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants, the *x*-coordinate of the vertex is $-\frac{b}{2a}$. For the equation $f(x) = x^2 + 5x - 266$, $a = 1$, $b = 5$, and $c = -266$. It follows that the *x*-coordinate of the vertex is $-\frac{5}{2(1)}$, or $-\frac{5}{2}$. Therefore, $f(x)$ reaches its minimum when the value of x is $-\frac{5}{2}$.

Alternate approach: The value of x for the vertex of a parabola is the *x*-value of the midpoint between the two *x*-intercepts of the parabola. Since it's given that $f(x) = (x - 14)(x + 19)$, it follows that the two *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane occur when $x = 14$ and $x = -19$, or at the points $(14, 0)$ and $(-19, 0)$. The midpoint between two points, (x_1, y_1) and (x_2, y_2), is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Therefore, the midpoint between $(14, 0)$ and $(-19, 0)$ is $\left(\frac{14+(-19)}{2}, \frac{0+0}{2}\right)$, or $\left(-\frac{5}{2}, 0\right)$. It follows that $f(x)$ reaches its minimum when the value of x is $-\frac{5}{2}$.

Choice A is incorrect. This is the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane.

Choice B is incorrect. This is one of the *x*-coordinates of the *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 033e2be3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 033e2be3

The functions f and g are defined by the given equations.

$$f(x) = 3 + \left| -2x - x^2 \right|$$
$$g(w) = \left| \frac{-w}{w-1} \right| - w + 5$$

If $f(-4) = c$, where c is a constant, what is the value of $g(c)$?

ID: 033e2be3 Answer

Correct Answer: -4.9, -49/10

Rationale

The correct answer is -4.9. The value of $f(-4)$ is the value of $f(x)$ when $x = -4$. Substituting -4 for x in the equation $f(x) = 3 + \left|-2x - x^2\right|$ yields $f(-4) = 3 + \left|-2(-4) - (-4)^2\right|$, or $f(-4) = 3 + |-8|$, which is equivalent to $f(-4) = 3 + 8$, or $f(-4) = 11$. Since it's given that $f(-4) = c$, it follows that $c = 11$ and the value of $g(c)$ is the value of $g(11)$. Substituting 11 for w in the equation $g(w) = \left|\frac{-w}{w-1}\right| - w + 5$ yields $g(11) = \left|\frac{-11}{11-1}\right| - 11 + 5$, or $g(11) = |-1.1| - 6$, which is equivalent to $g(11) = 1.1 - 6$, or $g(11) = -4.9$. Note that -4.9 and -49/10 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID b7f055bc

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: b7f055bc

The function h is defined by $h\left(x\right)=a^x+b$, where a and b are positive constants. The graph of $y=h\left(x\right)$ in the xy-plane passes through the points $(0, 10)$ and $\left(-2, \frac{325}{36}\right)$. What is the value of ab?

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 54

D. 60

ID: b7f055bc Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the function h is defined by $h(x)=a^x+b$ and that the graph of $y=h(x)$ in the *xy*-plane passes through the points $(0, 10)$ and $\left(-2, \frac{325}{36}\right)$. Substituting 0 for x and 10 for $h(x)$ in the equation $h(x)=a^x+b$ yields $10=a^0+b$, or $10=1+b$. Subtracting 1 from both sides of this equation yields $9=b$. Substituting -2 for x and $\frac{325}{36}$ for $h(x)$ in the equation $h(x)=a^x+9$ yields $\frac{325}{36}=a^{-2}+9$. Subtracting 9 from both sides of this equation yields $\frac{1}{36}=a^{-2}$, which can be rewritten as $a^2=36$. Taking the square root of both sides of this equation yields $a=6$ and $a=-6$, but because it's given that a is a positive constant, a must equal 6. Because the value of a is 6 and the value of b is 9, the value of ab is $(6)(9)$, or 54.

Choice A is incorrect and may result from finding the value of $a^{-2}b$ rather than the value of ab.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from correctly finding the value of a as 6, but multiplying it by the *y*-value in the first ordered pair rather than by the value of b.

Question Difficulty: Hard

Question ID d02e610e

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: d02e610e

The function f is defined by $f(x) = a\left(2.2^x + 2.2^b\right)$, where a and b are integer constants and $0 < a < b$. The functions g and h are equivalent to function f, where k and m are constants. Which of the following equations displays the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane as a constant or coefficient?

I. $g(x) = a(2.2^x + k)$
II. $h(x) = a(2.2)^x + m$

A. I only

B. II only

C. I and II

D. Neither I nor II

ID: d02e610e Answer

Correct Answer: D

Rationale

Choice D is correct. A *y*-intercept of a graph in the *xy*-plane is a point where the graph intersects the *y*-axis, or a point where $x = 0$. Substituting 0 for x in the equation defining function f yields $f(0) = a\left(2.2^0 + 2.2^b\right)$, or $f(0) = a\left(1 + 2.2^b\right)$. So, the *y*-coordinate of the *y*-intercept of the graph is $a\left(1 + 2.2^b\right)$, or equivalently, $a + a(2.2)^b$. It's given that function g is equivalent to function f, where $0 < a < b$. It follows that $k = 2.2^b$. Since $a(2.2)^b$ can't be equal to 0, the coefficient a can't be equal to $a + a(2.2)^b$. Since $0 < a$, the constant k, which is equal to 2.2^b, can't be equal to $a + a(2.2)^b$. Therefore, function g doesn't display the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane as a constant or coefficient. It's also given that function h is equivalent to function f, where $0 < a < b$. The equation defining f can be rewritten as $f(x) = a(2.2)^x + a(2.2)^b$. It follows that $m = a(2.2)^b$. Since $a(2.2)^b$ can't be equal to 0, the coefficient a can't be equal to $a + a(2.2)^b$. Since $0 < a$, the constant m, which is equal to $a(2.2)^b$, can't be equal to $a + a(2.2)^b$. Therefore, function h doesn't display the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane as a constant or coefficient. Thus, neither function g nor function h displays the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane as a constant or coefficient.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID aea3b524

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: aea3b524

Square P has a side length of x inches. Square Q has a perimeter that is 176 inches greater than the perimeter of square P. The function f gives the area of square Q, in square inches. Which of the following defines f?

A. $f(x) = (x + 44)^2$

B. $f(x) = (x + 176)^2$

C. $f(x) = (176x + 44)^2$

D. $f(x) = (176x + 176)^2$

ID: aea3b524 Answer

Correct Answer: A

Rationale

Choice A is correct. Let x represent the side length, in inches, of square P. It follows that the perimeter of square P is $4x$ inches. It's given that square Q has a perimeter that is 176 inches greater than the perimeter of square P. Thus, the perimeter of square Q is 176 inches greater than $4x$ inches, or $4x + 176$ inches. Since the perimeter of a square is 4 times the side length of the square, each side length of Q is $\frac{4x+176}{4}$, or $x + 44$ inches. Since the area of a square is calculated by multiplying the length of two sides, the area of square Q is $(x + 44)(x + 44)$, or $(x + 44)^2$ square inches. It follows that function f is defined by $f(x) = (x + 44)^2$.

Choice B is incorrect. This function represents a square with side lengths $(x + 176)$ inches.

Choice C is incorrect. This function represents a square with side lengths $(176x + 44)$ inches.

Choice D is incorrect. This function represents a square with side lengths $(176x + 176)$ inches.

Question Difficulty: Hard

Question ID a9e93fa1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: a9e93fa1

$$f(x) = 4x^2 + 64x + 262$$

The function g is defined by $g(x) = f(x+5)$. For what value of x does $g(x)$ reach its minimum?

A. -13

B. -8

C. -5

D. -3

ID: a9e93fa1 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $g(x) = f(x+5)$. Since $f(x) = 4x^2 + 64x + 262$, it follows that $f(x+5) = 4(x+5)^2 + 64(x+5) + 262$. Expanding the quantity $(x+5)^2$ in this equation yields $f(x+5) = 4(x^2 + 10x + 25) + 64(x+5) + 262$. Distributing the 4 and the 64 yields $f(x+5) = 4x^2 + 40x + 100 + 64x + 320 + 262$. Combining like terms yields $f(x+5) = 4x^2 + 104x + 682$. Therefore, $g(x) = 4x^2 + 104x + 682$. For a quadratic function defined by an equation of the form $g(x) = a(x-h)^2 + k$, where a, h, and k are constants and a is positive, $g(x)$ reaches its minimum, k, when the value of x is h. The equation $g(x) = 4x^2 + 104x + 682$ can be rewritten in this form by completing the square. This equation is equivalent to $g(x) = 4(x^2 + 26) + 682$, or $g(x) = 4(x^2 + 26x + 169 - 169) + 682$. This equation can be rewritten as $g(x) = 4\left((x+13)^2 - 169\right) + 682$, or $g(x) = 4(x+13)^2 - 4(169) + 682$, which is equivalent to $g(x) = 4(x+13)^2 + 6$. This equation is in the form $g(x) = a(x-h)^2 + k$, where $a = 4$, $h = -13$, and $k = 6$. Therefore, $g(x)$ reaches its minimum when the value of x is -13.

Choice B is incorrect. This is the value of x for which $f(x)$, rather than $g(x)$, reaches its minimum.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This is the value of x for which $f(x-5)$, rather than $f(x+5)$, reaches its minimum.

Question Difficulty: Hard

Question ID af1aea31

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: af1aea31

Which of the following functions has(have) a minimum value at -3?

I. $f(x) = -6(3)^x - 3$

II. $g(x) = -3(6)^x$

A. I only

B. II only

C. I and II

D. Neither I nor II

ID: af1aea31 Answer

Correct Answer: D

Rationale

Choice D is correct. A function of the form $f(x) = a(b)^x + c$, where $a < 0$ and $b > 1$, is a decreasing function. Both of the given functions are of this form; therefore, both are decreasing functions. If a function f is decreasing as the value of x increases, the corresponding value of $f(x)$ decreases; therefore, the function doesn't have a minimum value. Thus, neither of the given functions has a minimum value.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 0bb72c19

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: 0bb72c19

The area of a rectangular banner is $2{,}661$ square inches. The banner's length x, in inches, is 24 inches longer than its width, in inches. Which equation represents this situation?

A. $0 = x^2 - 24x - 2{,}661$

B. $0 = x^2 - 24x + 2{,}661$

C. $0 = x^2 + 24x - 2{,}661$

D. $0 = x^2 + 24x + 2{,}661$

ID: 0bb72c19 Answer

Correct Answer: A

Rationale

Choice A is correct. It’s given that the banner’s length x, in inches, is 24 inches longer than its width, in inches. It follows that the banner’s width, in inches, can be represented by the expression $x - 24$. The area of a rectangle is the product of its length and its width. It's given that the area of the banner is $2{,}661$ square inches, so it follows that $2{,}661 = x(x - 24)$, or $2{,}661 = x^2 - 24x$. Subtracting $2{,}661$ from each side of this equation yields $0 = x^2 - 24x - 2{,}661$. Therefore, the equation that represents this situation is $0 = x^2 - 24x - 2{,}661$.

Choice B is incorrect and may result from representing the width, in inches, of the banner as $24 - x$, rather than $x - 24$.

Choice C is incorrect and may result from representing the width, in inches, of the banner as $x + 24$, rather than $x - 24$.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID c3e3b12e

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: c3e3b12e

The function f is defined by $f(x) = ax^2 + bx + c$, where a, b, and c are constants. The graph of $y = f(x)$ in the *xy*-plane passes through the points $(7, 0)$ and $(-3, 0)$. If a is an integer greater than 1, which of the following could be the value of $a + b$?

A. -6

B. -3

C. 4

D. 5

ID: c3e3b12e Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the graph of $y = f(x)$ in the *xy*-plane passes through the points $(7, 0)$ and $(-3, 0)$. It follows that when the value of x is either 7 or -3, the value of $f(x)$ is 0. It's also given that the function f is defined by $f(x) = ax^2 + bx + c$, where a, b, and c are constants. It follows that the function f is a quadratic function and, therefore, may be written in factored form as $f(x) = a(x - u)(x - v)$, where the value of $f(x)$ is 0 when x is either u or v. Since the value of $f(x)$ is 0 when the value of x is either 7 or -3, and the value of $f(x)$ is 0 when the value of x is either u or v, it follows that u and v are equal to 7 and -3. Substituting 7 for u and -3 for v in the equation $f(x) = a(x - u)(x - v)$ yields $f(x) = a(x - 7)(x - (-3))$, or $f(x) = a(x - 7)(x + 3)$. Distributing the right-hand side of this equation yields $f(x) = a\left(x^2 - 7x + 3x - 21\right)$, or $f(x) = ax^2 - 4ax - 21a$. Since it's given that $f(x) = ax^2 + bx + c$, it follows that $b = -4a$. Adding a to each side of this equation yields $a + b = -3a$. Since $a + b = -3a$, if a is an integer, the value of $a + b$ must be a multiple of 3. If a is an integer greater than 1, it follows that $a \geq 2$. Therefore, $-3a \leq -3(2)$. It follows that the value of $a + b$ is less than or equal to $-3(2)$, or -6. Of the given choices, only -6 is a multiple of 3 that's less than or equal to -6.

Choice B is incorrect. This is the value of $a + b$ if a is equal to, not greater than, 1.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID c9e4bac1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	Hard

ID: c9e4bac1

A quadratic function models a projectile's height, in meters, above the ground in terms of the time, in seconds, after it was launched. The model estimates that the projectile was launched from an initial height of 7 meters above the ground and reached a maximum height of 51.1 meters above the ground 3 seconds after the launch. How many seconds after the launch does the model estimate that the projectile will return to a height of 7 meters?

A. 3

B. 6

C. 7

D. 9

ID: c9e4bac1 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that a quadratic function models the projectile's height, in meters, above the ground in terms of the time, in seconds, after it was launched. It follows that an equation representing the model can be written in the form $f(x) = a(x-h)^2 + k$, where $f(x)$ is the projectile's estimated height above the ground, in meters, x seconds after the launch, a is a constant, and k is the maximum height above the ground, in meters, the model estimates the projectile reached h seconds after the launch. It's given that the model estimates the projectile reached a maximum height of 51.1 meters above the ground 3 seconds after the launch. Therefore, $k = 51.1$ and $h = 3$. Substituting 51.1 for k and 3 for h in the equation $f(x) = a(x-h)^2 + k$ yields $f(x) = a(x-3)^2 + 51.1$. It's also given that the model estimates that the projectile was launched from an initial height of 7 meters above the ground. Therefore, when $x = 0$, $f(x) = 7$. Substituting 0 for x and 7 for $f(x)$ in the equation $f(x) = a(x-3)^2 + 51.1$ yields $7 = a(0-3)^2 + 51.1$, or $7 = 9a + 51.1$. Subtracting 51.1 from both sides of this equation yields $-44.1 = 9a$. Dividing both sides of this equation by 9 yields $-4.9 = a$. Substituting -4.9 for a in the equation $f(x) = a(x-3)^2 + 51.1$ yields $f(x) = -4.9(x-3)^2 + 51.1$. Therefore, the equation $f(x) = -4.9(x-3)^2 + 51.1$ models the projectile's height, in meters, above the ground x seconds after it was launched. The number of seconds after the launch that the model estimates that the projectile will return to a height of 7 meters is the value of x when $f(x) = 7$. Substituting 7 for $f(x)$ in $f(x) = -4.9(x-3)^2 + 51.1$ yields $7 = -4.9(x-3)^2 + 51.1$. Subtracting 51.1 from both sides of this equation yields $-44.1 = -4.9(x-3)^2$. Dividing both sides of this equation by -4.9 yields $9 = (x-3)^2$. Taking the square root of both sides of this equation yields two equations: $3 = x - 3$ and $-3 = x - 3$. Adding 3 to both sides of the equation $3 = x - 3$ yields $6 = x$. Adding 3 to both sides of the equation $-3 = x - 3$ yields $0 = x$. Since 0 seconds after the launch represents the time at which the projectile was launched, 6 must be the number of seconds the model estimates that the projectile will return to a height of 7 meters.

Alternate approach: It's given that a quadratic function models the projectile's height, in meters, above the ground in terms of the time, in seconds, after it was launched. It's also given that the model estimates that the projectile was launched from an initial height of 7 meters above the ground and reached a maximum height of 51.1 meters above the ground 3 seconds after the launch. Since the model is quadratic, and quadratic functions are symmetric, the model estimates that for any given height less than the maximum height, the time the projectile takes to travel from the given height to the maximum height is the same as the time the projectile takes to travel from the maximum height back to the given height. Thus, since the model estimates the projectile took 3 seconds to travel from 7 meters above the ground to its maximum height of 51.1 meters above the ground, the model also estimates the projectile will take 3 more seconds to travel from its maximum height of 51.1 meters above the ground back to 7 meters above the ground. Thus, the model estimates that the projectile will return to a height of 7 meters 3 seconds after it reaches its maximum height, which is 6 seconds after the launch.

Choice A is incorrect. The model estimates that 3 seconds after the launch, the projectile reached a height of 51.1 meters, not 7 meters.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard