Question ID 2d1e5eff

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 2d1e5eff

$$y = 2x^2 - 21x + 64$$
$$y = 3x + a$$

In the given system of equations, a is a constant. The graphs of the equations in the given system intersect at exactly one point, (x, y), in the *xy*-plane. What is the value of x?

A. -8

B. -6

C. 6

D. 8

ID: 2d1e5eff Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the graphs of the equations in the given system intersect at exactly one point, (x, y), in the *xy*-plane. Therefore, (x, y) is the only solution to the given system of equations. The given system of equations can be solved by subtracting the second equation, $y = 3x + a$, from the first equation, $y = 2x^2 - 21x + 64$. This yields $y - y = (2x^2 - 21x + 64) - (3x + a)$, or $0 = 2x^2 - 24x + 64 - a$. Since the given system has only one solution, this equation has only one solution. A quadratic equation in the form $rx^2 + sx + t = 0$, where r, s, and t are constants, has one solution if and only if the discriminant, $s^2 - 4rt$, is equal to zero. Substituting 2 for r, -24 for s, and $-a + 64$ for t in the expression $s^2 - 4rt$ yields $(-24)^2 - (4)(2)(64 - a)$. Setting this expression equal to zero yields $(-24)^2 - (4)(2)(64 - a) = 0$, or $8a + 64 = 0$. Subtracting 64 from both sides of this equation yields $8a = -64$. Dividing both sides of this equation by 8 yields $a = -8$. Substituting -8 for a in the equation $0 = 2x^2 - 24x + 64 - a$ yields $0 = 2x^2 - 24x + 64 + 8$, or $0 = 2x^2 - 24x + 72$. Factoring 2 from the right-hand side of this equation yields $0 = 2(x^2 - 12x + 36)$. Dividing both sides of this equation by 2 yields $0 = x^2 - 12x + 36$, which is equivalent to $0 = (x - 6)(x - 6)$, or $0 = (x - 6)^2$. Taking the square root of both sides of this equation yields $0 = x - 6$. Adding 6 to both sides of this equation yields $x = 6$.

Choice A is incorrect. This is the value of a, not x.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 68298043

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 68298043

$$y + k = x + 26$$
$$y - k = x^2 - 5x$$

In the given system of equations, k is a constant. The system has exactly one distinct real solution. What is the value of k?

ID: 68298043 Answer

Correct Answer: 17.5, 35/2

Rationale

The correct answer is $\frac{35}{2}$. Subtracting the second equation from the first equation yields $(y+k)-(y-k)=x+26-\left(x^2-5x\right)$, or $2k=-x^2+6x+26$. This is equivalent to $x^2-6x+(2k-26)=0$. It's given that the system has exactly one distinct real solution; therefore, this equation has exactly one distinct real solution. An equation of the form $ax^2+bx+c=0$, where a, b, and c are constants, has exactly one distinct real solution when the discriminant, b^2-4ac, is equal to 0. The equation $x^2-6x+\left(2k-26\right)=0$ is of this form, where $a=1$, $b=-6$, and $c=2k-26$. Substituting these values into the discriminant, b^2-4ac, yields $(-6)^2-4(1)(2k-26)$. Setting the discriminant equal to 0 yields $(-6)^2-4(1)(2k-26)=0$, or $-8k+140=0$. Subtracting 140 from both sides of this equation yields $-8k=-140$. Dividing both sides of this equation by -8 yields $k=\frac{35}{2}$. Note that 35/2 and 17.5 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 65244c8d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 65244c8d

$$\sqrt{(x-2)^2} = \sqrt{3x+34}$$

What is the smallest solution to the given equation?

ID: 65244c8d Answer

Correct Answer: -3

Rationale

The correct answer is -3. Squaring both sides of the given equation yields $(x-2)^2 = 3x+34$, which can be rewritten as $x^2 - 4x + 4 = 3x + 34$. Subtracting $3x$ and 34 from both sides of this equation yields $x^2 - 7x - 30 = 0$. This quadratic equation can be rewritten as $(x-10)(x+3) = 0$. According to the zero product property, $(x-10)(x+3)$ equals zero when either $x - 10 = 0$ or $x + 3 = 0$. Solving each of these equations for x yields $x = 10$ or $x = -3$. Therefore, the given equation has two solutions, 10 and -3. Of these two solutions, -3 is the smallest solution to the given equation.

Question Difficulty: Hard

Question ID 8217606b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 8217606b

$$64x^2 - (16a + 4b)x + ab = 0$$

In the given equation, a and b are positive constants. The sum of the solutions to the given equation is $k(4a + b)$, where k is a constant. What is the value of k?

ID: 8217606b Answer

Correct Answer: .0625, 1/16

Rationale

The correct answer is $\frac{1}{16}$. Let p and q represent the solutions to the given equation. Then, the given equation can be rewritten as $64(x - p)(x - q) = 0$, or $64x^2 - 64(p + q) + pq = 0$. Since this equation is equivalent to the given equation, it follows that $-(16a + 4b) = -64(p + q)$. Dividing both sides of this equation by -64 yields $\frac{16a+4b}{64} = p + q$, or $\frac{1}{16}(4a + b) = p + q$. Therefore, the sum of the solutions to the given equation, $p + q$, is equal to $\frac{1}{16}(4a + b)$. Since it's given that the sum of the solutions to the given equation is $k(4a + b)$, where k is a constant, it follows that $k = \frac{1}{16}$. Note that 1/16, .0625, 0.062, and 0.063 are examples of ways to enter a correct answer.

Alternate approach: The given equation can be rewritten as $64x^2 - 4(4a + b)x + ab = 0$, where a and b are positive constants. Dividing both sides of this equation by 4 yields $16x^2 - (4a + b)x + \frac{ab}{4} = 0$. The solutions for a quadratic equation in the form $Ax^2 + Bx + C = 0$, where A, B, and C are constants, can be calculated using the quadratic formula, $x = \frac{-B+\sqrt{B^2-4AC}}{2A}$ and $x = \frac{-B-\sqrt{B^2-4AC}}{2A}$. It follows that the sum of the solutions to a quadratic equation in the form $Ax^2 + Bx + C = 0$ is $\frac{-B+\sqrt{B^2-4AC}}{2A} + \frac{-B-\sqrt{B^2-4AC}}{2A}$, which can be rewritten as $\frac{-B+-B+\sqrt{B^2-4AC}-\sqrt{B^2-4AC}}{2A}$, which is equivalent to $\frac{-2B}{2A}$, or $-\frac{B}{A}$. In the equation $16x^2 - (4a + b)x + \frac{ab}{4} = 0$, $A = 16$, $B = -(4a + b)$, and $C = \frac{ab}{4}$. Substituting 16 for A and $-(4a + b)$ for B in $-\frac{B}{A}$ yields $-\frac{-(4a+b)}{16}$, which can be rewritten as $\frac{1}{16}(4a + b)$. Thus, the sum of the solutions to the given equation is $\frac{1}{16}(4a + b)$. Since it's given that the sum of the solutions to the given equation is $k(4a + b)$, where k is a constant, it follows that $k = \frac{1}{16}$.

Question Difficulty: Hard

Question ID 0e4cd7da

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 0e4cd7da

Which quadratic equation has no real solutions?

A. $x^2 + 14x - 49 = 0$

B. $x^2 - 14x + 49 = 0$

C. $5x^2 - 14x - 49 = 0$

D. $5x^2 - 14x + 49 = 0$

ID: 0e4cd7da Answer

Correct Answer: D

Rationale

Choice D is correct. The number of solutions to a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, can be determined by the value of the discriminant, $b^2 - 4ac$. If the value of the discriminant is greater than zero, then the quadratic equation has two distinct real solutions. If the value of the discriminant is equal to zero, then the quadratic equation has exactly one real solution. If the value of the discriminant is less than zero, then the quadratic equation has no real solutions. For the quadratic equation in choice D, $5x^2 - 14x + 49 = 0$, $a = 5$, $b = -14$, and $c = 49$. Substituting 5 for a, -14 for b, and 49 for c in $b^2 - 4ac$ yields $(-14)^2 - 4(5)(49)$, or -784. Since -784 is less than zero, it follows that the quadratic equation $5x^2 - 14x + 49 = 0$ has no real solutions.

Choice A is incorrect. The value of the discriminant for this quadratic equation is 392. Since 392 is greater than zero, it follows that this quadratic equation has two real solutions.

Choice B is incorrect. The value of the discriminant for this quadratic equation is 0. Since zero is equal to zero, it follows that this quadratic equation has exactly one real solution.

Choice C is incorrect. The value of the discriminant for this quadratic equation is $1{,}176$. Since $1{,}176$ is greater than zero, it follows that this quadratic equation has two real solutions.

Question Difficulty: Hard

Question ID 536832c0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 536832c0

In the xy-plane, a line with equation $2y = 4.5$ intersects a parabola at exactly one point. If the parabola has equation $y = -4x^2 + bx$, where b is a positive constant, what is the value of b?

ID: 536832c0 Answer

Correct Answer: 6

Rationale

The correct answer is 6. It's given that a line with equation $2y = 4.5$ intersects a parabola with equation $y = -4x^2 + bx$, where b is a positive constant, at exactly one point in the *xy*-plane. It follows that the system of equations consisting of $2y = 4.5$ and $y = -4x^2 + bx$ has exactly one solution. Dividing both sides of the equation of the line by 2 yields $y = 2.25$. Substituting 2.25 for y in the equation of the parabola yields $2.25 = -4x^2 + bx$. Adding $4x^2$ and subtracting bx from both sides of this equation yields $4x^2 - bx + 2.25 = 0$. A quadratic equation in the form of $ax^2 + bx + c = 0$, where a, b, and c are constants, has exactly one solution when the discriminant, $b^2 - 4ac$, is equal to zero. Substituting 4 for a and 2.25 for c in the expression $b^2 - 4ac$ and setting this expression equal to 0 yields $b^2 - 4(4)(2.25) = 0$, or $b^2 - 36 = 0$. Adding 36 to each side of this equation yields $b^2 = 36$. Taking the square root of each side of this equation yields $b = \pm 6$. It's given that b is positive, so the value of b is 6.

Question Difficulty: Hard

Question ID 6c28bdc9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 6c28bdc9

$$x(x+1)-56=4x(x-7)$$

What is the sum of the solutions to the given equation?

ID: 6c28bdc9 Answer

Correct Answer: 29/3, 9.666, 9.667

Rationale

The correct answer is $\frac{29}{3}$. Applying the distributive property to the left-hand side of the given equation, $x(x+1)-56$, yields x^2+x-56. Applying the distributive property to the right-hand side of the given equation, $4x(x-7)$, yields $4x^2-28x$. Thus, the equation becomes $x^2+x-56=4x^2-28x$. Combining like terms on the left- and right-hand sides of this equation yields $0=\left(4x^2-x^2\right)+(-28x-x)+56$, or $3x^2-29x+56=0$. For a quadratic equation in the form $ax^2+bx+c=0$, where a, b, and c are constants, the quadratic formula gives the solutions to the equation in the form $x=\frac{\left(-b\pm\sqrt{b^2-4ac}\right)}{2a}$. Substituting 3 for a, -29 for b, and 56 for c from the equation $3x^2-29x+56=0$ into the quadratic formula yields $x=\frac{\left(29\pm\sqrt{(-29)^2-4(3)(56)}\right)}{2(3)}$, or $x=\frac{29}{6}\pm\frac{13}{6}$. It follows that the solutions to the given equation are $\frac{29}{6}+\frac{13}{6}$ and $\frac{29}{6}-\frac{13}{6}$. Adding these two solutions gives the sum of the solutions: $\frac{29}{6}+\frac{13}{6}+\frac{29}{6}-\frac{13}{6}$, which is equivalent to $\frac{29}{6}+\frac{29}{6}$, or $\frac{29}{3}$. Note that 29/3, 9.666, and 9.667 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID c9d2651d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: c9d2651d

If $3x^2 - 18x - 15 = 0$, what is the value of $x^2 - 6x$?

ID: c9d2651d Answer

Correct Answer: 5

Rationale

The correct answer is 5. Dividing each side of the given equation by 3 yields $x^2 - 6x - 5 = 0$. Adding 5 to each side of this equation yields $x^2 - 6x = 5$. Therefore, if $3x^2 - 18x - 15 = 0$, the value of $x^2 - 6x$ is 5.

Question Difficulty: Hard

Question ID b40b491b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: b40b491b

$$\frac{14x}{7y} = 2\sqrt{w+19}$$

The given equation relates the distinct positive real numbers w, x, and y. Which equation correctly expresses w in terms of x and y?

A. $w = \sqrt{\frac{x}{y}} - 19$

B. $w = \sqrt{\frac{28x}{14y}} - 19$

C. $w =$ msup $- 19$

D. $w =$ msup $- 19$

ID: b40b491b Answer

Correct Answer: C

Rationale

Choice C is correct. Dividing each side of the given equation by 2 yields $\frac{14x}{14y} = \frac{2\sqrt{w+19}}{2}$, or $\frac{x}{y} = \sqrt{w+19}$. Because it's given that each of the variables is positive, squaring each side of this equation yields the equivalent equation $\left(\frac{x}{y}\right)^2 = w + 19$. Subtracting 19 from each side of this equation yields $\left(\frac{x}{y}\right)^2 - 19 = w$, or $w = \left(\frac{x}{y}\right)^2 - 19$.

Choice A is incorrect. This equation isn't equivalent to the given equation.

Choice B is incorrect. This equation isn't equivalent to the given equation.

Choice D is incorrect. This equation isn't equivalent to the given equation.

Question Difficulty: Hard

Question ID d9799723

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: d9799723

$$x^2 - 40x - 10 = 0$$

What is the sum of the solutions to the given equation?

A. 0

B. 5

C. 10

D. 40

ID: d9799723 Answer

Correct Answer: D

Rationale

Choice D is correct. Adding 10 to each side of the given equation yields $x^2 - 40x = 10$. To complete the square, adding $\left(\frac{40}{2}\right)^2$, or 20^2, to each side of this equation yields $x^2 - 40x + 20^2 = 10 + 20^2$, or $(x-20)^2 = 410$. Taking the square root of each side of this equation yields $x - 20 = \pm\sqrt{410}$. Adding 20 to each side of this equation yields $x = 20 \pm \sqrt{410}$. Therefore, the solutions to the given equation are $x = 20 + \sqrt{410}$ and $x = 20 - \sqrt{410}$. The sum of these solutions is $\left(20 + \sqrt{410}\right) + \left(20 - \sqrt{410}\right)$, or 40.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 9298a52e

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 9298a52e

$$x^2 + y + 7 = 7$$
$$20x + 100 - y = 0$$

The solution to the given system of equations is (x, y). What is the value of x?

ID: 9298a52e Answer

Correct Answer: -10

Rationale

The correct answer is -10. Adding y to both sides of the second equation in the given system yields $20x + 100 = y$. Substituting $20x + 100$ for y in the first equation in the given system yields $x^2 + 20x + 100 + 7 = 7$. Subtracting 7 from both sides of this equation yields $x^2 + 20x + 100 = 0$. Factoring the left-hand side of this equation yields $(x + 10)(x + 10) = 0$, or $(x + 10)^2 = 0$. Taking the square root of both sides of this equation yields $x + 10 = 0$. Subtracting 10 from both sides of this equation yields $x = -10$. Therefore, the value of x is -10.

Question Difficulty: Hard

Question ID 8e46ba71

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 8e46ba71

$$\sqrt{k-x}=58-x$$

In the given equation, k is a constant. The equation has exactly one real solution. What is the minimum possible value of $4k$?

ID: 8e46ba71 Answer

Correct Answer: 231

Rationale

The correct answer is 231. It's given that $\sqrt{k-x}=58-x$. Squaring both sides of this equation yields $k-x=(58-x)^2$, which is equivalent to the given equation if $58-x>0$. It follows that if a solution to the equation $k-x=(58-x)^2$ satisfies $58-x>0$, then it's also a solution to the given equation; if not, it's extraneous. The equation $k-x=(58-x)^2$ can be rewritten as $k-x=3{,}364-116x+x^2$. Adding x to both sides of this equation yields $k=x^2-115x+3{,}364$. Subtracting k from both sides of this equation yields $0=x^2-115x+\left(3{,}364-k\right)$. The number of solutions to a quadratic equation in the form $0=ax^2+bx+c$, where a, b, and c are constants, can be determined by the value of the discriminant, b^2-4ac. Substituting -115 for b, 1 for a, and $3{,}364-k$ for c in b^2-4ac yields $(-115)^2-4(1)(3{,}364-k)$, or $4k-231$. The equation $0=x^2-115x+\left(3{,}364-k\right)$ has exactly one real solution if the discriminant is equal to zero, or $4k-231=0$. Subtracting 231 from both sides of this equation yields $4k=231$. Dividing both sides of this equation by 4 yields $k=57.75$. Therefore, if $k=57.75$, then the equation $0=x^2-115x+\left(3{,}364-k\right)$ has exactly one real solution. Substituting 57.75 for k in this equation yields $0=x^2-115x+\left(3{,}364-57.75\right)$, or $0=x^2-115x+3{,}306.25$, which is equivalent to $0=(x-57.5)^2$. Taking the square root of both sides of this equation yields $0=x-57.5$. Adding 57.5 to both sides of this equation yields $57.5=x$. To check whether this solution satisfies $58-x>0$, the solution, 57.5, can be substituted for x in $58-x>0$, which yields $58-57.5>0$, or $0.5>0$. Since 0.5 is greater than 0, it follows that if $k=57.75$, or $4k=231$, then the given equation has exactly one real solution. If $4k<231$, then the discriminant, $4k-231$, is negative and the given equation has no solutions. Therefore, the minimum possible value of $4k$ is 231.

Question Difficulty: Hard

Question ID e1774551

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: e1774551

$$|-5x+13|=73$$

What is the sum of the solutions to the given equation?

A. $-\frac{146}{5}$

B. -12

C. 0

D. $\frac{26}{5}$

ID: e1774551 Answer

Correct Answer: D

Rationale

Choice D is correct. By the definition of absolute value, if $|-5x+13|=73$, then $-5x+13=73$ or $-5x+13=-73$. Subtracting 13 from both sides of the equation $-5x+13=73$ yields $-5x=60$. Dividing both sides of this equation by -5 yields $x=-12$. Subtracting 13 from both sides of the equation $-5x+13=-73$ yields $-5x=-86$. Dividing both sides of this equation by -5 yields $x=\frac{86}{5}$. Therefore, the solutions to the given equation are -12 and $\frac{86}{5}$, and it follows that the sum of the solutions to the given equation is $-12+\frac{86}{5}$, or $\frac{26}{5}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. This is a solution, not the sum of the solutions, to the given equation.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID cde831b3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: cde831b3

$$x^2 - 2x - 9 = 0$$

One solution to the given equation can be written as $1 + \sqrt{k}$, where k is a constant. What is the value of k?

A. 8

B. 10

C. 20

D. 40

ID: cde831b3 Answer

Correct Answer: B

Rationale

Choice B is correct. Adding 9 to each side of the given equation yields $x^2 - 2x = 9$. To complete the square, adding 1 to each side of this equation yields $x^2 - 2x + 1 = 9 + 1$, or $(x-1)^2 = 10$. Taking the square root of each side of this equation yields $x - 1 = \pm\sqrt{10}$. Adding 1 to each side of this equation yields $x = 1 \pm \sqrt{10}$. Since it's given that one of the solutions to the equation can be written as $1 + \sqrt{k}$, the value of k must be 10.

Alternate approach: It's given that $1 + \sqrt{k}$ is a solution to the given equation. It follows that $x = 1 + \sqrt{k}$. Substituting $1 + \sqrt{k}$ for x in the given equation yields $\left(1 + \sqrt{k}\right)^2 - 2\left(1 + \sqrt{k}\right) - 9 = 0$, or $\left(1 + \sqrt{k}\right)\left(1 + \sqrt{k}\right) - 2\left(1 + \sqrt{k}\right) - 9 = 0$. Expanding the products on the left-hand side of this equation yields $1 + 2\sqrt{k} + k - 2 - 2\sqrt{k} - 9 = 0$, or $k - 10 = 0$. Adding 10 to each side of this equation yields $k = 10$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 95b69a20

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 95b69a20

$$\frac{x^2}{\sqrt{x^2-c^2}}=\frac{c^2}{\sqrt{x^2-c^2}}+39$$

In the given equation, c is a positive constant. Which of the following is one of the solutions to the given equation?

A. $-c$

B. $-c^2-39^2$

C. $-\sqrt{39^2-c^2}$

D. $-\sqrt{c^2+39^2}$

ID: 95b69a20 Answer

Correct Answer: D

Rationale

Choice D is correct. If $x^2-c^2\leq 0$, then neither side of the given equation is defined and there can be no solution. Therefore, $x^2-c^2>0$. Subtracting $\frac{c^2}{\sqrt{x^2-c^2}}$ from both sides of the given equation yields $\frac{x^2}{\sqrt{x^2-c^2}}-\frac{c^2}{\sqrt{x^2-c^2}}=39$, or $\frac{x^2-c^2}{\sqrt{x^2-c^2}}=39$. Squaring both sides of this equation yields $\left(\frac{x^2-c^2}{\sqrt{x^2-c^2}}\right)^2=39^2$, or $\frac{\left(x^2-c^2\right)\left(x^2-c^2\right)}{x^2-c^2}=39^2$. Since x^2-c^2 is positive and, therefore, nonzero, the expression $\frac{x^2-c^2}{x^2-c^2}$ is defined and equivalent to 1. It follows that the equation $\frac{\left(x^2-c^2\right)\left(x^2-c^2\right)}{x^2-c^2}=39^2$ can be rewritten as $\left(\frac{x^2-c^2}{x^2-c^2}\right)\left(x^2-c^2\right)=39^2$, or $(1)\left(x^2-c^2\right)=39^2$, which is equivalent to $x^2-c^2=39^2$. Adding c^2 to both sides of this equation yields $x^2=c^2+39^2$. Taking the square root of both sides of this equation yields two solutions: $x=\sqrt{c^2+39^2}$ and $x=-\sqrt{c^2+39^2}$. Therefore, of the given choices, $-\sqrt{c^2+39^2}$ is one of the solutions to the given equation.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID b939a904

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: b939a904

$$64x^2 + bx + 25 = 0$$

In the given equation, b is a constant. For which of the following values of b will the equation have more than one real solution?

A. -91

B. -80

C. 5

D. 40

ID: b939a904 Answer

Correct Answer: A

Rationale

Choice A is correct. A quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has either no real solutions, exactly one real solution, or exactly two real solutions. That is, for the given equation to have more than one real solution, it must have exactly two real solutions. When the value of the discriminant, or $b^2 - 4ac$, is greater than 0, the given equation has exactly two real solutions. In the given equation, $64x^2 + bx + 25 = 0$, $a = 64$ and $c = 25$. Therefore, the given equation has exactly two real solutions when $(b)^2 - 4(64)(25) > 0$, or $b^2 - 6{,}400 > 0$. Adding $6{,}400$ to both sides of this inequality yields $b^2 > 6{,}400$. Taking the square root of both sides of $b^2 > 6{,}400$ yields two possible inequalities: $b < -80$ or $b > 80$. Of the choices, only choice A satisfies $b < -80$ or $b > 80$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 1844a2ab

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 1844a2ab

$$y = -2.5$$
$$y = x^2 + 8x + k$$

In the given system of equations, k is a positive integer constant. The system has no real solutions. What is the least possible value of k?

ID: 1844a2ab Answer

Correct Answer: 14

Rationale

The correct answer is 14. It's given by the first equation of the system of equations that $y = -2.5$. Substituting -2.5 for y in the second given equation, $y = x^2 + 8x + k$, yields $-2.5 = x^2 + 8x + k$. Adding 2.5 to both sides of this equation yields $0 = x^2 + 8x + k + 2.5$. A quadratic equation of the form $0 = ax^2 + bx + c$, where a, b, and c are constants, has no real solutions if and only if its discriminant, $b^2 - 4ac$, is negative. In the equation $0 = x^2 + 8x + k + 2.5$, where k is a positive integer constant, $a = 1$, $b = 8$, and $c = k + 2.5$. Substituting 1 for a, 8 for b, and $k + 2.5$ for c in $b^2 - 4ac$ yields $8^2 - 4(1)(k + 2.5)$, or $64 - 4(k + 2.5)$. Since this value must be negative, $64 - 4(k + 2.5) < 0$. Adding $4(k + 2.5)$ to both sides of this inequality yields $64 < 4(k + 2.5)$. Dividing both sides of this inequality by 4 yields $16 < k + 2.5$. Subtracting 2.5 from both sides of this inequality yields $13.5 < k$. Since k is a positive integer constant, the least possible value of k is 14.

Question Difficulty: Hard

Question ID 98a35f81

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 98a35f81

$$x(kx - 56) = -16$$

In the given equation, k is an integer constant. If the equation has no real solution, what is the least possible value of k?

ID: 98a35f81 Answer

Correct Answer: 50

Rationale

The correct answer is 50. An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has no real solutions if and only if its discriminant, $b^2 - 4ac$, is negative. Applying the distributive property to the left-hand side of the equation $x(kx - 56) = -16$ yields $kx^2 - 56x = -16$. Adding 16 to each side of this equation yields $kx^2 - 56x + 16 = 0$. Substituting k for a, -56 for b, and 16 for c in $b^2 - 4ac$ yields a discriminant of $(-56)^2 - 4(k)(16)$, or $3{,}136 - 64k$. If the given equation has no real solution, it follows that the value of $3{,}136 - 64k$ must be negative. Therefore, $3{,}136 - 64k < 0$. Adding $64k$ to both sides of this inequality yields $3{,}136 < 64k$. Dividing both sides of this inequality by 64 yields $49 < k$, or $k > 49$. Since it's given that k is an integer, the least possible value of k is 50.

Question Difficulty: Hard

Question ID 2d8f1f6a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 2d8f1f6a

$$8x + y = -11$$
$$2x^2 = y + 341$$

The graphs of the equations in the given system of equations intersect at the point (x, y) in the xy-plane. What is a possible value of x?

A. -15

B. -11

C. 2

D. 8

ID: 2d8f1f6a Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the graphs of the equations in the given system of equations intersect at the point (x, y). Therefore, this intersection point is a solution to the given system. The solution can be found by isolating y in each equation. The given equation $8x + y = -11$ can be rewritten to isolate y by subtracting $8x$ from both sides of the equation, which gives $y = -8x - 11$. The given equation $2x^2 = y + 341$ can be rewritten to isolate y by subtracting 341 from both sides of the equation, which gives $2x^2 - 341 = y$. With each equation solved for y, the value of y from one equation can be substituted into the other, which gives $2x^2 - 341 = -8x - 11$. Adding $8x$ and 11 to both sides of this equation results in $2x^2 + 8x - 330 = 0$. Dividing both sides of this equation by 2 results in $x^2 + 4x - 165 = 0$. This equation can be rewritten by factoring the left-hand side, which yields $(x + 15)(x - 11) = 0$. By the zero-product property, if $(x + 15)(x - 11) = 0$, then $(x + 15) = 0$, or $(x - 11) = 0$. It follows that $x = -15$, or $x = 11$. Since only -15 is given as a choice, a possible value of x is -15.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 962eb92e

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 962eb92e

$$\frac{12}{n} - \frac{2}{t} = -\frac{2}{w}$$

The given equation relates the variables n, t, and w, where $n > 0$, $t > 0$, and $w > t$. Which expression is equivalent to n?

A. $12tw$

B. $6(t - w)$

C. $\frac{w-t}{6tw}$

D. $\frac{6tw}{w-t}$

ID: 962eb92e Answer

Correct Answer: D

Rationale

Choice D is correct. Adding $\frac{2}{t}$ to each side of the given equation yields $\frac{12}{n} = -\frac{2}{w} + \frac{2}{t}$. The fractions on the right side of this equation have a common denominator of tw; therefore, the equation can be written as $\frac{12}{n} = \frac{2w}{tw} - \frac{2t}{tw}$, or $\frac{12}{n} = \frac{2w-2t}{tw}$, which is equivalent to $\frac{12}{n} = \frac{2(w-t)}{tw}$. Dividing each side of this equation by 2 yields $\frac{6}{n} = \frac{w-t}{tw}$. Since n, t, w, and $w - t$ are all positive quantities, taking the reciprocal of each side of the equation $\frac{6}{n} = \frac{w-t}{tw}$ yields an equivalent equation: $\frac{n}{6} = \frac{tw}{w-t}$. Multiplying each side of this equation by 6 yields $n = \frac{6tw}{w-t}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is equivalent to $\frac{1}{n}$ rather than n.

Question Difficulty: Hard

Question ID edcedac7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: edcedac7

The solutions to $x^2 + 6x + 7 = 0$ are r and s, where $r < s$. The solutions to $x^2 + 8x + 8 = 0$ are t and u, where $t < u$. The solutions to $x^2 + 14x + c = 0$, where c is a constant, are $r + t$ and $s + u$. What is the value of c?

ID: edcedac7 Answer

Correct Answer: 31

Rationale

The correct answer is 31. Subtracting 7 from both sides of the equation $x^2 + 6x + 7 = 0$ yields $x^2 + 6x = -7$. To complete the square, adding $\left(\frac{6}{2}\right)^2$, or 3^2, to both sides of this equation yields $x^2 + 6x + 3^2 = -7 + 3^2$, or $(x + 3)^2 = 2$. Taking the square root of both sides of this equation yields $x + 3 = \pm\sqrt{2}$. Subtracting 3 from both sides of this equation yields $x = -3 \pm \sqrt{2}$. Therefore, the solutions r and s to the equation $x^2 + 6x + 7 = 0$ are $-3 - \sqrt{2}$ and $-3 + \sqrt{2}$. Since $r < s$, it follows that $r = -3 - \sqrt{2}$ and $s = -3 + \sqrt{2}$. Subtracting 8 from both sides of the equation $x^2 + 8x + 8 = 0$ yields $x^2 + 8x = -8$. To complete the square, adding $\left(\frac{8}{2}\right)^2$, or 4^2, to both sides of this equation yields $x^2 + 8x + 4^2 = -8 + 4^2$, or $(x + 4)^2 = 8$. Taking the square root of both sides of this equation yields $x + 4 = \pm\sqrt{8}$, or $x + 4 = \pm 2\sqrt{2}$. Subtracting 4 from both sides of this equation yields $x = -4 \pm 2\sqrt{2}$. Therefore, the solutions t and u to the equation $x^2 + 8x + 8 = 0$ are $-4 - 2\sqrt{2}$ and $-4 + 2\sqrt{2}$. Since $t < u$, it follows that $t = -4 - 2\sqrt{2}$ and $u = -4 + 2\sqrt{2}$. It's given that the solutions to $x^2 + 14x + c = 0$, where c is a constant, are $r + t$ and $s + u$. It follows that this equation can be written as $(x - (r + t))(x - (s + u)) = 0$, which is equivalent to $x^2 - (r + t + s + u)x + (r + t)(s + u) = 0$. Therefore, the value of c is $(r + t)(s + u)$. Substituting $-3 - \sqrt{2}$ for r, $-4 - 2\sqrt{2}$ for t, $-3 + \sqrt{2}$ for s, and $-4 + 2\sqrt{2}$ for u in this equation yields $\left(\left(-3 - \sqrt{2}\right) + \left(-4 - 2\sqrt{2}\right)\right)\left(\left(-3 + \sqrt{2}\right) + \left(-4 + 2\sqrt{2}\right)\right)$, which is equivalent to $\left(-7 - 3\sqrt{2}\right)\left(-7 + 3\sqrt{2}\right)$, or $(-7)(-7) - \left(3\sqrt{2}\right)\left(3\sqrt{2}\right)$, which is equivalent to $49 - 18$, or 31. Therefore, the value of c is 31.

Question Difficulty: Hard

Question ID 14fe10e5

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 14fe10e5

$$|x-9|+45=63$$

What is the sum of the solutions to the given equation?

ID: 14fe10e5 Answer

Correct Answer: 18

Rationale

The correct answer is 18. Subtracting 45 from each side of the given equation yields $|x-9|=18$. By the definition of absolute value, if $|x-9|=18$, then $x-9=18$ or $x-9=-18$. Adding 9 to each side of the equation $x-9=18$ yields $x=27$. Adding 9 to each side of the equation $x-9=-18$ yields $x=-9$. Therefore, the solutions to the given equation are 27 and -9, and it follows that the sum of the solutions to the given equation is $27+(-9)$, or 18.

Question Difficulty: Hard

Question ID 960aabc0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 960aabc0

In the *xy*-plane, a line with equation $2y = c$ for some constant c intersects a parabola at exactly one point. If the parabola has equation $y = -2x^2 + 9x$, what is the value of c?

ID: 960aabc0 Answer

Correct Answer: 20.25, 81/4

Rationale

The correct answer is $\frac{81}{4}$. The given linear equation is $2y = c$. Dividing both sides of this equation by 2 yields $y = \frac{c}{2}$. Substituting $\frac{c}{2}$ for y in the equation of the parabola yields $\frac{c}{2} = -2x^2 + 9x$. Adding $2x^2$ and $-9x$ to both sides of this equation yields $2x^2 - 9x + \frac{c}{2} = 0$. Since it's given that the line and the parabola intersect at exactly one point, the equation $2x^2 - 9x + \frac{c}{2} = 0$ must have exactly one solution. An equation of the form $Ax^2 + Bx + C = 0$, where A, B, and C are constants, has exactly one solution when the discriminant, $B^2 - 4AC$, is equal to 0. In the equation $2x^2 - 9x + \frac{c}{2} = 0$, where $A = 2$, $B = -9$, and $C = \frac{c}{2}$, the discriminant is $(-9)^2 - 4(2)\left(\frac{c}{2}\right)$. Setting the discriminant equal to 0 yields $(-9)^2 - 4(2)\left(\frac{c}{2}\right) = 0$, or $81 - 4c = 0$. Adding $4c$ to both sides of this equation yields $81 = 4c$. Dividing both sides of this equation by 4 yields $c = \frac{81}{4}$. Note that 81/4 and 20.25 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 2b7d8635

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 2b7d8635

$$y = -1.50$$
$$y = x^2 + 8x + a$$

In the given system of equations, a is a positive constant. The system has exactly one distinct real solution. What is the value of a?

ID: 2b7d8635 Answer

Correct Answer: 14.5, 29/2

Rationale

The correct answer is $\frac{29}{2}$. According to the first equation in the given system, the value of y is -1.5. Substituting -1.5 for y in the second equation in the given system yields $-1.5 = x^2 + 8x + a$. Adding 1.5 to both sides of this equation yields $0 = x^2 + 8x + a + 1.5$. If the given system has exactly one distinct real solution, it follows that $0 = x^2 + 8x + a + 1.5$ has exactly one distinct real solution. A quadratic equation in the form $0 = px^2 + qx + r$, where p, q, and r are constants, has exactly one distinct real solution if and only if the discriminant, $q^2 - 4pr$, is equal to 0. The equation $0 = x^2 + 8x + a + 1.5$ is in this form, where $p = 1$, $q = 8$, and $r = a + 1.5$. Therefore, the discriminant of the equation $0 = x^2 + 8x + a + 1.5$ is $(8)^2 - 4(1)(a + 1.5)$, or $58 - 4a$. Setting the discriminant equal to 0 to solve for a yields $58 - 4a = 0$. Adding $4a$ to both sides of this equation yields $58 = 4a$. Dividing both sides of this equation by 4 yields $\frac{58}{4} = a$, or $\frac{29}{2} = a$. Therefore, if the given system of equations has exactly one distinct real solution, the value of a is $\frac{29}{2}$. Note that 29/2 and 14.5 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 59cf1dd3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 59cf1dd3

$$(x-1)^2 = -4$$

How many distinct real solutions does the given equation have?

A. Exactly one

B. Exactly two

C. Infinitely many

D. Zero

ID: 59cf1dd3 Answer

Correct Answer: D

Rationale

Choice D is correct. Any quantity that is positive or negative in value has a positive value when squared. Therefore, the left-hand side of the given equation is either positive or zero for any value of x. Since the right-hand side of the given equation is negative, there is no value of x for which the given equation is true. Thus, the number of distinct real solutions for the given equation is zero.

Choices A, B, and C are incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 33cc7555

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 33cc7555

$$2|4-x|+3|4-x|=25$$

What is the positive solution to the given equation?

ID: 33cc7555 Answer

Correct Answer: 9

Rationale

The correct answer is 9. The given equation can be rewritten as $5|4-x|=25$. Dividing each side of this equation by 5 yields $|4-x|=5$. By the definition of absolute value, if $|4-x|=5$, then $4-x=5$ or $4-x=-5$. Subtracting 4 from each side of the equation $4-x=5$ yields $-x=1$. Dividing each side of this equation by -1 yields $x=-1$. Similarly, subtracting 4 from each side of the equation $4-x=-5$ yields $-x=-9$. Dividing each side of this equation by -1 yields $x=9$. Therefore, since the two solutions to the given equation are -1 and 9, the positive solution to the given equation is 9.

Question Difficulty: Hard

Question ID a4b12e2f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: a4b12e2f

$$-9x^2 + 30x + c = 0$$

In the given equation, c is a constant. The equation has exactly one solution. What is the value of c?

A. 3

B. 0

C. -25

D. -53

ID: a4b12e2f Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the equation $-9x^2 + 30x + c = 0$ has exactly one solution. A quadratic equation of the form $ax^2 + bx + c = 0$ has exactly one solution if and only if its discriminant, $-4ac + b^2$, is equal to zero. It follows that for the given equation, $a = -9$ and $b = 30$. Substituting -9 for a and 30 for b into $b^2 - 4ac$ yields $30^2 - 4(-9)(c)$, or $900 + 36c$. Since the discriminant must equal zero, $900 + 36c = 0$. Subtracting $36c$ from both sides of this equation yields $900 = -36c$. Dividing each side of this equation by -36 yields $-25 = c$. Therefore, the value of c is -25.

Choice A is incorrect. If the value of c is 3, this would yield a discriminant that is greater than zero. Therefore, the given equation would have two solutions, rather than exactly one solution.

Choice B is incorrect. If the value of c is 0, this would yield a discriminant that is greater than zero. Therefore, the given equation would have two solutions, rather than exactly one solution.

Choice D is incorrect. If the value of c is -53, this would yield a discriminant that is less than zero. Therefore, the given equation would have no real solutions, rather than exactly one solution.

Question Difficulty: Hard

Question ID 9f13fad1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 9f13fad1

$$-16x^2 - 8x + c = 0$$

In the given equation, c is a constant. The equation has exactly one solution. What is the value of c?

ID: 9f13fad1 Answer

Correct Answer: -1

Rationale

The correct answer is -1. A quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has exactly one solution when its discriminant, $b^2 - 4ac$, is equal to 0. In the given equation, $-16x^2 - 8x + c = 0$, $a = -16$ and $b = -8$. Substituting -16 for a and -8 for b in $b^2 - 4ac$ yields $(-8)^2 - 4(-16)(c)$, or $64 + 64c$. Since the given equation has exactly one solution, $64 + 64c = 0$. Subtracting 64 from both sides of this equation yields $64c = -64$. Dividing both sides of this equation by 64 yields $c = -1$. Therefore, the value of c is -1.

Question Difficulty: Hard

Question ID b2e26a55

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: b2e26a55

In the *xy*-plane, the graph of the equation $y = -x^2 + 9x - 100$ intersects the line $y = c$ at exactly one point. What is the value of c?

A. $-\frac{481}{4}$

B. -100

C. $-\frac{319}{4}$

D. $-\frac{9}{2}$

ID: b2e26a55 Answer

Correct Answer: C

Rationale

Choice C is correct. In the *xy*-plane, the graph of the line $y = c$ is a horizontal line that crosses the *y*-axis at $y = c$ and the graph of the quadratic equation $y = -x^2 + 9x - 100$ is a parabola. A parabola can intersect a horizontal line at exactly one point only at its vertex. Therefore, the value of c should be equal to the *y*-coordinate of the vertex of the graph of the given equation. For a quadratic equation in vertex form, $y = a(x-h)^2 + k$, the vertex of its graph in the *xy*-plane is (h, k). The given quadratic equation, $y = -x^2 + 9x - 100$, can be rewritten as $y = -\left(x^2 - 2\left(\frac{9}{2}\right)x + \left(\frac{9}{2}\right)^2\right) + \left(\frac{9}{2}\right)^2 - 100$, or $y = -\left(x - \frac{9}{2}\right)^2 + \left(-\frac{319}{4}\right)$. Thus, the value of c is equal to $-\frac{319}{4}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 032caee7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 032caee7

$$x^2 - 34x + c = 0$$

In the given equation, c is a constant. The equation has no real solutions if $c > n$. What is the least possible value of n?

ID: 032caee7 Answer

Correct Answer: 289

Rationale

The correct answer is 289. A quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has no real solutions when the value of the discriminant, $b^2 - 4ac$, is less than 0. In the given equation, $x^2 - 34x + c = 0$, $a = 1$ and $b = -34$. Therefore, the discriminant of the given equation can be expressed as $(-34)^2 - 4(1)(c)$, or $1{,}156 - 4c$. It follows that the given equation has no real solutions when $1{,}156 - 4c < 0$. Adding $4c$ to both sides of this inequality yields $1{,}156 < 4c$. Dividing both sides of this inequality by 4 yields $289 < c$, or $c > 289$. It's given that the equation $x^2 - 34x + c = 0$ has no real solutions when $c > n$. Therefore, the least possible value of n is 289.

Question Difficulty: Hard

Question ID 54fecb11

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 54fecb11

$$2x^2 - 8x - 7 = 0$$

One solution to the given equation can be written as $\frac{8-\sqrt{k}}{4}$, where k is a constant. What is the value of k?

ID: 54fecb11 Answer

Correct Answer: 120

Rationale

The correct answer is 120. The solutions to a quadratic equation of the form $ax^2 + bx + c = 0$ can be calculated using the quadratic formula and are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The given equation is in the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -8$, and $c = -7$. It follows that the solutions to the given equation are $x = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(-7)}}{2(2)}$, which is equivalent to $x = \frac{8 \pm \sqrt{64+56}}{4}$, or $x = \frac{8 \pm \sqrt{120}}{4}$. It's given that one solution to the equation $2x^2 - 8x - 7 = 0$ can be written as $\frac{8-\sqrt{k}}{4}$. The solution $\frac{8-\sqrt{120}}{4}$ is in this form. Therefore, the value of k is 120.

Question Difficulty: Hard

Question ID 14787dca

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 14787dca

$$x - 29 = (x - a)(x - 29)$$

Which of the following are solutions to the given equation, where a is a constant and $a > 30$?

I. a

II. $a + 1$

III. 29

A. I and II only

B. I and III only

C. II and III only

D. I, II, and III

ID: 14787dca Answer

Correct Answer: C

Rationale

Choice C is correct. Subtracting the expression $(x - 29)$ from both sides of the given equation yields $0 = (x - a)(x - 29) - (x - 29)$, which can be rewritten as $0 = (x - a)(x - 29) + (-1)(x - 29)$. Since the two terms on the right-hand side of this equation have a common factor of $(x - 29)$, it can be rewritten as $0 = (x - 29)(x - a + (-1))$, or $0 = (x - 29)(x - a - 1)$. Since $x - a - 1$ is equivalent to $x - (a + 1)$, the equation $0 = (x - 29)(x - a - 1)$ can be rewritten as $0 = (x - 29)(x - (a + 1))$. By the zero product property, it follows that $x - 29 = 0$ or $x - (a + 1) = 0$. Adding 29 to both sides of the equation $x - 29 = 0$ yields $x = 29$. Adding $a + 1$ to both sides of the equation $x - (a + 1) = 0$ yields $x = a + 1$. Therefore, the two solutions to the given equation are 29 and $a + 1$. Thus, only $a + 1$ and 29, not a, are solutions to the given equation.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 409b7ab8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 409b7ab8

$$y = 18$$
$$y = -3(x-18)^2 + 15$$

If the given equations are graphed in the *xy*-plane, at how many points do the graphs of the equations intersect?

A. Exactly one

B. Exactly two

C. Infinitely many

D. Zero

ID: 409b7ab8 Answer

Correct Answer: D

Rationale

Choice D is correct. A point (x, y) is a solution to a system of equations if it lies on the graphs of both equations in the *xy*-plane. In other words, a solution to a system of equations is a point (x, y) at which the graphs intersect. It's given that the first equation is $y = 18$. Substituting 18 for y in the second equation yields $18 = -3(x-18)^2 + 15$. Subtracting 15 from each side of this equation yields $3 = -3(x-18)^2$. Dividing each side of this equation by -3 yields $-1 = (x-18)^2$. Since the square of a real number is at least 0, this equation can't have any real solutions. Therefore, the graphs of the equations intersect at zero points.

Alternate approach: The graph of the second equation is a parabola that opens downward and has a vertex at $(18, 15)$. Therefore, the maximum value of this parabola occurs when $y = 15$. The graph of the first equation is a horizontal line at 18 on the *y*-axis, or $y = 18$. Since 18 is greater than 15, or the horizontal line is above the vertex of the parabola, the graphs of these equations intersect at zero points.

Choice A is incorrect. The graph of $y = 15$, not $y = 18$, and the graph of the second equation intersect at exactly one point.

Choice B is incorrect. The graph of any horizontal line such that the value of y is less than 15, not greater than 15, and the graph of the second equation intersect at exactly two points.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID e7f2ab9c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: e7f2ab9c

$$57x^2 + (57b + a)x + ab = 0$$

In the given equation, a and b are positive constants. The product of the solutions to the given equation is kab, where k is a constant. What is the value of k?

A. $\frac{1}{57}$

B. $\frac{1}{19}$

C. 1

D. 57

ID: e7f2ab9c Answer

Correct Answer: A

Rationale

Choice A is correct. The left-hand side of the given equation is the expression $57x^2 + (57b + a)x + ab$. Applying the distributive property to this expression yields $57x^2 + 57bx + ax + ab$. Since the first two terms of this expression have a common factor of $57x$ and the last two terms of this expression have a common factor of a, this expression can be rewritten as $57x(x + b) + a(x + b)$. Since the two terms of this expression have a common factor of $(x + b)$, it can be rewritten as $(x + b)(57x + a)$. Therefore, the given equation can be rewritten as $(x + b)(57x + a) = 0$. By the zero product property, it follows that $x + b = 0$ or $57x + a = 0$. Subtracting b from both sides of the equation $x + b = 0$ yields $x = -b$. Subtracting a from both sides of the equation $57x + a = 0$ yields $57x = -a$. Dividing both sides of this equation by 57 yields $x = \frac{-a}{57}$. Therefore, the solutions to the given equation are $-b$ and $\frac{-a}{57}$. It follows that the product of the solutions of the given equation is $(-b)\left(\frac{-a}{57}\right)$, or $\frac{ab}{57}$. It's given that the product of the solutions of the given equation is kab. It follows that $\frac{ab}{57} = kab$, which can also be written as $ab\left(\frac{1}{57}\right) = ab(k)$. It's given that a and b are positive constants. Therefore, dividing both sides of the equation $ab\left(\frac{1}{57}\right) = ab(k)$ by ab yields $\frac{1}{57} = k$. Thus, the value of k is $\frac{1}{57}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2aaaec85

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 2aaaec85

$$-x^2 + bx - 676 = 0$$

In the given equation, b is a positive integer. The equation has no real solution. What is the greatest possible value of b?

ID: 2aaaec85 Answer

Correct Answer: 51

Rationale

The correct answer is 51. A quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has no real solution if and only if its discriminant, $-4ac + b^2$, is negative. In the given equation, $a = -1$ and $c = -676$. Substituting -1 for a and -676 for c in this expression yields a discriminant of $b^2 - 4(-1)(-676)$, or $b^2 - 2{,}704$. Since this value must be negative, $b^2 - 2{,}704 < 0$, or $b^2 < 2{,}704$. Taking the positive square root of each side of this inequality yields $b < 52$. Since b is a positive integer, and the greatest integer less than 52 is 51, the greatest possible value of b is 51.

Question Difficulty: Hard

Question ID 36ca6037

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 36ca6037

$$\frac{20}{p}=\frac{20}{q}-\frac{20}{r}-\frac{20}{s}$$

The given equation relates the positive variables p, q, r, and s. Which of the following is equivalent to q?

A. $p+r+s$

B. $20(p+r+s)$

C. $\frac{prs}{pr+ps+rs}$

D. $\frac{prs}{20p+20r+20s}$

ID: 36ca6037 Answer

Correct Answer: C

Rationale

Choice C is correct. Multiplying each side of the given equation by $\frac{1}{20}$ yields $\frac{1}{20}\left(\frac{20}{p}\right)=\frac{1}{20}\left(\frac{20}{q}-\frac{20}{r}-\frac{20}{s}\right)$. Distributing $\frac{1}{20}$ on each side of this equation yields $\frac{20}{20p}=\frac{20}{20q}-\frac{20}{20r}-\frac{20}{20s}$, or $\frac{1}{p}=\frac{1}{q}-\frac{1}{r}-\frac{1}{s}$. Adding $\frac{1}{r}+\frac{1}{s}$ to each side of this equation yields $\frac{1}{s}+\frac{1}{r}+\frac{1}{p}=\frac{1}{q}$. Multiplying $\frac{1}{s}$ by $\frac{pr}{pr}$, $\frac{1}{r}$ by $\frac{ps}{ps}$, and $\frac{1}{p}$ by $\frac{rs}{rs}$ yields $\frac{pr}{prs}+\frac{ps}{prs}+\frac{rs}{prs}=\frac{1}{q}$, which is equivalent to $\frac{pr+ps+rs}{prs}=\frac{1}{q}$. Since $\frac{pr+ps+rs}{prs}=\frac{1}{q}$, and it's given that p, q, r, and s are positive, it follows that the reciprocals of each side of this equation are also equal. Thus, $\frac{prs}{pr+ps+rs}=\frac{q}{1}$, or $\frac{prs}{pr+ps+rs}=q$. Therefore, $\frac{prs}{pr+ps+rs}$ is equivalent to q.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 1aec2be9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 1aec2be9

$$y = x + 9$$
$$y = x^2 + 16x + 63$$

A solution to the given system of equations is (x, y). What is the greatest possible value of x?

A. -6

B. 7

C. 9

D. 63

ID: 1aec2be9 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $y = x + 9$ and $y = x^2 + 16x + 63$; therefore, it follows that $x + 9 = x^2 + 16x + 63$. This equation can be rewritten as $x + 9 = (x + 9)(x + 7)$. Subtracting $(x + 9)$ from both sides of this equation yields $0 = (x + 9)(x + 7) - (x + 9)$. This equation can be rewritten as $0 = (x + 9)((x + 7) - 1)$, or $0 = (x + 9)(x + 6)$. By the zero product property, $x + 9 = 0$ or $x + 6 = 0$. Subtracting 9 from both sides of the equation $x + 9 = 0$ yields $x = -9$. Subtracting 6 from both sides of the equation $x + 6 = 0$ yields $x = -6$. Therefore, the given system of equations has solutions, (x, y), that occur when $x = -9$ and $x = -6$. Since -6 is greater than -9, the greatest possible value of x is -6.

Choice B is incorrect. This is the negative of the greatest possible value of x when $y = 0$ for the second equation in the given system of equations.

Choice C is incorrect. This is the value of y when $x = 0$ for the first equation in the given system of equations.

Choice D is incorrect. This is the value of y when $x = 0$ for the second equation in the given system of equations.

Question Difficulty: Hard

Question ID 9a182495

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: 9a182495

$$5x^2 + 10x + 16 = 0$$

How many distinct real solutions does the given equation have?

A. Exactly one

B. Exactly two

C. Infinitely many

D. Zero

ID: 9a182495 Answer

Correct Answer: D

Rationale

Choice D is correct. The number of solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, can be determined by the value of the discriminant, $b^2 - 4ac$. If the value of the discriminant is positive, then the quadratic equation has exactly two distinct real solutions. If the value of the discriminant is equal to zero, then the quadratic equation has exactly one real solution. If the value of the discriminant is negative, then the quadratic equation has zero real solutions. In the given equation, $5x^2 + 10x + 16 = 0$, $a = 5$, $b = 10$, and $c = 16$. Substituting these values for a, b, and c in $b^2 - 4ac$ yields $(10)^2 - 4(5)(16)$, or -220. Since the value of its discriminant is negative, the given equation has zero real solutions. Therefore, the number of distinct real solutions the given equation has is zero.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID dba8a697

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	Hard

ID: dba8a697

$$5(x+7) = 15(x-17)(x+7)$$

What is the sum of the solutions to the given equation?

ID: dba8a697 Answer

Correct Answer: 10.33, 31/3

Rationale

The correct answer is $\frac{31}{3}$. Subtracting $5(x+7)$ from each side of the given equation yields $0 = 15(x-17)(x+7) - 5(x+7)$. Since $5(x+7)$ is a common factor of each of the terms on the right-hand side of this equation, it can be rewritten as $0 = 5(x+7)(3(x-17)-1)$. This is equivalent to $0 = 5(x+7)(3x-51-1)$, or $0 = 5(x+7)(3x-52)$. Dividing both sides of this equation by 5 yields $0 = (x+7)(3x-52)$. Since a product of two factors is equal to 0 if and only if at least one of the factors is 0, either $x+7=0$ or $3x-52=0$. Subtracting 7 from both sides of the equation $x+7=0$ yields $x=-7$. Adding 52 to both sides of the equation $3x-52=0$ yields $3x=52$. Dividing both sides of this equation by 3 yields $x=\frac{52}{3}$. Therefore, the solutions to the given equation are -7 and $\frac{52}{3}$. It follows that the sum of the solutions to the given equation is $-7+\frac{52}{3}$, which is equivalent to $-\frac{21}{3}+\frac{52}{3}$, or $\frac{31}{3}$. Note that 31/3 and 10.33 are examples of ways to enter a correct answer.

Question Difficulty: Hard