The particle flux at distance x at arrival time T is given by integrating the particle velocity distribution $f(v,t)$:

$$S(x,T) = \int_{-\infty}^{+\infty} dt \int_0^{\infty} dv f(v,t) \delta\left(t - T + \frac{x}{v}\right) \ . \tag{1}$$

If we discretise (v,t) space such that $f(v,t)$ can be assumed uniform within each bin then the following equation is obtained:

$$S(x,T) = \sum_k \sum_l P_{kl} f_{kl} \ , \tag{2}$$

$$P_{kl} = \int_{t_{l-1/2}}^{t_{l+1/2}} dt \int_{v_{k-1/2}}^{v_{k+1/2}} dv \delta\left(t - T + \frac{x}{v}\right) \ , \tag{3}$$

$$f_{kl} = f(v_k, t_l) \ . \tag{4}$$

If we also discretise the arrival time domain and include a small finite number of detector locations, x_i, we arrive at a full set of linear equations:

$$S_{ij} = \sum_k \sum_l P_{kl}^{ij} f_{kl} \ , \tag{5}$$

$$S_{ij} = \frac{1}{T_{j+1/2} - T_{j-1/2}} \int_{T_{j-1/2}}^{T_{j+1/2}} S(x_i, T) dT \ , \tag{6}$$

$$P_{kl}^{ij} = \frac{1}{T_{j+1/2} - T_{j-1/2}} \int_{T_{j-1/2}}^{T_{j+1/2}} dT \int_{t_{l-1/2}}^{t_{l+1/2}} dt \int_{v_{k-1/2}}^{v_{k+1/2}} dv \delta\left(t - T + \frac{x_i}{v}\right) \ . \tag{7}$$

The "transfer" matrix, P_{kl}^{ij}, can be evaluated analytically:

$$P_{kl}^{ij} = \frac{x_i}{\Delta T_j} \sum_{n=0}^{3} (-1)^n \left[\Theta(y_{n,k+1/2}) g_n(y_{n,k+1/2}) - \Theta(y_{n,k-1/2}) g_n(y_{n,k-1/2})\right] \ , \tag{8}$$

$$g_n(y_{n,k}) = \log\left(\frac{a_n - y_{n,k}}{a_n}\right) + \frac{a_n}{a_n - y_{n,k}} \ , \tag{9}$$

$$y_{n,k} = a_n - \frac{x_i}{v_k} \ , \tag{10}$$

$$a_n = \begin{cases} T_{j+1/2} - t_{l-1/2} \ , \ n = 0 \\ T_{j+1/2} - t_{l+1/2} \ , \ n = 1 \\ T_{j-1/2} - t_{l+1/2} \ , \ n = 2 \\ T_{j-1/2} - t_{l-1/2} \ , \ n = 3 \end{cases} \tag{11}$$

where $\Theta(y)$ is the Heaviside step function and $\Delta T_j = T_{j+1/2} - T_{j-1/2}$. By stacking dimensions, the linear equations can be written in the more compact form:

$$S_m = \sum_n P_{mn} f_n + \epsilon_m \ , \tag{12}$$

where we have introduced the noise function ϵ. With measurements, S_m, we wish to invert this equation to obtain f_n.