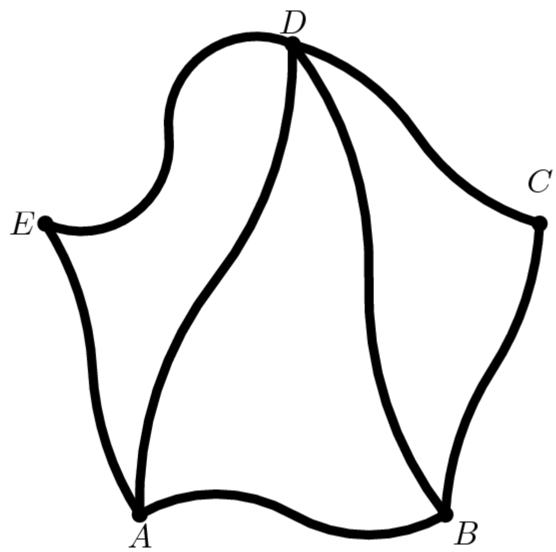
Problem Set - 6 Dec 2023

PROBLEM 1 (2013 AMC 12B #12)

Cities A, B, C, D, and E are connected by roads $\widetilde{AB}$, $\widetilde{AD}$, $\widetilde{AE}$, $\widetilde{BC}$, $\widetilde{BD}$, $\widetilde{CD}$, and $\widetilde{DE}$. How many different routes are there from A to B that use each road exactly once? (Such a route will necessarily visit some cities more than once.)



(A) 7 **(B)** 9 **(C)** 12 **(D)** 16 **(E)** 18

PROBLEM 2 (2020 AMC 12A #13)

There are integers a, b, and c, each greater than 1, such that

$$\sqrt[a]{N\sqrt[b]{N\sqrt[c]{N}}} = \sqrt[36]{N^{25}}$$

for all $N \neq 1$. What is b?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

PROBLEM 3 (2013 AMC 10B #18)

The number 2013 has the property that its units digit is the sum of its other digits, that is $2 + 0 + 1 = 3$. How many integers less than 2013 but greater than 1000 have this property?

(A) 33 **(B)** 34 **(C)** 45 **(D)** 46 **(E)** 58

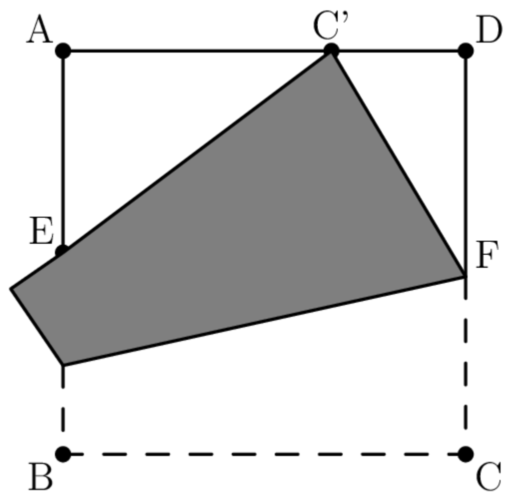
PROBLEM 4 (2015 AIME II #1)

Let N be the least positive integer that is both 22 percent less than one integer and 16 percent greater than another integer. Find the remainder when N is divided by 1000.

PROBLEM 5 (2021 AMC 10B #21)

A square piece of paper has side length 1 and vertices A, B, C, and D in that order. As shown in the figure, the paper is folded so that vertex C meets edge $\overline{AD}$ at point C', and edge $\overline{BC}$ intersects edge $\overline{AB}$ at point E. Suppose that $C'D = \frac{1}{3}$. What is the perimeter of triangle $\triangle AEC'$?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 1+\frac{2}{3}\sqrt{3} \qquad \textbf{(C)}\ \frac{13}{6} \qquad \textbf{(D)}\ 1+\frac{3}{4}\sqrt{3} \qquad \textbf{(E)}\ \frac{7}{3}$



PROBLEM 6 (2014 AMC 10A #21)

Positive integers a and b are such that the graphs of $y = ax + 5$ and $y = 3x + b$ intersect the x-axis at the same point. What is the sum of all possible x-coordinates of these points of intersection?

$\textbf{(A)}\ -20 \qquad \textbf{(B)}\ -18 \qquad \textbf{(C)}\ -15 \qquad \textbf{(D)}\ -12 \qquad \textbf{(E)}\ -8$

PROBLEM 7 (2017 AMC 12B #22)

Abby, Bernardo, Carl, and Debra play a game in which each of them starts with four coins. The game consists of four rounds. In each round, four balls are placed in an urn---one green, one red, and two white. The players each draw a ball at random without replacement. Whoever gets the green ball gives one coin to whoever gets the red ball. What is the probability that, at the end of the fourth round, each of the players has four coins?

$\textbf{(A)}\ \dfrac{7}{576} \qquad \textbf{(B)}\ \dfrac{5}{192} \qquad \textbf{(C)}\ \dfrac{1}{36} \qquad \textbf{(D)}\ \dfrac{5}{144} \qquad \textbf{(E)}\ \dfrac{7}{48}$

PROBLEM 8 (2010 AIME II #9)

Let $ABCDEF$ be a regular hexagon. Let G, H, I, J, K, and L be the midpoints of sides AB, BC, CD, DE, EF, and AF, respectively. The segments $\overline{AH}$, $\overline{BI}$, $\overline{CJ}$, $\overline{DK}$, $\overline{EL}$, and $\overline{FG}$ bound a smaller regular hexagon. Let the ratio of the area of the smaller hexagon to the area of $ABCDEF$ be expressed as a fraction $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m+n$.

PROBLEM 9 (2022 AIME I #8)

Equilateral triangle $\triangle ABC$ is inscribed in circle ω with radius 18. Circle ω_A is tangent to sides $\overline{AB}$ and $\overline{AC}$ and is internally tangent to ω. Circles ω_B and ω_C are defined analogously. Circles ω_A, ω_B, and ω_C meet in six points---two points for each pair of circles. The three intersection points closest to the vertices of $\triangle ABC$ are the vertices of a large equilateral triangle in the interior of $\triangle ABC$, and the other three intersection points are the vertices of a smaller equilateral triangle in the interior of $\triangle ABC$. The side length of the smaller equilateral triangle can be written as $\sqrt{a}-\sqrt{b}$, where a and b are positive integers. Find $a+b$.

PROBLEM 10 (2010 AIME I #10)

Let N be the number of ways to write 2010 in the form $2010 = a_3 \cdot 10^3 + a_2 \cdot 10^2 + a_1 \cdot 10 + a_0$, where the a_i's are integers, and $0 \le a_i \le 99$. An example of such a representation is $1 \cdot 10^3 + 3 \cdot 10^2 + 67 \cdot 10^1 + 40 \cdot 10^0$. Find N.