Randomizing Over Lipton

	X	Y	Z
Agent 1	0	0.3	1.7
Agent 2	0.3	0.2	1.5
Agent 3	0.9	0.5	0.6

In the envy graph algorithm, we take an arbitrary ordering of the items. While there are unassigned items, we ensure that there is an unenvied agent and give the next item to the unenvied agent. We then remove cycles in the envy graph.

At each step, we randomize equally between all possible choices. Doing this does not result in pre-ante envy-freeness, as Agent 1 envies Agent 2 even in the long run. The intuition for this is that Agent 1, when assigned X, will often trade it with Agent 2 for Y. This results in Agent 2 acquiring the very valuable Z from Agent 3 more often than Agent 1, causing envy in the long run.

Outcome trees are included on the following pages. I use shorthand and describe the assignment of good g to agent i with $a_i : g$. Upon examination, we find that a randomized envy graph algorithm results in allocation

- $a_1 : Y,\ a_2 : Z,\ a_3 : X$ with expectation 7/12
- $a_1 : Z,\ a_2 : X,\ a_3 : Y$ with expectation 5/36
- $a_1 : Z,\ a_2 : Y,\ a_3 : X$ with expectation 10/36

Thus, a_1 values their own bundle in expectation $\frac{7}{12} * .3 + \frac{5}{12} * 1.7 = .8833$.

a_1 values a_2's bundle in expectation $\frac{7}{12} * 1.7 + \frac{5}{36} * 0 + \frac{5}{18} * .3 = 1.075$.

It follows that envy does not disappear in expectation or in the long run.

We consider each possible ordering of goods (each with probability 1/6).

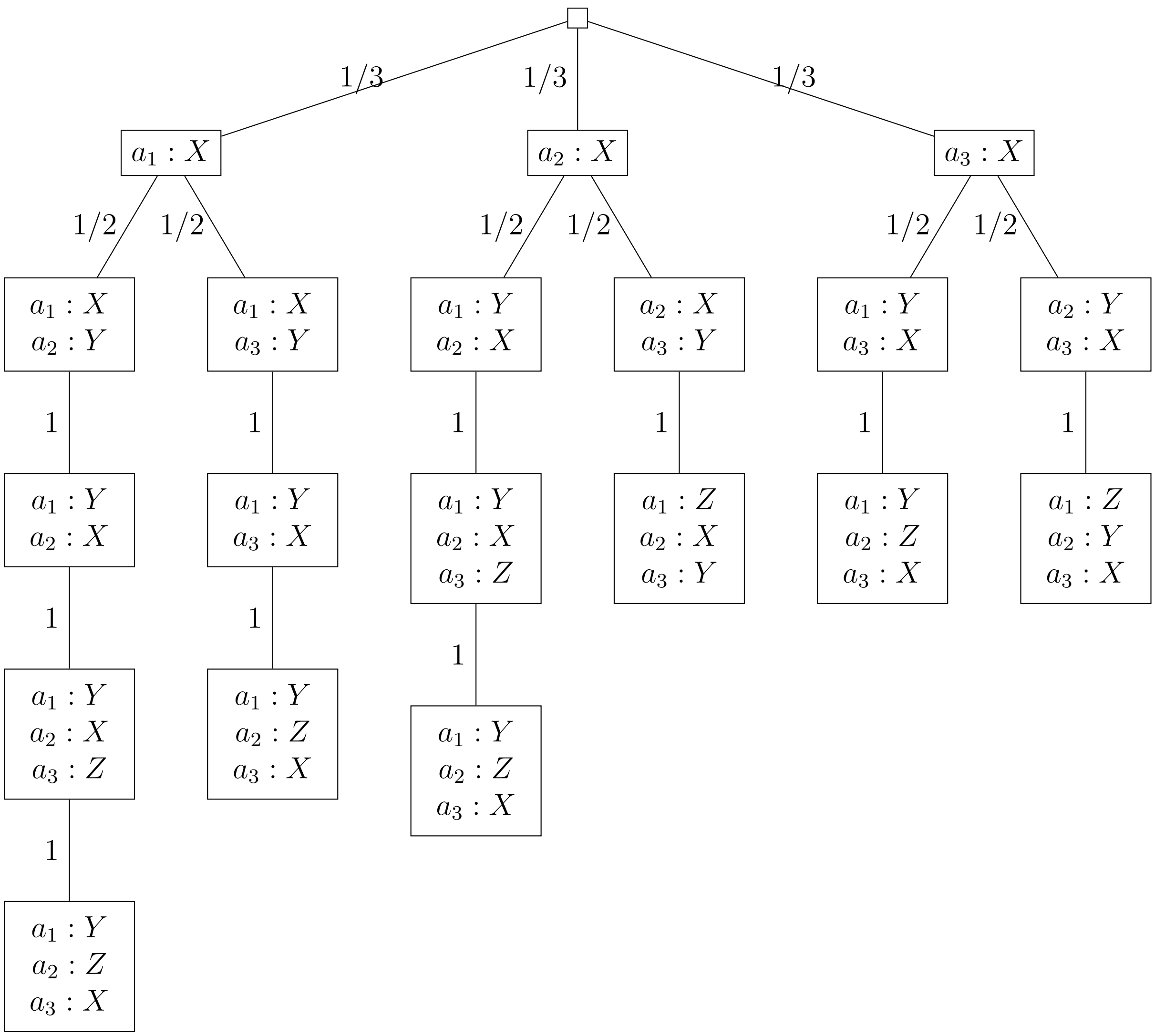

Figure 1: The outcomes resulting from ordering XYZ

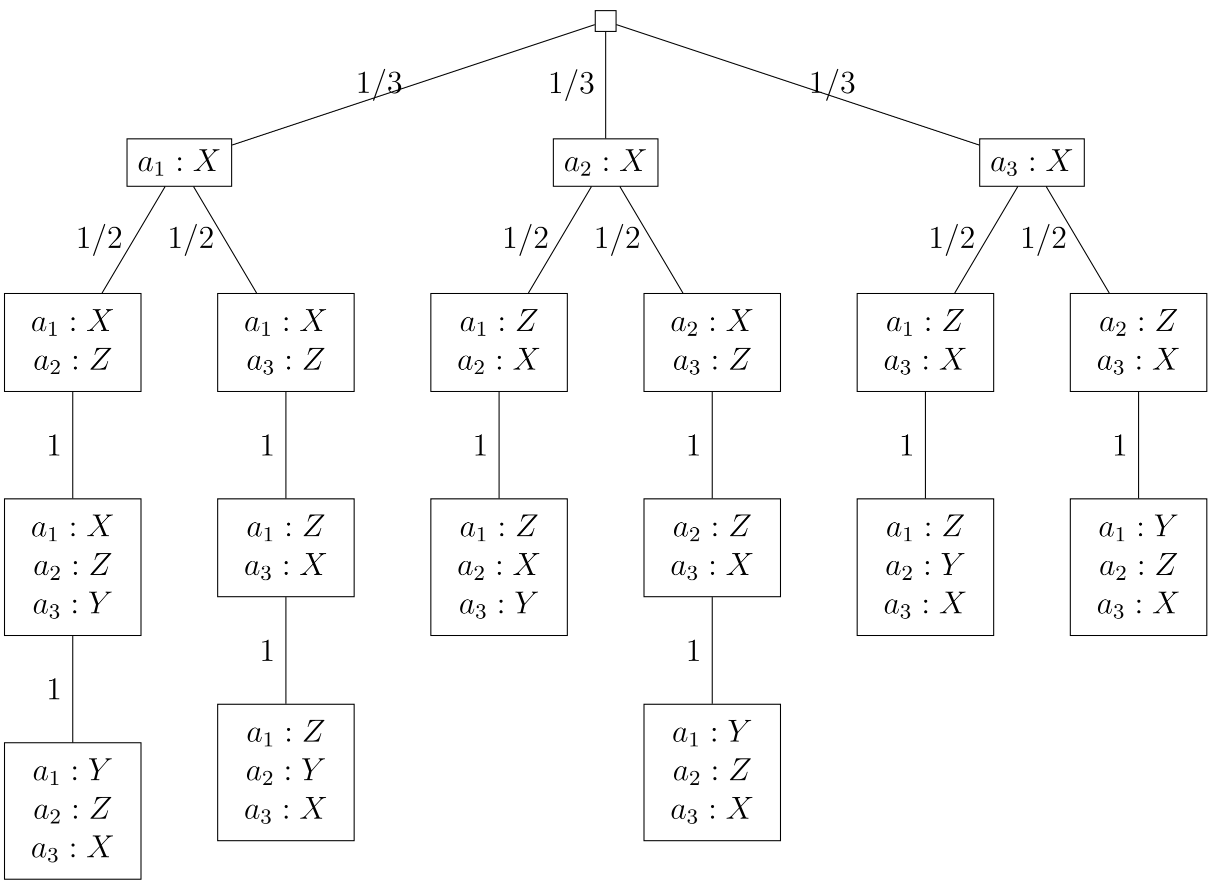

Figure 2: Outcomes resulting from the ordering XZY.

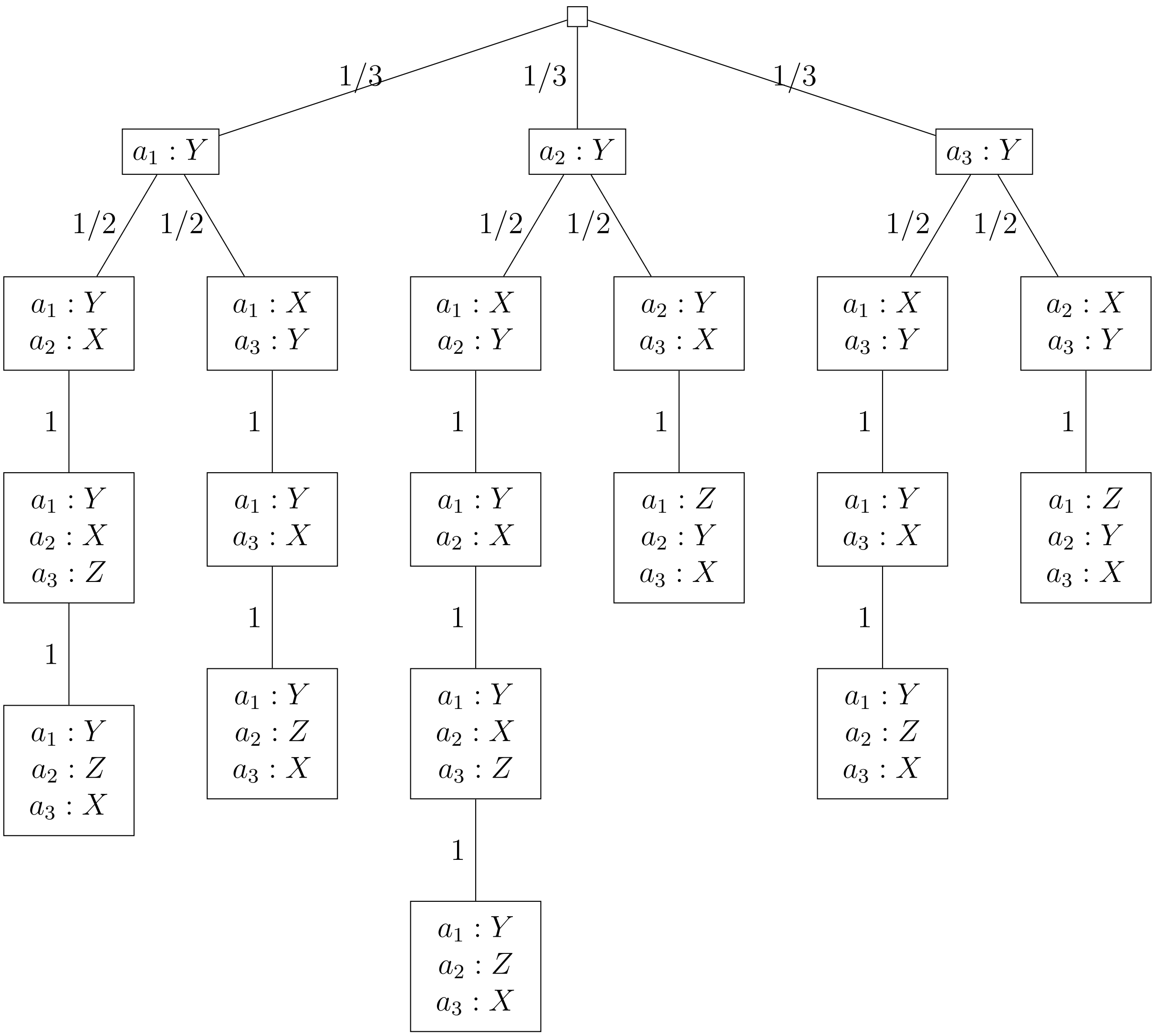

Figure 3: Outcomes from the ordering YXZ

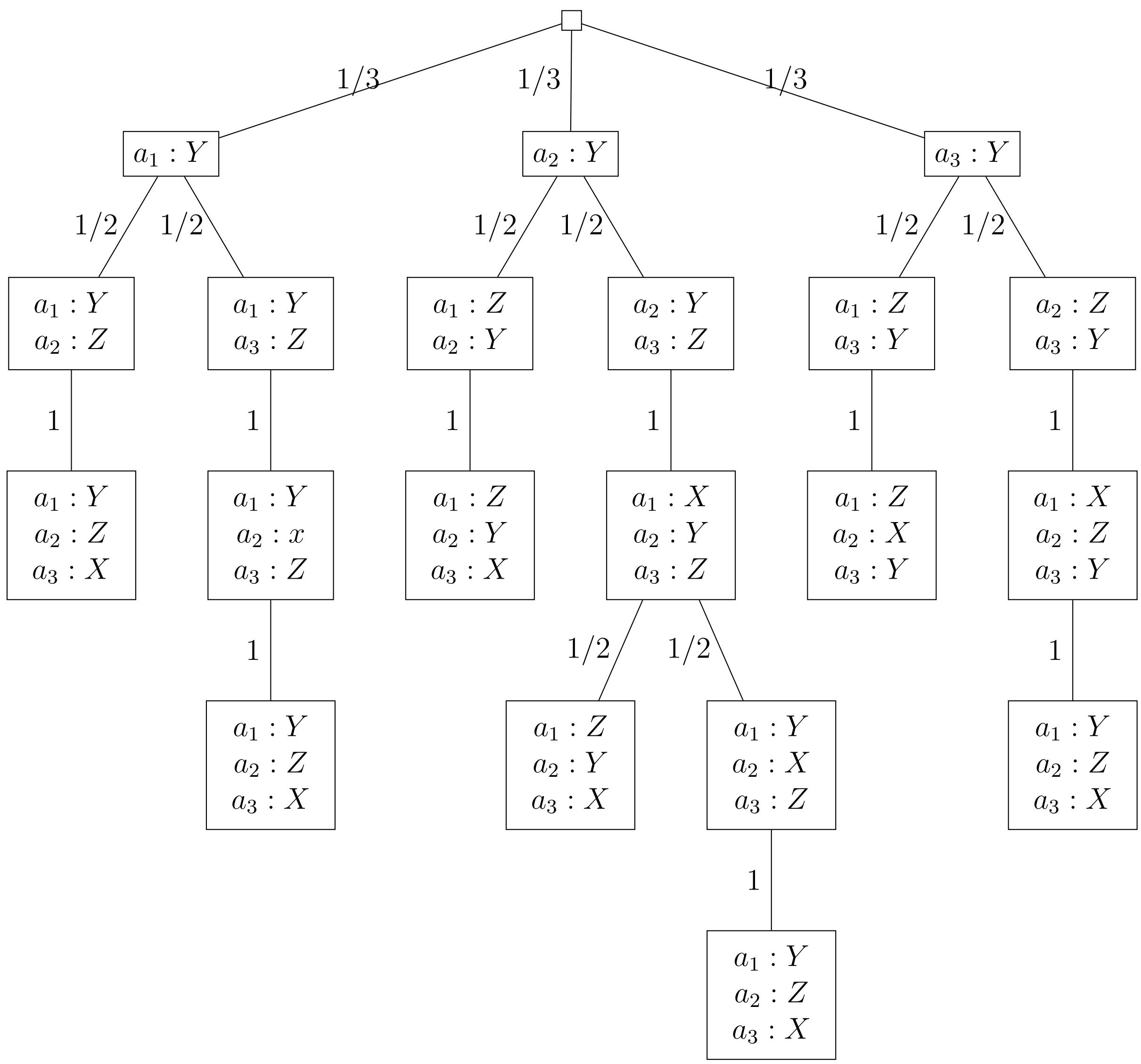

Figure 4: Outcomes from the ordering YZX.

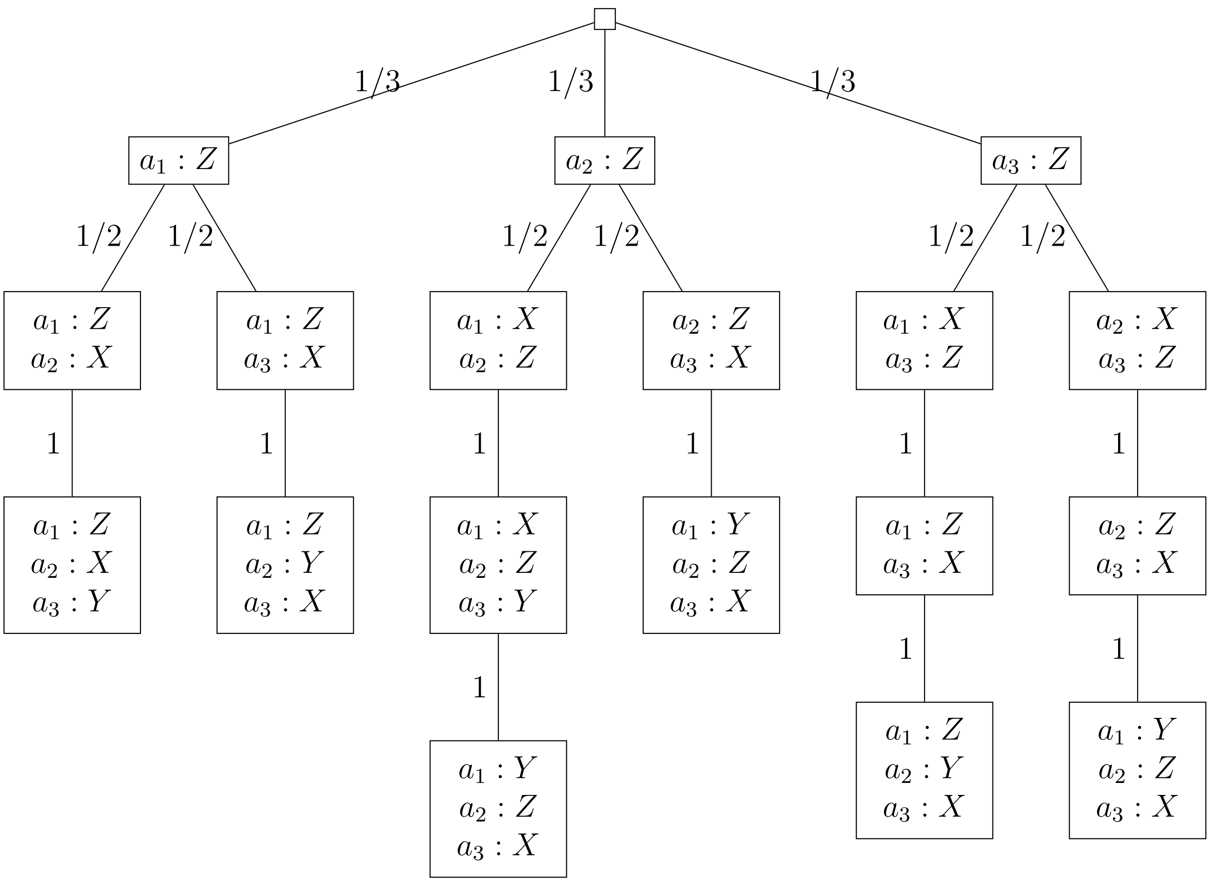

Figure 5: Outcomes from the ordering ZXY

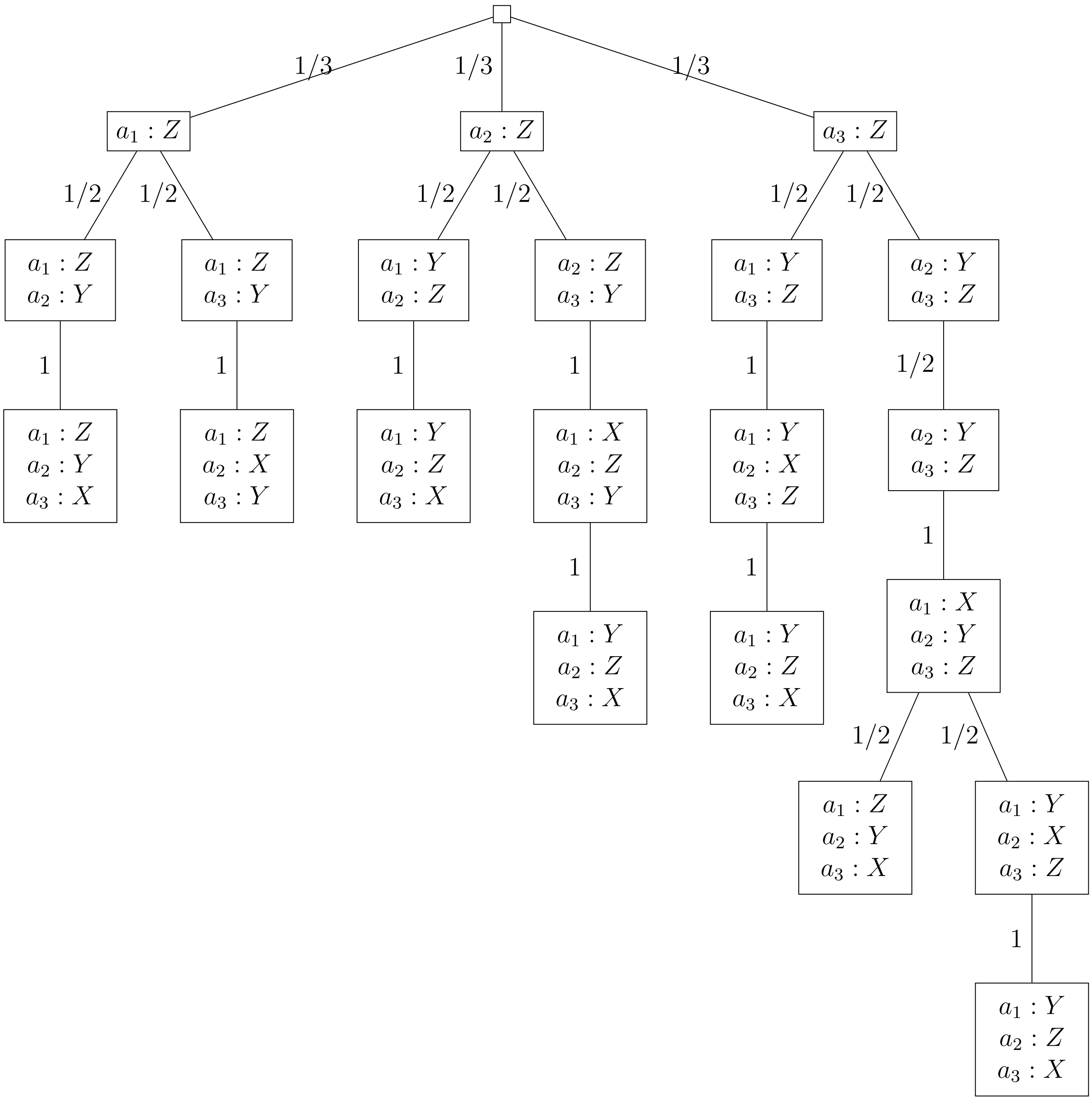

Figure 6: Outcomes from the ordering ZYX.