HW2 solution

1.

1. (a) $x(n) = \{\underset{\uparrow}{1}, 2, 0, -1\}$ $\qquad$ $h(n) = \{2, \underset{\uparrow}{0}, 2\}$

$y_1(n) = x(n) * h(n)$ can be computed using the table method

	$\underset{\uparrow}{1}$	2	0	-1
2	2	4	0	-2
$\underset{\uparrow}{0}$	$\underset{\uparrow}{0}$	0	0	0
2	2	4	0	-2

$y_1(n) = x(n) * h(n) = \{2, \underset{\uparrow}{4}, 2, 2, 0, -2\}$

It can also be expressed as $x(n) * h(n) = 2\delta(n+1) + 4\delta(n) + 2\delta(n-1) + 2\delta(n-2) + (-2)\delta(n-4)$

(b) $y_2(n) = x(n+2) * h(n)$

$x(n+2) = \{1, 2, \underset{\uparrow}{0}, -1\}$

	1	2	$\underset{\uparrow}{0}$	-1
2	2	4	0	-2
$\underset{\uparrow}{0}$	0	0	$\underset{\uparrow}{0}$	0
2	2	4	0	-2

$y_2(n) = x(n+2) * h(n)$
$= \{2, 4, 2, \underset{\uparrow}{2}, 0, -2\}$

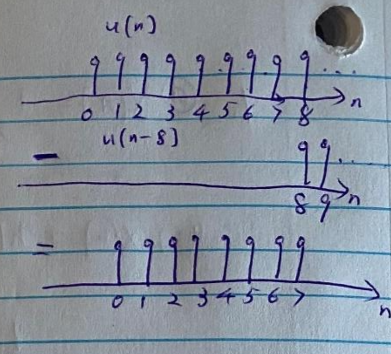
(c) $h(n+2) = \{2, 0, 2, \underset{\uparrow}{0}\}$

	$\underset{\uparrow}{1}$	2	0	-1
2	2	4	0	-2
0	0	0	0	0
2	2	4	0	-2
$\underset{\uparrow}{0}$	$\underset{\uparrow}{0}$	0	0	0

$y_3(n) = \{2, 4, 2, \underset{\uparrow}{2}, 0, -2\}$

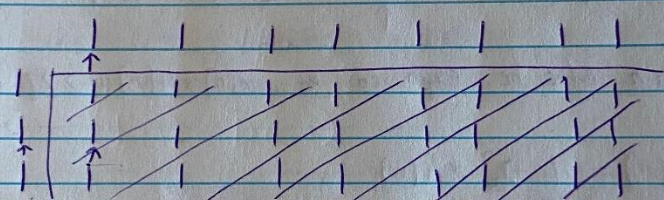
2. & 3.

2. $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1, 1, 1, 1\}$



$h(n) = \{1, \underset{\uparrow}{1}, 1\}$

$x(n) * h(n) = \{1, \underset{\uparrow}{2}, 3, 3, 3, 3, 3, 3, 2, 1\}$



3. ⓐ Based on difference equation. $y(n) = 2x(n) - x(n-1) + 2x(n-3)$

the impulse response can be derived:

$h(n) = 2\delta(n) - \delta(n-1) + 2\delta(n-3) = \{\underset{\uparrow}{2}, -1, 0, 2\}$

ⓑ the system is <u>causal</u> since $h(n) = 0$ for $n < 0$

ⓒ the system is BIBO stable since

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{3} |h(n)| = 2+1+2 = 5 < \infty$$

4.

4. $h_2[n] * h_2[n] = (u[n] - u[n-2]) * (u[n] - u[n-2])$

$= (\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1])$

$= \delta[n] * \delta[n] + \delta[n-1] * \delta[n] + \delta[n] * \delta[n-1] + \delta[n-1] * \delta[n-1]$

$= \delta[n] + \delta[n-1] + \delta[n-1] + \delta[n-2]$

$= \delta[n] + 2\delta[n-1] + \delta[n-2]$

$h[n] = h_1[n] * h_2[n] * h_2[n] = h_1[n] * (\delta[n] + 2\delta[n-1] + \delta[n-2])$

$= h_1[n] + 2h_1[n-1] + h_1[n-2]$

from figure 2.24(b) we know the amplitude of overall impulse response $h[n]$ at all time indexes.

$h[0] = h_1[0] + 2h_1[-1] + h_1[-2] = 1$

$\because$ the system is causal

$\therefore$ $h[n] = 0$ for $n < 0$

$h[0] = h_1[0] = 1$

$h_1[0] = 1$

$h[1] = h_1[1] + 2h_1[0] + h_1[-1] = 5$

$h_1[1] = 5 - 2h_1[0] = 3$

$h[2] = h_1[2] + 2h_1[1] + h_1[0] = 10$

$h_1[2] = 10 - 2h_1[1] - h_1[0] = 10 - 6 - 1 = 3$

$h[3] = h_1[3] + 2h_1[2] + h_1[1] = 11$

$h_1[3] = 11 - 6 - 3 = 2$

$h[4] = h_1[4] + 2h_1[3] + h_1[2] = 8$

$h_1[4] = 8 - 4 - 3 = 1$

$h[5] = h_1[5] + 2h_1[4] + h_1[3] = 4$ $\quad h_1[5] = 4 - 2 - 2 = 0$

$h[6] = h_1[6] + 2h_1[5] + h_1[4] = 1$ $\quad h_1[6] = 1 - 0 - 1 = 0$

$h[7] = h_1[7] + 2h_1[6] + h_1[5] = 0$ $\quad h_1[7] = 0 - 0 - 0 = 0$

$\vdots$

$h_1[n] = \{\underset{\uparrow}{1}, 3, 3, 2, 1\}$

5.

5. $\because x(n) = n$

$\therefore y_p(n) = C_1 n + C_2$

take it in the difference equation

$C_1 n + C_2 = -5(C_1(n-1) + C_2) + n$

$C_1 n + C_2 = -5C_1 n + 5C_1 - 5C_2 + n$

$6C_1 n + 6C_2 = n + 5C_1$

$\begin{cases} 6C_1 = 1 \\ 6C_2 = 5C_1 \end{cases} \Rightarrow \begin{cases} C_1 = \frac{1}{6} \\ C_2 = \frac{5}{36} \end{cases}$

$y_p(n) = \frac{1}{6}n + \frac{5}{36}$

based on the difference equation

$r + 5 = 0 \Rightarrow r = -5$

$y_h(n) = D(-5)^n$

$y(n) = y_p(n) + y_h(n) = \frac{1}{6}n + \frac{5}{36} + D(-5)^n$

☆ when you use $y(n) = y_p(n) + y_h(n)$, the $n \geq 0$

so we cannot take $y(-1)$ in the above function to solve D.

$y(0) = -5y(-1) + n \quad \because n = 0$

$y(0) = -5y(-1) + 0 = 0$

$y(0) = \frac{1}{6}n + \frac{5}{36} + D(-5)^n = 0$

$= 0 + \frac{5}{36} + D = 0$

$D = -\frac{5}{36}$

$y(n) = -\frac{5}{36}(-5)^n + \frac{1}{6}n + \frac{5}{36}$