On Error Free Transformations and Applications to Polynomial Equations

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We present modifications of root finding methods which extend the working precision of both methods through floating-point techniques. We use Knuth's algorithm for summing two floating point numbers as well as the Veltkamp-Dekker method for splitting and then multiplying floating point numbers for error free floating point sums and products. Through applying these methods we find that root finding methods become much more accurate, especially in the case of ill-conditioned roots.

Through accounting for the error of floating point operations such as summing and multiplying two floating point numbers, we show that we can work in fixed precision when summing or multiplying floating point numbers. Following the work of Stef Graillat, we show that the application of these techniques reduces the error in the Ruffini-Horner method, resulting in more accurate polynomial evaluation, with full precision so long as the condition number is less than approximately 10^{16}. For condition numbers greater than 10^{16}, the error begins to increase, closely bounded by $eps + \widetilde{\gamma}_{2n}^2 cond(p, x)$, in which eps is the positive difference from the absolute value of a floating point number to the next biggest floating point number of the same precision, and γ_n is given by $\frac{neps}{1-neps}$ for $n \in \mathbb{N}$. Finally, we show that this method can similarly be applied to root finding algorithms, increasing their working precision for arbitrarily ill-conditioned roots.