Homework 1

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3.5.9

To show that

$$\nabla \cdot \left(\vec{a} \times \vec{b} \right) = \vec{b} \cdot (\nabla \times \vec{a}) - \vec{a} \cdot \left(\nabla \times \vec{b} \right)$$

We can use a similar proof to showing the BAC-CAB rule, utiziling the Levi-Cevita symbol.

$$\nabla \cdot \left(\vec{a} \times \vec{b} \right) = \sum_{i} \nabla_{i} \cdot \sum_{jk} \epsilon_{ijk} a_{j} b_{k}$$
$$= \sum_{i}$$

3.6.7

The identity

$$\boldsymbol{A}\times\left(\nabla\times\boldsymbol{A}\right)=\frac{1}{2}\nabla\left(\boldsymbol{A}^{2}\right)-\left(\boldsymbol{A}\cdot\nabla\right)\boldsymbol{A}$$

Can be verified by making note of the fact that $A^2 = \mathbf{A} \cdot \mathbf{A}$, letting us say

$$=\frac{1}{2}\nabla\left(\boldsymbol{A}\cdot\boldsymbol{A}\right)-\left(\boldsymbol{A}\cdot\nabla\right)\boldsymbol{A}$$

=

- 3.6.9
- 3.8.10
- 3.10.4
- 3.10.10