Question 1

$$\frac{dS}{dt} = -\beta I S \qquad \frac{dE}{dt} = \beta I S - \kappa E$$

$$\frac{dI}{dt} = \kappa E - \alpha I \qquad \frac{dr}{dt} = \alpha I$$

a) $$\frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$$

$$\frac{d(S+E+I+R)}{dt} = 0$$

Let $S+E+I+R = N$

$$\frac{dN}{dt} = 0$$

$$N = \int 0 \, dt = C \; \#$$

b) $\frac{dS}{dt} = -\beta I S$ (decreasing) so we assume $\lim_{t \to \infty} S(t) = S_\infty \geq 0$

$\frac{dR}{dt} = \alpha I$ (increasing) so we assume $\lim_{t \to \infty} R(t) = R_\infty \leq N$

Is it $S_\infty \geq 0$ or $S_\infty > 0$?

Let $$\frac{dS}{dR} = \frac{dS}{dt} \div \frac{dR}{dt} = \frac{-\beta I S}{\alpha I} = \frac{-\beta}{\alpha} S$$

hence we know that dS/dR is solveable

Sub ① into $S(R)$

$$S(R) = S(0)\, e^{-\beta/\alpha\, S}$$

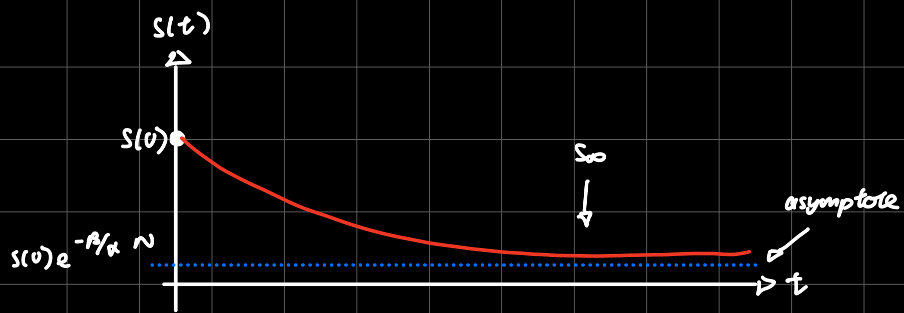
* Note that $R \le N = -R \ge -N = \beta/\alpha(-R) > \beta/\alpha(-N)$

so $S(R) = S(0)\, e^{-\beta/\alpha\, R} \ge \underbrace{S(0)\, e^{-\beta/\alpha\, N}} > 0$

=> positive integers

=> independent of time

Thus $S(t) = S(R(t))$:



$\therefore$ $S_\infty > 0$, so in the long-term, not everyone gets the disease

c) $$\frac{dS}{dt} = -\beta S(t) I(t)$$

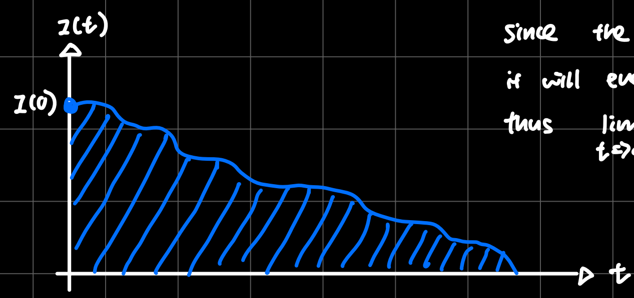
$$\int_0^\infty \frac{dS}{dt} = -\beta \int_0^\infty S(t)\, I(t)\, dt$$

$$S(t) \Big]_0^\infty = -\beta \int_0^\infty S(t)\, I(t)\, dt \ge -\beta \int_0^\infty S_\infty\, I(t)\, dt$$

$$S_\infty - S_0 \ge -\beta S_\infty \int_0^\infty I(t)\, dt$$

$$\int_0^\infty I(t)\,dt \leq \frac{S_0 - S_\infty}{\beta S_\infty} < \infty \quad \text{since} \quad S_\infty \neq 0$$

hence $\int_0^\infty I(t)\,dt < \infty$



Since the area is finite, it will eventually hit 0, thus $\lim_{t \to \infty} I(t) = 0$

$\therefore \lim_{t \to \infty} I(t) = 0$ where the epedemic eventually ends

d) In the long run, those who got exposed to the disease will eventually got infected, hence $E_\infty = 0$. Furthermore, those who get infected will eventually recover & fall into the removed class, at least in the SEIR model. Thus, the population N within this model will consist of people who either have no encounter with the disease (S) or people who have gotten it & eventually recovered (R), assuming the disease isn't terminal & there are no deaths. So $S_\infty + R_\infty = N$.

e) $r_0 = \frac{R(0)}{N}$ $\qquad s_\infty = \frac{S_\infty}{N}$

Fraction of people who are infected at some point t^* $= \frac{\#\text{ of infected @ } t^*}{\text{population}} = \frac{I_{t^*}}{N}$

hence $1 - \frac{R(0)}{N} - \frac{S_\infty}{N} = \frac{I_{t^*}}{N}$

$$\frac{N - R(0) - S_\infty}{N} = \frac{I_{t^*}}{N}$$

$$N - R(0) - S_\infty = I_{t^*}$$

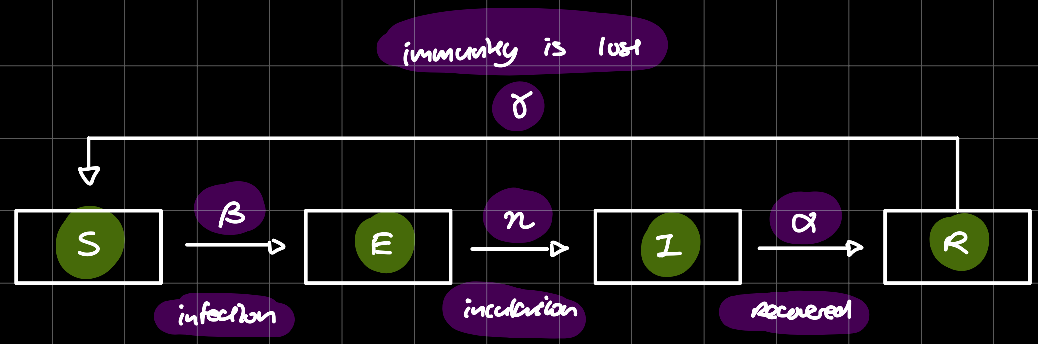
=> Assume that $N - R(0)$ is the # of people who may get the disease

=> Assume that S_∞ is the # of people who didn't get the disease

=> By dividing the equation by N, we will obtain the fraction of those who actually got the disease for $t > 0$

Question 2

a)



<u>ODE's</u>

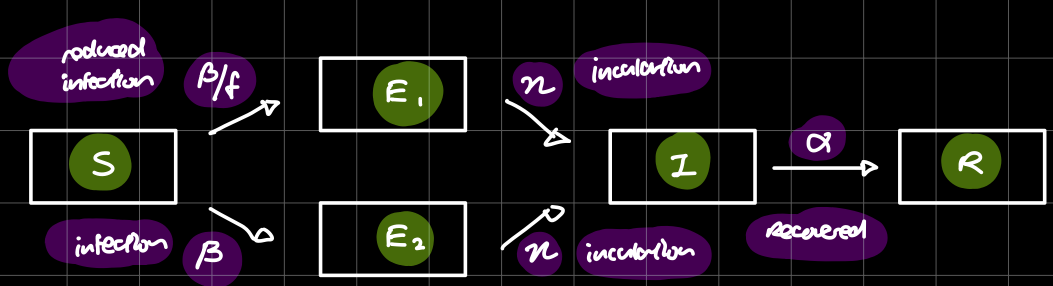
$$S' = -\beta I S + \gamma R$$

$$E' = \beta I S - \eta E$$

$$I' = \eta E - \alpha I$$

$$R' = \alpha I - \gamma R$$

b)



<u>ODE's</u>

$$S' = -p\frac{\beta}{f} S I - (p-1)\beta S I$$

$$E_1' = p\frac{\beta}{f} S I - \eta E_1$$

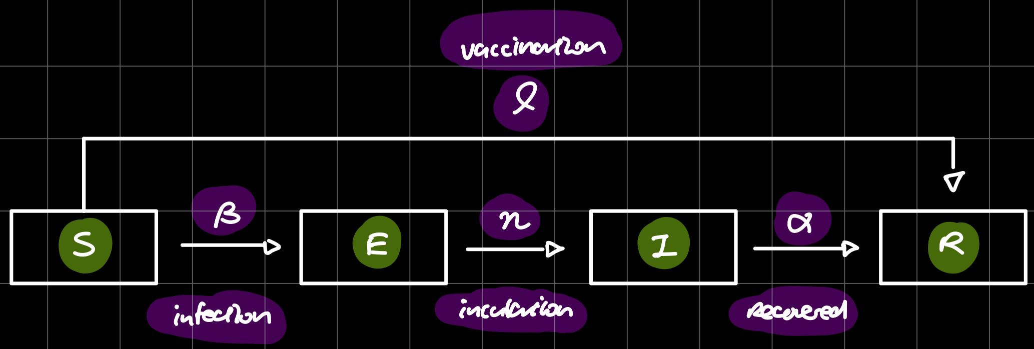
$$E_2' = (p-1)\beta S I - \eta E_2$$

$$I' = \eta E_1 + \eta E_2 - \alpha I$$

$$R' = \alpha I$$

⇒ Assume p is the # of people who self-isolated

⇒ Assume $(1-p)$ is the # of people who didn't self-isolate c)



<u>ODE's</u>

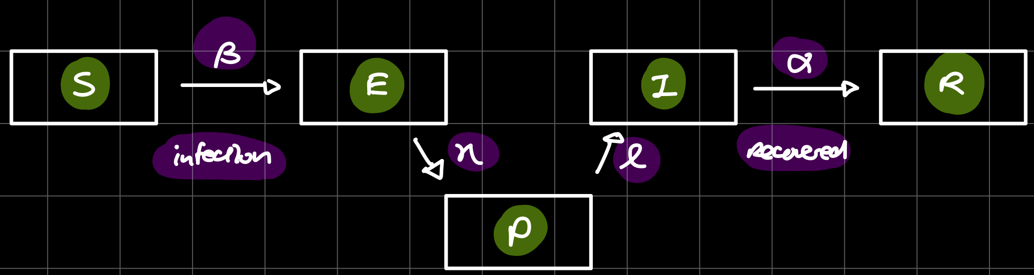
$S' = -\beta I S - \gamma S$

$E' = \beta I S - n E$

$I' = n E - \alpha I$

$R' = \alpha I + \gamma S$

d)



<u>ODE's</u>

$S' = -\beta I S$

$E' = \beta I S - n E$

$P' = n E - \ell P$

$I' = \ell P - \alpha I$

$R' = \alpha I$

Question 3

⇒ I will be investigating the 2009 H1N1 Swine Flu pandemic within Malaysia

⇒ The pandemic in Malaysia lasted from May 2009 to June 2010, a total of 13 months

⇒ On average, H1N1 influenza lasts for 8 days, hence...

$$\alpha = \frac{1}{\text{mean time spent in infectious class}}$$

$$= \frac{1}{8 \text{ days}}$$

$$= 1/8$$

⇒ In the first month of the outbreak, 642 individuals were infected

⇒ Assume that everyone in the population was vulnerable to the disease, hence we shall use the statistic 28.2 million in 2009

⇒ so for S_0, we shall assume (28.2 million - 642)

⇒ At the end of the pandemic, a total of 14,912 cases were reported

⇒ From research, we know that after getting infected, our bodies will have temporary immunity for 1 - 2 years. Since the pandemic only lasted for 13 months, we can assume that no one got the virus again so the recovered cannot be susceptible for our model

⇒ Thus, S_∞ will be (28.2 million - 14,912)

$$\beta = \frac{\alpha \ln (S_0/S_\infty)}{S_0 + I_0 - S_\infty}$$

$$\beta = \frac{(1/8) \ln \left({}^{28.2\text{ mil} - 642}/_{28.2\text{ mil} - 14{,}912} \right)}{(28.2\text{ mil} - 642) + 642 - (28.2\text{ mil} - 14{,}912)}$$

$$= \frac{6.3271 \times 10^{-5}}{14{,}912}$$

$$= 4.24296 \times 10^{-9}$$

#