# Problem Set - 14 Dec 2023

## PROBLEM 1 (2015 AMC 10A #15)

Consider the set of all fractions $\frac{x}{y}$, where $x$ and $y$ are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ \text{infinitely many}$

## PROBLEM 2 (2018 AMC 10A #18)

How many nonnegative integers can be written in the form

$$a_7\cdot3^7+a_6\cdot3^6+a_5\cdot3^5+a_4\cdot3^4+a_3\cdot3^3+a_2\cdot3^2+a_1\cdot3^1+a_0\cdot3^0,$$

where $a_i\in \{-1,0,1\}$ for $0\le i \le 7$?

$\textbf{(A)}\ 512 \qquad \textbf{(B)}\ 729 \qquad \textbf{(C)}\ 1094 \qquad \textbf{(D)}\ 3281 \qquad \textbf{(E)}\ 59,048$

## PROBLEM 3 (2016 AMC 10A #20)

For some particular value of $N$, when $(a+b+c+d+1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables $a, b$, $c$, and $d$, each to some positive power. What is $N$?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 19$

## PROBLEM 4 (2012 AMC 12B #18)

Let $(a_1,a_2,\dots,a_{10})$ be a list of the first 10 positive integers such that for each $2\le i\le 10$ either $a_{i}+1$ or $a_i-1$ or both appear somewhere before $a_i$ in the list. How many such lists are there?

$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 512 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 181,440 \qquad \textbf{(E)}\ 362,880$

## PROBLEM 5 (2017 AMC 12A #20)

How many ordered pairs $(a,b)$ such that $a$ is a positive real number and $b$ is an integer between 2 and 200, inclusive, satisfy the equation $(\log_b a)^{2017}=\log_b(a^{2017})$?

$\textbf{(A)}\ 198 \qquad \textbf{(B)}\ 199 \qquad \textbf{(C)}\ 398 \qquad \textbf{(D)}\ 399 \qquad \textbf{(E)}\ 597$

## PROBLEM 6 (2018 AMC 10A #21)

Which of the following describes the set of values of $a$ for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real $xy$-plane intersect at exactly 3 points?

$\textbf{(A)}\ a = \frac{1}{4} \qquad \textbf{(B)}\ \frac{1}{4} < a < \frac{1}{2} \qquad \textbf{(C)}\ a > \frac{1}{4} \qquad \textbf{(D)}\ a = \frac{1}{2} \qquad \textbf{(E)}\ a > \frac{1}{2}$

## PROBLEM 7 (2019 AIME II #6)

In a Martian civilization, all logarithms whose bases are not specified are assumed to be base $b$, for some fixed $b \geq 2$. A Martian student writes down

$$3\log(\sqrt{x}\log x) = 56$$

$$\log_{\log x}(x) = 54$$

and finds that this system of equations has a single real number solution $x > 1$. Find $b$.

## PROBLEM 8 (2012 AMC 10A #24)

Let $a$, $b$, and $c$ be positive integers with $a \geq b \geq c$ such that $a^2 - b^2 - c^2 + ab = 2011$ and $a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997$.

What is $a$?

$\textbf{(A)}\ 249 \qquad \textbf{(B)}\ 250 \qquad \textbf{(C)}\ 251 \qquad \textbf{(D)}\ 252 \qquad \textbf{(E)}\ 253$

## PROBLEM 9 (2017 AIME II #7)

Find the number of integer values of $k$ in the closed interval $[-500, 500]$ for which the equation $\log(kx) = 2\log(x+2)$ has exactly one real solution.

## PROBLEM 10 (2012 AMC 10A #24)

Let $a$, $b$, and $c$ be positive integers with $a \geq b \geq c$ such that $a^2 - b^2 - c^2 + ab = 2011$ and $a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997$.

What is $a$?

$\textbf{(A)}\ 249 \qquad \textbf{(B)}\ 250 \qquad \textbf{(C)}\ 251 \qquad \textbf{(D)}\ 252 \qquad \textbf{(E)}\ 253$