Question ID 669f307b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 669f307b

$$RS = 20$$
$$ST = 48$$
$$TR = 52$$

The side lengths of right triangle RST are given. Triangle RST is similar to triangle UVW, where S corresponds to V and T corresponds to W. What is the value of $\tan W$?

A. $\frac{5}{13}$

B. $\frac{5}{12}$

C. $\frac{12}{13}$

D. $\frac{12}{5}$

ID: 669f307b Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that right triangle RST is similar to triangle UVW, where S corresponds to V and T corresponds to W. It's given that the side lengths of the right triangle RST are $RS = 20$, $ST = 48$, and $TR = 52$. Corresponding angles in similar triangles are equal. It follows that the measure of angle T is equal to the measure of angle W. The hypotenuse of a right triangle is the longest side. It follows that the hypotenuse of triangle RST is side TR. The hypotenuse of a right triangle is the side opposite the right angle. Therefore, angle S is a right angle. The adjacent side of an acute angle in a right triangle is the side closest to the angle that is not the hypotenuse. It follows that the adjacent side of angle T is side ST. The opposite side of an acute angle in a right triangle is the side across from the acute angle. It follows that the opposite side of angle T is side RS. The tangent of an acute angle in a right triangle is the ratio of the length of the opposite side to the length of the adjacent side. Therefore, $\tan\ T = \frac{RS}{ST}$. Substituting 20 for RS and 48 for ST in this equation yields $\tan\ T = \frac{20}{48}$, or $\tan\ T = \frac{5}{12}$. The tangents of two acute angles with equal measures are equal. Since the measure of angle T is equal to the measure of angle W, it follows that $\tan\ T = \tan\ W$. Substituting $\frac{5}{12}$ for $\tan\ T$ in this equation yields $\frac{5}{12} = \tan\ W$. Therefore, the value of $\tan\ W$ is $\frac{5}{12}$.

Choice A is incorrect. This is the value of $\sin\ W$.

Choice C is incorrect. This is the value of $\cos\ W$.

Choice D is incorrect. This is the value of $\frac{1}{\tan\ W}$.

Question Difficulty: Hard

Question ID c10968c1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: c10968c1

Triangle ABC is similar to triangle DEF, where angle A corresponds to angle D and angles C and F are right angles. The length of $\overline{AB}$ is 2.9 times the length of $\overline{DE}$. If $\tan A = \frac{21}{20}$, what is the value of $\sin D$?

ID: c10968c1 Answer

Correct Answer: .7241, 21/29

Rationale

The correct answer is $\frac{21}{29}$. It's given that triangle ABC is similar to triangle DEF, where angle A corresponds to angle D and angles C and F are right angles. In similar triangles, the tangents of corresponding angles are equal. Therefore, if $\tan A = \frac{21}{20}$, then $\tan D = \frac{21}{20}$. In a right triangle, the tangent of an acute angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. Therefore, in triangle DEF, if $\tan D = \frac{21}{20}$, the ratio of the length of $\overline{EF}$ to the length of $\overline{DF}$ is $\frac{21}{20}$. If the lengths of $\overline{EF}$ and $\overline{DF}$ are 21 and 20, respectively, then the ratio of the length of $\overline{EF}$ to the length of $\overline{DF}$ is $\frac{21}{20}$. In a right triangle, the sine of an acute angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse. Therefore, the value of $\sin D$ is the ratio of the length of $\overline{EF}$ to the length of $\overline{DE}$. The length of $\overline{DE}$ can be calculated using the Pythagorean theorem, which states that if the lengths of the legs of a right triangle are a and b and the length of the hypotenuse is c, then $a^2 + b^2 = c^2$. Therefore, if the lengths of $\overline{EF}$ and $\overline{DF}$ are 21 and 20, respectively, then $(21)^2 + (20)^2 = (DE)^2$, or $841 = (DE)^2$. Taking the positive square root of both sides of this equation yields $29 = DE$. Therefore, if the lengths of $\overline{EF}$ and $\overline{DF}$ are 21 and 20, respectively, then the length of $\overline{DE}$ is 29 and the ratio of the length of $\overline{EF}$ to the length of $\overline{DE}$ is $\frac{21}{29}$. Thus, if $\tan A = \frac{21}{20}$, the value of $\sin D$ is $\frac{21}{29}$. Note that 21/29, .7241, and 0.724 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 3f2b93ef

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 3f2b93ef

A rectangle is inscribed in a circle, such that each vertex of the rectangle lies on the circumference of the circle. The diagonal of the rectangle is twice the length of the shortest side of the rectangle. The area of the rectangle is $1{,}089\sqrt{3}$ square units. What is the length, in units, of the diameter of the circle?

ID: 3f2b93ef Answer

Correct Answer: 66

Rationale

The correct answer is 66. It's given that each vertex of the rectangle lies on the circumference of the circle. Therefore, the length of the diameter of the circle is equal to the length of the diagonal of the rectangle. The diagonal of a rectangle forms a right triangle with the shortest and longest sides of the rectangle, where the shortest side and the longest side of the rectangle are the legs of the triangle and the diagonal of the rectangle is the hypotenuse of the triangle. Let s represent the length, in units, of the shortest side of the rectangle. Since it's given that the diagonal is twice the length of the shortest side, $2s$ represents the length, in units, of the diagonal of the rectangle. By the Pythagorean theorem, if a right triangle has a hypotenuse with length c and legs with lengths a and b, then $a^2+b^2=c^2$. Substituting s for a and $2s$ for c in this equation yields $s^2+b^2=(2s)^2$, or $s^2+b^2=4s^2$. Subtracting s^2 from both sides of this equation yields $b^2=3s^2$. Taking the positive square root of both sides of this equation yields $b=s\sqrt{3}$. Therefore, the length, in units, of the rectangle's longest side is $s\sqrt{3}$. The area of a rectangle is the product of the length of the shortest side and the length of the longest side. The lengths, in units, of the shortest and longest sides of the rectangle are represented by s and $s\sqrt{3}$, and it's given that the area of the rectangle is $1{,}089\sqrt{3}$ square units. It follows that $1{,}089\sqrt{3}=s\left(s\sqrt{3}\right)$, or $1{,}089\sqrt{3}=s^2\sqrt{3}$. Dividing both sides of this equation by $\sqrt{3}$ yields $1{,}089=s^2$. Taking the positive square root of both sides of this equation yields $33=s$. Since the length, in units, of the diagonal is represented by $2s$, it follows that the length, in units, of the diagonal is $2(33)$, or 66. Since the length of the diameter of the circle is equal to the length of the diagonal of the rectangle, the length, in units, of the diameter of the circle is 66.

Question Difficulty: Hard

Question ID 381eefb8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 381eefb8

A right triangle has legs with lengths of 24 centimeters and 21 centimeters. If the length of this triangle's hypotenuse, in centimeters, can be written in the form $3\sqrt{d}$, where d is an integer, what is the value of d?

ID: 381eefb8 Answer

Correct Answer: 113

Rationale

The correct answer is 113. It's given that the legs of a right triangle have lengths 24 centimeters and 21 centimeters. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs. It follows that if h represents the length, in centimeters, of the hypotenuse of the right triangle, $h^2 = 24^2 + 21^2$. This equation is equivalent to $h^2 = 1{,}017$. Taking the square root of each side of this equation yields $h = \sqrt{1{,}017}$. This equation can be rewritten as $h = \sqrt{9 \cdot 113}$, or $h = \sqrt{9} \cdot \sqrt{113}$. This equation is equivalent to $h = 3\sqrt{113}$. It's given that the length of the triangle's hypotenuse, in centimeters, can be written in the form $3\sqrt{d}$. It follows that the value of d is 113.

Question Difficulty: Hard

Question ID d32d4957

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: d32d4957

Triangle ABC is similar to triangle DEF, where A corresponds to D and C corresponds to F. Angles C and F are right angles. If $\tan(A) = \sqrt{3}$ and $DF = 125$, what is the length of $\overline{DE}$?

A. $125\frac{\sqrt{3}}{3}$

B. $125\frac{\sqrt{3}}{2}$

C. $125\sqrt{3}$

D. 250

ID: d32d4957 Answer

Correct Answer: D

Rationale

Choice D is correct. Corresponding angles in similar triangles have equal measures. It's given that triangle ABC is similar to triangle DEF, where A corresponds to D, so the measure of angle A is equal to the measure of angle D. Therefore, if $\tan(A) = \sqrt{3}$, then $\tan(D) = \sqrt{3}$. It's given that angles C and F are right angles, so triangles ABC and DEF are right triangles. The adjacent side of an acute angle in a right triangle is the side closest to the angle that is not the hypotenuse. It follows that the adjacent side of angle D is side DF. The opposite side of an acute angle in a right triangle is the side across from the acute angle. It follows that the opposite side of angle D is side EF. The tangent of an acute angle in a right triangle is the ratio of the length of the opposite side to the length of the adjacent side. Therefore, $\tan(D) = \frac{EF}{DF}$. If $DF = 125$, the length of side EF can be found by substituting $\sqrt{3}$ for $\tan(D)$ and 125 for DF in the equation $\tan(D) = \frac{EF}{DF}$, which yields $\sqrt{3} = \frac{EF}{125}$. Multiplying both sides of this equation by 125 yields $125\sqrt{3} = EF$. Since the length of side EF is $\sqrt{3}$ times the length of side DF, it follows that triangle DEF is a special right triangle with angle measures 30°, 60°, and 90°. Therefore, the length of the hypotenuse, $\overline{DE}$, is 2 times the length of side DF, or $DE = 2(DF)$. Substituting 125 for DF in this equation yields $DE = 2(125)$, or $DE = 250$. Thus, if $\tan(A) = \sqrt{3}$ and $DF = 125$, the length of $\overline{DE}$ is 250.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the length of $\overline{EF}$, not $\overline{DE}$.

Question Difficulty: Hard

Question ID d8a0b327

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: d8a0b327

In triangle XYZ, angle Y is a right angle, the measure of angle Z is 33°, and the length of $\overline{YZ}$ is 26 units. If the area, in square units, of triangle XYZ can be represented by the expression $k \tan 33^{\circ}$, where k is a constant, what is the value of k?

ID: d8a0b327 Answer

Correct Answer: 338

Rationale

The correct answer is 338. The tangent of an acute angle in a right triangle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. In triangle XYZ, it's given that angle Y is a right angle. Thus, $\overline{XY}$ is the leg opposite of angle Z and $\overline{YZ}$ is the leg adjacent to angle Z. It follows that $\tan Z = \frac{XY}{YZ}$. It's also given that the measure of angle Z is 33° and the length of $\overline{YZ}$ is 26 units. Substituting 33° for Z and 26 for YZ in the equation $\tan Z = \frac{XY}{YZ}$ yields $\tan 33^{\circ} = \frac{XY}{26}$. Multiplying each side of this equation by 26 yields $26 \tan 33^{\circ} = XY$. Therefore, the length of $\overline{XY}$ is $26 \tan 33^{\circ}$. The area of a triangle is half the product of the lengths of its legs. Since the length of $\overline{YZ}$ is 26 and the length of $\overline{XY}$ is $26 \tan 33^{\circ}$, it follows that the area of triangle XYZ is $\frac{1}{2}(26)(26 \tan 33^{\circ})$ square units, or $338 \tan 33^{\circ}$ square units. It's given that the area, in square units, of triangle XYZ can be represented by the expression $k \tan 33^{\circ}$, where k is a constant. Therefore, 338 is the value of k.

Question Difficulty: Hard

Question ID 08cbd418

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 08cbd418

A square is inscribed in a circle. The radius of the circle is $\frac{20\sqrt{2}}{2}$ inches. What is the side length, in inches, of the square?

A. 20

B. $\frac{20\sqrt{2}}{2}$

C. $20\sqrt{2}$

D. 40

ID: 08cbd418 Answer

Correct Answer: A

Rationale

Choice A is correct. When a square is inscribed in a circle, a diagonal of the square is a diameter of the circle. It's given that a square is inscribed in a circle and the length of a radius of the circle is $\frac{20\sqrt{2}}{2}$ inches. Therefore, the length of a diameter of the circle is $2\left(\frac{20\sqrt{2}}{2}\right)$ inches, or $20\sqrt{2}$ inches. It follows that the length of a diagonal of the square is $20\sqrt{2}$ inches. A diagonal of a square separates the square into two right triangles in which the legs are the sides of the square and the hypotenuse is a diagonal. Since a square has 4 congruent sides, each of these two right triangles has congruent legs and a hypotenuse of length $20\sqrt{2}$ inches. Since each of these two right triangles has congruent legs, they are both 45-45-90 triangles. In a 45-45-90 triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg. Let s represent the length of a leg of one of these 45-45-90 triangles. It follows that $20\sqrt{2}=\sqrt{2}(s)$. Dividing both sides of this equation by $\sqrt{2}$ yields $20=s$. Therefore, the length of a leg of one of these 45-45-90 triangles is 20 inches. Since the legs of these two 45-45-90 triangles are the sides of the square, it follows that the side length of the square is 20 inches.

Choice B is incorrect. This is the length of a radius, in inches, of the circle.

Choice C is incorrect. This is the length of a diameter, in inches, of the circle.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2ab5f0fd

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 2ab5f0fd

The length of a rectangle's diagonal is $3\sqrt{17}$, and the length of the rectangle's shorter side is 3. What is the length of the rectangle's longer side?

ID: 2ab5f0fd Answer

Correct Answer: 12

Rationale

The correct answer is 12. The diagonal of a rectangle forms a right triangle, where the shorter side and the longer side of the rectangle are the legs of the triangle and the diagonal of the rectangle is the hypotenuse of the triangle. It's given that the length of the rectangle's diagonal is $3\sqrt{17}$ and the length of the rectangle's shorter side is 3. Thus, the length of the hypotenuse of the right triangle formed by the diagonal is $3\sqrt{17}$ and the length of one of the legs is 3. By the Pythagorean theorem, if a right triangle has a hypotenuse with length c and legs with lengths a and b, then $a^2 + b^2 = c^2$. Substituting $3\sqrt{17}$ for c and 3 for b in this equation yields $a^2 + (3)^2 = \left(3\sqrt{17}\right)^2$, or $a^2 + 9 = 153$. Subtracting 9 from both sides of this equation yields $a^2 = 144$. Taking the square root of both sides of this equation yields $a = \pm\sqrt{144}$, or $a = \pm 12$. Since a represents a length, which must be positive, the value of a is 12. Thus, the length of the rectangle's longer side is 12.

Question Difficulty: Hard

Question ID 7d7d80b2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 7d7d80b2

In triangle JKL, $\cos(K) = \frac{24}{51}$ and angle J is a right angle. What is the value of $\cos(L)$?

ID: 7d7d80b2 Answer

Correct Answer: .8823, .8824, 15/17

Rationale

The correct answer is $\frac{15}{17}$. It's given that angle J is the right angle in triangle JKL. Therefore, the acute angles of triangle JKL are angle K and angle L. The hypotenuse of a right triangle is the side opposite its right angle. Therefore, the hypotenuse of triangle JKL is side KL. The cosine of an acute angle in a right triangle is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse. It's given that $\cos(K) = \frac{24}{51}$. This can be written as $\cos(K) = \frac{8}{17}$. Since the cosine of angle K is a ratio, it follows that the length of the side adjacent to angle K is $8n$ and the length of the hypotenuse is $17n$, where n is a constant. Therefore, $JK = 8n$ and $KL = 17n$. The Pythagorean theorem states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides. For triangle JKL, it follows that $(JK)^2 + (JL)^2 = (KL)^2$. Substituting $8n$ for JK and $17n$ for KL yields $(8n)^2 + (JL)^2 = (17n)^2$. This is equivalent to $64n^2 + (JL)^2 = 289n^2$. Subtracting $64n^2$ from each side of this equation yields $(JL)^2 = 225n^2$. Taking the square root of each side of this equation yields $JL = 15n$. Since $\cos(L) = \frac{JL}{KL}$, it follows that $\cos(L) = \frac{15n}{17n}$, which can be rewritten as $\cos(L) = \frac{15}{17}$. Note that 15/17, .8824, .8823, and 0.882 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID f8e6e6c6

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: f8e6e6c6

An isosceles right triangle has a hypotenuse of length 58 inches. What is the perimeter, in inches, of this triangle?

A. $29\sqrt{2}$

B. $58\sqrt{2}$

C. $58+58\sqrt{2}$

D. $58+116\sqrt{2}$

ID: f8e6e6c6 Answer

Correct Answer: C

Rationale

Choice C is correct. Since the triangle is an isosceles right triangle, the two sides that form the right angle must be the same length. Let x be the length, in inches, of each of those sides. The Pythagorean theorem states that in a right triangle, $a^2+b^2=c^2$, where c is the length of the hypotenuse and a and b are the lengths of the other two sides. Substituting x for a, x for b, and 58 for c in this equation yields $x^2+x^2=58^2$, or $2x^2=58^2$. Dividing each side of this equation by 2 yields $x^2=\frac{58^2}{2}$, or $x^2=\frac{2\cdot 58^2}{4}$. Taking the square root of each side of this equation yields two solutions: $x=\frac{58\sqrt{2}}{2}$ and $x=-\frac{58\sqrt{2}}{2}$. The value of x must be positive because it represents a side length. Therefore, $x=\frac{58\sqrt{2}}{2}$, or $x=29\sqrt{2}$. The perimeter, in inches, of the triangle is $58+x+x$, or $58+2x$. Substituting $29\sqrt{2}$ for x in this expression gives a perimeter, in inches, of $58+2\left(29\sqrt{2}\right)$, or $58+58\sqrt{2}$.

Choice A is incorrect. This is the length, in inches, of each of the congruent sides of the triangle, not the perimeter, in inches, of the triangle.

Choice B is incorrect. This is the sum of the lengths, in inches, of the congruent sides of the triangle, not the perimeter, in inches, of the triangle.

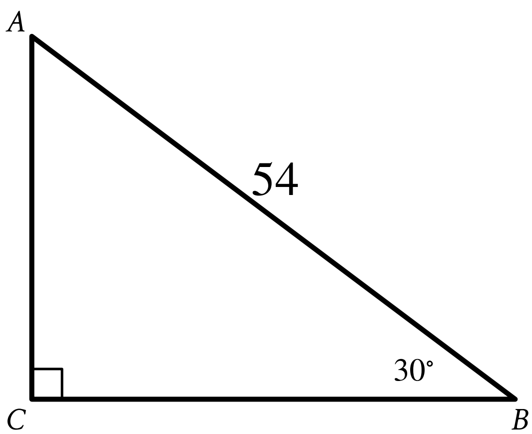
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 1b0b382b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 1b0b382b



Note: Figure not drawn to scale.

Right triangle ABC is shown. What is the value of $\tan A$?

A. $\frac{\sqrt{3}}{54}$

B. $\frac{1}{\sqrt{3}}$

C. $\sqrt{3}$

D. $27\sqrt{3}$

ID: 1b0b382b Answer

Correct Answer: C

Rationale

Choice C is correct. In the triangle shown, the measure of angle B is 30° and angle C is a right angle, which means that it has a measure of 90°. Since the sum of the angles in a triangle is equal to 180°, the measure of angle A is equal to $180^\circ - (30 + 90)^\circ$, or 60°. In a right triangle whose acute angles have measures 30° and 60°, the lengths of the legs can be represented by the expressions x, $x\sqrt{3}$, and $2x$, where x is the length of the leg opposite the angle with measure 30°, $x\sqrt{3}$ is the length of the leg opposite the angle with measure 60°, and $2x$ is the length of the hypotenuse. In the triangle shown, the hypotenuse has a length of 54. It follows that $2x = 54$, or $x = 27$. Therefore, the length of the leg opposite angle B is 27 and the length of the leg opposite angle A is $27\sqrt{3}$. The tangent of an acute angle in a right triangle is defined as the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. The length of the leg opposite angle A is $27\sqrt{3}$ and the length of the leg adjacent to angle A is 27. Therefore, the value of $\tan A$ is $\frac{27\sqrt{3}}{27}$, or $\sqrt{3}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. This is the value of $\frac{1}{\tan A}$, not the value of $\tan A$.

Choice D is incorrect. This is the length of the leg opposite angle A, not the value of $\tan A$.

Question Difficulty: Hard

Question ID e1137c5a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: e1137c5a

In triangle XYZ, angle Z is a right angle and the length of $\overline{YZ}$ is 24 units. If $\tan X = \frac{12}{35}$, what is the perimeter, in units, of triangle XYZ?

A. 188

B. 168

C. 84

D. 71

ID: e1137c5a Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that angle Z in triangle XYZ is a right angle. Thus, side YZ is the leg opposite angle X and side XZ is the leg adjacent to angle X. The tangent of an acute angle in a right triangle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle. It follows that $\tan X = \frac{YZ}{XZ}$. It's given that $\tan X = \frac{12}{35}$ and the length of side YZ is 24 units. Substituting $\frac{12}{35}$ for $\tan X$ and 24 for YZ in the equation $\tan X = \frac{YZ}{XZ}$ yields $\frac{12}{35} = \frac{24}{XZ}$. Multiplying both sides of this equation by $35(XZ)$ yields $12(XZ) = 24(35)$, or $12(XZ) = 840$. Dividing both sides of this equation by 12 yields $XZ = 70$. The length XY can be calculated using the Pythagorean theorem, which states that if a right triangle has legs with lengths of a and b and a hypotenuse with length c, then $a^2 + b^2 = c^2$. Substituting 70 for a and 24 for b in this equation yields $70^2 + 24^2 = c^2$, or $5{,}476 = c^2$. Taking the square root of both sides of this equation yields $\pm 74 = c$. Since the length of the hypotenuse must be positive, $74 = c$. Therefore, the length of XY is 74 units. The perimeter of a triangle is the sum of the lengths of all sides. Thus, $(74 + 70 + 24)$ units, or 168 units, is the perimeter of triangle XYZ.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This would be the perimeter, in units, for a right triangle where the length of side YZ is 12 units, not 24 units.

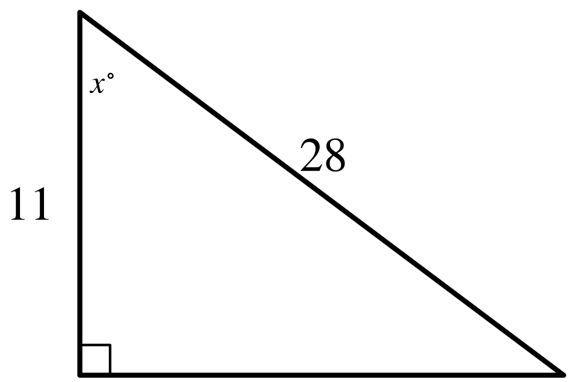
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 8aeff54c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 8aeff54c



Note: Figure not drawn to scale.

In the triangle shown, what is the value of $\cos x^\circ$?

ID: 8aeff54c Answer

Correct Answer: .3928, .3929, 11/28

Rationale

The correct answer is $\frac{11}{28}$. The cosine of an acute angle in a right triangle is defined as the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse. In the triangle shown, the length of the leg adjacent to the angle with measure x° is 11 units and the length of the hypotenuse is 28 units. Therefore, the value of $\cos x^\circ$ is $\frac{11}{28}$. Note that 11/28, .3928, .3929, 0.392, and 0.393 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID cb6de2ae

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: cb6de2ae

The perimeter of an equilateral triangle is 852 centimeters. The three vertices of the triangle lie on a circle. The radius of the circle is $w\sqrt{3}$ centimeters. What is the value of w?

ID: cb6de2ae Answer

Correct Answer: 284/3, 94.66, 94.67

Rationale

The correct answer is $\frac{284}{3}$. Since the perimeter of a triangle is the sum of the lengths of its sides, and the given triangle is equilateral, the length of each side is $\frac{852}{3}$, or 284, centimeters (cm). Right triangle AMO can be formed, where M is the midpoint of one of the triangle's sides, A is one of this side's endpoints, and O is the center of the circle. It follows that AM is $\frac{284}{2}$, or 142, cm. Additionally, triangle AMO has angles measuring 30°, 60°, and 90°, where the measure of angle OMA is 90° and the measure of angle OAM is 30°. It follows that the length of side MO is half the length of hypotenuse AO, and the length of side AM is $\sqrt{3}$ times the length of side MO. It's given that $AO = w\sqrt{3}$ cm. Therefore, $MO = \frac{w\sqrt{3}}{2}$ cm and $AM = \frac{w\sqrt{3}\sqrt{3}}{2}$ cm, which is equivalent to $AM = \frac{3w}{2}$ cm. Since $AM = 142$ cm, it follows that $\frac{3w}{2} = 142$. Multiplying both sides of this equation by 2 yields $3w = 284$. Dividing both sides of this equation by 3 yields $w = \frac{284}{3}$. Note that 284/3, 94.66, and 94.67 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 50cd2366

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 50cd2366

An isosceles right triangle has a perimeter of $94+94\sqrt{2}$ inches. What is the length, in inches, of one leg of this triangle?

A. 47

B. $47\sqrt{2}$

C. 94

D. $94\sqrt{2}$

ID: 50cd2366 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the right triangle is isosceles. In an isosceles right triangle, the two legs have equal lengths, and the length of the hypotenuse is $\sqrt{2}$ times the length of one of the legs. Let ℓ represent the length, in inches, of each leg of the isosceles right triangle. It follows that the length of the hypotenuse is $\ell\sqrt{2}$ inches. The perimeter of a figure is the sum of the lengths of the sides of the figure. Therefore, the perimeter of the isosceles right triangle is $\ell+\ell+\ell\sqrt{2}$ inches. It's given that the perimeter of the triangle is $94+94\sqrt{2}$ inches. It follows that $\ell+\ell+\ell\sqrt{2}=94+94\sqrt{2}$. Factoring the left-hand side of this equation yields $\left(1+1+\sqrt{2}\right)\ell=94+94\sqrt{2}$, or $\left(2+\sqrt{2}\right)\ell=94+94\sqrt{2}$. Dividing both sides of this equation by $2+\sqrt{2}$ yields $\ell=\frac{94+94\sqrt{2}}{2+\sqrt{2}}$. Rationalizing the denominator of the right-hand side of this equation by multiplying the right-hand side of the equation by $\frac{2-\sqrt{2}}{2-\sqrt{2}}$ yields $\ell=\frac{\left(94+94\sqrt{2}\right)\left(2-\sqrt{2}\right)}{\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)}$. Applying the distributive property to the numerator and to the denominator of the right-hand side of this equation yields $\ell=\frac{188-94\sqrt{2}+188\sqrt{2}-94\sqrt{4}}{4-2\sqrt{2}+2\sqrt{2}-\sqrt{4}}$. This is equivalent to $\ell=\frac{94\sqrt{2}}{2}$, or $\ell=47\sqrt{2}$. Therefore, the length, in inches, of one leg of the isosceles right triangle is $47\sqrt{2}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the length, in inches, of the hypotenuse.

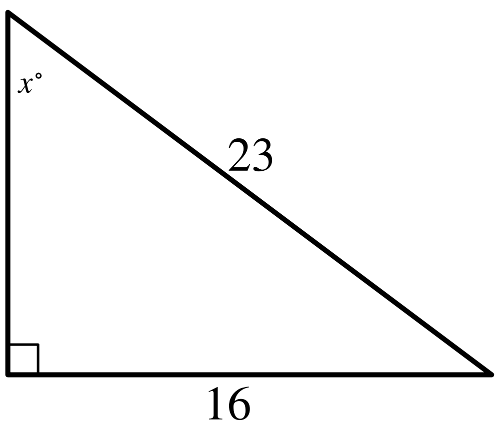
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 1dbbea6b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 1dbbea6b



Note: Figure not drawn to scale.

In the triangle shown, what is the value of $\sin x^\circ$?

ID: 1dbbea6b Answer

Correct Answer: .6956, .6957, 16/23

Rationale

The correct answer is $\frac{16}{23}$. In a right triangle, the sine of an acute angle is defined as the ratio of the length of the side opposite the angle to the length of the hypotenuse. In the triangle shown, the length of the side opposite the angle with measure x° is 16 units and the length of the hypotenuse is 23 units. Therefore, the value of $\sin x^\circ$ is $\frac{16}{23}$. Note that 16/23, .6956, .6957, 0.695, and 0.696 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 8970ec84

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 8970ec84

The perimeter of an equilateral triangle is 624 centimeters. The height of this triangle is $k\sqrt{3}$ centimeters, where k is a constant. What is the value of k?

ID: 8970ec84 Answer

Correct Answer: 104

Rationale

The correct answer is 104. An equilateral triangle is a triangle in which all three sides have the same length and all three angles have a measure of 60°. The height of the triangle, $k\sqrt{3}$, is the length of the altitude from one vertex. The altitude divides the equilateral triangle into two congruent 30-60-90 right triangles, where the altitude is the side across from the 60° angle in each 30-60-90 right triangle. Since the altitude has a length of $k\sqrt{3}$, it follows from the properties of 30-60-90 right triangles that the side across from each 30° angle has a length of k and each hypotenuse has a length of $2k$. In this case, the hypotenuse of each 30-60-90 right triangle is a side of the equilateral triangle; therefore, each side length of the equilateral triangle is $2k$. The perimeter of a triangle is the sum of the lengths of each side. It's given that the perimeter of the equilateral triangle is 624; therefore, $2k + 2k + 2k = 624$, or $6k = 624$. Dividing both sides of this equation by 6 yields $k = 104$.

Question Difficulty: Hard

Question ID 1215eb0a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 1215eb0a

Which of the following expressions is equivalent to $(\sin 24^\circ)(\cos 66^\circ)+(\cos 24^\circ)(\sin 66^\circ)$?

A. $2(\cos 66^\circ)(\sin 24^\circ)$

B. $2(\cos 66^\circ)+2(\cos 24^\circ)$

C. msup $+(\cos 24^\circ)^2$

D. msup $+(\sin 24^\circ)^2$

ID: 1215eb0a Answer

Correct Answer: C

Rationale

Choice C is correct. The sine of an angle is equal to the cosine of its complementary angle. Since angles with measures 24° and 66° are complementary to each other, $\sin 24^\circ$ is equal to $\cos 66^\circ$ and $\sin 66^\circ$ is equal to $\cos 24^\circ$. Substituting $\cos 66^\circ$ for $\sin 24^\circ$ and $\cos 24^\circ$ for $\sin 66^\circ$ in the given expression yields $(\cos 66^\circ)(\cos 66^\circ)+(\cos 24^\circ)(\cos 24^\circ)$, or $(\cos 66^\circ)^2+(\cos 24^\circ)^2$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID acb49e4b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: acb49e4b

$$RS = 440$$
$$ST = 384$$
$$TR = 584$$

The side lengths of right triangle RST are given. Triangle RST is similar to triangle UVW, where S corresponds to V and T corresponds to W. What is the value of $\tan W$?

A. $\frac{48}{73}$

B. $\frac{55}{73}$

C. $\frac{48}{55}$

D. $\frac{55}{48}$

ID: acb49e4b Answer

Correct Answer: D

Rationale

Choice D is correct. The hypotenuse of triangle RST is the longest side and is across from the right angle. The longest side length given is 584, which is the length of side TR. Therefore, the hypotenuse of triangle RST is side TR, so the right angle is angle S. The tangent of an acute angle in a right triangle is the ratio of the length of the opposite side, which is the side across from the angle, to the length of the adjacent side, which is the side closest to the angle that is not the hypotenuse. It follows that the opposite side of angle T is side RS and the adjacent side of angle T is side ST. Therefore, $\tan T = \frac{RS}{ST}$. Substituting 440 for RS and 384 for ST in this equation yields $\tan T = \frac{440}{384}$. This is equivalent to $\tan T = \frac{55}{48}$. It's given that triangle RST is similar to triangle UVW, where S corresponds to V and T corresponds to W. It follows that R corresponds to U. Therefore, the hypotenuse of triangle UVW is side WU, which means $\tan W = \frac{UV}{VW}$. Since the lengths of corresponding sides of similar triangles are proportional, $\frac{RS}{ST} = \frac{UV}{VW}$. Therefore, $\tan W = \frac{UV}{VW}$ is equivalent to $\tan W = \frac{RS}{ST}$, or $\tan W = \tan T$. Thus, $\tan W = \frac{55}{48}$.

Choice A is incorrect. This is the value of $\cos W$, not $\tan W$.

Choice B is incorrect. This is the value of $\sin W$, not $\tan W$.

Choice C is incorrect. This is the value of $\frac{1}{\tan W}$, not $\tan W$.

Question Difficulty: Hard

Question ID 307d7ae0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: 307d7ae0

Triangle ABC is similar to triangle DEF, where angle A corresponds to angle D and angle C corresponds to angle F. Angles C and F are right angles. If $\tan(A) = \frac{50}{7}$, what is the value of $\tan(E)$?

ID: 307d7ae0 Answer

Correct Answer: .14, 7/50

Rationale

The correct answer is $\frac{7}{50}$. It's given that triangle ABC is similar to triangle DEF, where angle A corresponds to angle D and angle C corresponds to angle F. In similar triangles, the tangents of corresponding angles are equal. Since angle A and angle D are corresponding angles, if $\tan(A) = \frac{50}{7}$, then $\tan(D) = \frac{50}{7}$. It's also given that angles C and F are right angles. It follows that triangle DEF is a right triangle with acute angles D and E. The tangent of one acute angle in a right triangle is the inverse of the tangent of the other acute angle in the triangle. Therefore, $\tan(E) = \frac{1}{\tan(D)}$. Substituting $\frac{50}{7}$ for $\tan(D)$ in this equation yields $\tan(E) = \frac{1}{\frac{50}{7}}$, or $\tan(E) = \frac{7}{50}$. Thus, if $\tan(A) = \frac{50}{7}$, the value of $\tan(E)$ is $\frac{7}{50}$. Note that 7/50 and .14 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID a1ec8e47

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Right triangles and trigonometry	Hard

ID: a1ec8e47

In triangle ABC, angle B is a right angle. The length of side AB is $10\sqrt{37}$ and the length of side BC is $24\sqrt{37}$. What is the length of side AC?

A. $14\sqrt{37}$

B. $26\sqrt{37}$

C. $34\sqrt{37}$

D. $\sqrt{34 \cdot 37}$

ID: a1ec8e47 Answer

Correct Answer: B

Rationale

Choice B is correct. The Pythagorean theorem states that for a right triangle, $c^2 = a^2 + b^2$, where c represents the length of the hypotenuse and a and b represent the lengths of the legs. It's given that in triangle ABC, angle B is a right angle. Therefore, triangle ABC is a right triangle, where the hypotenuse is side AC and the legs are sides AB and BC. It's given that the lengths of sides AB and BC are $10\sqrt{37}$ and $24\sqrt{37}$, respectively. Substituting these values for a and b in the formula $c^2 = a^2 + b^2$ yields $c^2 = \left(10\sqrt{37}\right)^2 + \left(24\sqrt{37}\right)^2$, which is equivalent to $c^2 = 100(37) + 576(37)$, or $c^2 = 676(37)$. Taking the square root of both sides of this equation yields $c = \pm 26\sqrt{37}$. Since c represents the length of the hypotenuse, side AC, c must be positive. Therefore, the length of side AC is $26\sqrt{37}$.

Choice A is incorrect. This is the result of solving the equation $c = 24\sqrt{37} - 10\sqrt{37}$, not $c^2 = \left(10\sqrt{37}\right)^2 + \left(24\sqrt{37}\right)^2$.

Choice C is incorrect. This is the result of solving the equation $c = 10\sqrt{37} + 24\sqrt{37}$, not $c^2 = \left(10\sqrt{37}\right)^2 + \left(24\sqrt{37}\right)^2$.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard