Problem Set - 30 Nov 2023

PROBLEM 1 (2015 AMC 10A #15)

Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ \text{infinitely many}$

PROBLEM 2 (2019 AMC 10B #19)

Let S be the set of all positive integer divisors of $100,000$. How many numbers are the product of two distinct elements of S?

$\textbf{(A)}\ 98 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 117 \qquad \textbf{(D)}\ 119 \qquad \textbf{(E)}\ 121$

PROBLEM 3 (2021 AMC 12A #16)

In the following list of numbers, the integer n appears n times in the list for $1 \leq n \leq 200$.

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots, 200, 200, \ldots, 200$$

What is the median of the numbers in this list?

$\textbf{(A)}\ 100.5 \qquad \textbf{(B)}\ 134 \qquad \textbf{(C)}\ 142 \qquad \textbf{(D)}\ 150.5 \qquad \textbf{(E)}\ 167$

PROBLEM 4 (2020 AMC 12B #17)

How many polynomials of the form $x^5 + ax^4 + bx^3 + cx^2 + dx + 2020$, where a, b, c, and d are real numbers, have the property that whenever r is a root, so is $\frac{-1+i\sqrt{3}}{2} \cdot r$? (Note that $i=\sqrt{-1}$)

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

PROBLEM 5 (2016 AMC 12A #17)

Let $ABCD$ be a square. Let E, F, G and H be the centers, respectively, of equilateral triangles with bases $\overline{AB}, \overline{BC}, \overline{CD},$ and $\overline{DA}$, each exterior to the square. What is the ratio of the area of square $EFGH$ to the area of square $ABCD$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{2+\sqrt{3}}{3} \qquad \textbf{(C)}\ \sqrt{2} \qquad \textbf{(D)}\ \frac{\sqrt{2}+\sqrt{3}}{2} \qquad \textbf{(E)}\ \sqrt{3}$

PROBLEM 6 (2017 AMC 12A #18)

Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 1239 \qquad \textbf{(E)}\ 1265$

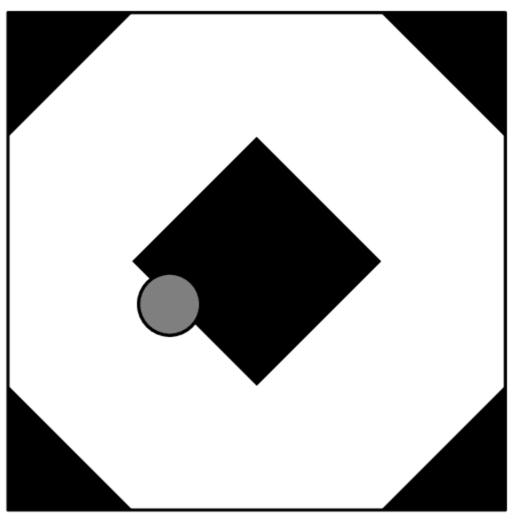
PROBLEM 7 (2013 AMC 10A #21)

A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

$\textbf{(A)}\ 720 \qquad \textbf{(B)}\ 1296 \qquad \textbf{(C)}\ 1728 \qquad \textbf{(D)}\ 1925 \qquad \textbf{(E)}\ 3850$

PROBLEM 8 (2021 AMC 10B #23)

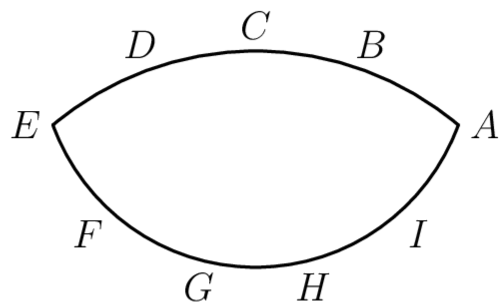
A square with side length 8 is colored white except for 4 black isosceles right triangular regions with legs of length 2 in each corner of the square and a black diamond with side length $2\sqrt{2}$ in the center of the square, as shown in the diagram. A circular coin with diameter 1 is dropped onto the square and lands in a random location where the coin is completely contained within the square. The probability that the coin will cover part of the black region of the square can be written as $\frac{1}{196}\left(a+b\sqrt{2}+\pi\right)$, where a and b are positive integers. What is $a+b$?



$\textbf{(A)}\ 64 \qquad \textbf{(B)}\ 66 \qquad \textbf{(C)}\ 68 \qquad \textbf{(D)}\ 70 \qquad \textbf{(E)}\ 72$

PROBLEM 9 (2015 AIME I #6)

Point A, B, C, D, and E are equally spaced on a minor arc of a circle. Points E, F, G, H, I and A are equally spaced on a minor arc of a second circle with center C as shown in the figure below. The angle $\angle ABD$ exceeds $\angle AHG$ by 12°. Find the degree measure of $\angle BAG$.



PROBLEM 10 (2017 AMC 12A #23)

For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

$\textbf{(A)}\ -9009 \qquad \textbf{(B)}\ -8008 \qquad \textbf{(C)}\ -7007 \qquad \textbf{(D)}\ -6006 \qquad \textbf{(E)}\ -5005$