Question 1

2.3. The simplest model of malaria assumes that the mosquito population is at equilibrium and models the proportion of the infected humans I with the following equation:

$$I' = \frac{\alpha\beta I}{\alpha I + r}(1 - I) - \mu I,$$

where r is the natural death rate of mosquitoes, μ is the death rate of humans, β is the transmission rate from infected mosquitoes to susceptible humans, and α is the transmission rate from humans to mosquitoes.

Finding the Equilibrium

$$\frac{dI}{dt} = \frac{\alpha\beta I(1-I)}{\alpha I + R} - \mu I$$

At equilibrium, let $\frac{dI}{dt} = 0$

$$\frac{\alpha\beta I - \alpha\beta I^2 - \mu I(\alpha I + R)}{\alpha I + R} = 0$$

$$\alpha\beta I - \alpha\beta I^2 - \alpha\mu I^2 - R\mu I = 0$$

$$I(\alpha\beta - \alpha\beta I - \alpha\mu I - R\mu) = 0$$

$$\alpha\beta - \alpha\beta I - \alpha\mu I - R\mu = 0$$

$$\alpha\beta I + \alpha\mu I = \alpha\beta - R\mu$$

$$I(\alpha\beta + \alpha\mu) = \alpha\beta - R\mu$$

$$I = \frac{\alpha\beta - R\mu}{\alpha(\beta + \mu)} = I^*$$

a) Case 1 infection dies out where $I^* \leq 0$

Simplifying the equation

$$\frac{dI}{dt} = \frac{\alpha\beta I(1-I)}{\alpha I + R} - \mu I = I\left(\frac{\alpha\beta(1-I)}{\alpha I + R} - \mu\right)$$

when $\frac{\alpha\beta(1-I)}{\alpha I + R} - \mu \leq 0$ ...

$$\frac{\alpha\beta - \alpha\beta I - \mu(\alpha I + R)}{\alpha I + R} \leq 0$$

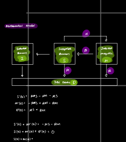
$$\frac{\alpha\beta - \alpha\beta I - \alpha I\mu - R\mu}{\alpha I + R} \leq 0$$

$$\alpha\beta - \alpha\beta I - \alpha I\mu - R\mu \leq 0$$

$$\alpha\beta I + \alpha I\mu \geq \alpha\beta - R\mu$$

$$I \geq \frac{\alpha\beta - R\mu}{\alpha\beta + \alpha\mu}$$

$\frac{\alpha\beta}{\alpha I + R}(1-I) - \mu$	$+$		$-$		$-$
	$\leftarrow$	I^*	$\rightarrow$	0	$\leftarrow$
I	$-$		$-$		$+$
dI/dt	$-$		$+$		$-$



b) Case 2 where infection has a positive equilibrium ($I^* > 0$)

when $$\frac{\alpha\beta(1-I)}{\alpha I + R} - \mu > 0 \ldots$$

$$\frac{\alpha\beta - \alpha\beta I - \mu(\alpha I + R)}{\alpha I + R} > 0$$

$$\frac{\alpha\beta - \alpha\beta I - \alpha I\mu - R\mu}{\alpha I + R} > 0$$

$$\alpha\beta - \alpha\beta I - \alpha I\mu - R\mu > 0$$

$$\alpha\beta I + \alpha I\mu < \alpha\beta - R\mu$$

$$I < \frac{\alpha\beta - R\mu}{\alpha\beta + \alpha\mu}$$

$\frac{\alpha\beta}{\alpha I + R}(1-I) - \mu$	$+$		$+$		$-$
	$\leftarrow$		$\rightarrow$		$\leftarrow$
I	$-$	0	$+$	I^*	$+$
dI/dt	$-$		$+$		$-$

Question 2

a) $$\alpha = \frac{1}{\text{mean time spent in infectious class}}$$

$$\alpha = \frac{1}{11 \text{ days}} = \frac{1}{11} \#$$

b) $$\beta = \frac{\alpha \ln(S_0/S_\infty)}{S_0 + I_0 - S_\infty}$$

$$= \frac{1/11 \ \ln(350/83)}{350 + 0 - 83} = \frac{\ln(350/83)}{2{,}937} = 0.00048 \#$$