PHYS417HW6

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This one was also a bit of a weird one.

1)

Given the publicly known key $P = 997$

a)

$N = 231$, so X is

$$X = 2^{231}\text{mod}(997)$$

$$= 72$$

b)

With $M = 337$, we have

$$Y = 2^{373}\text{mod}(997)$$

$$= 80$$

c)

I'll be real, I calculated the previous mod functions using Desmos. Desmos is blowing up when I try to calculate K_1 or K_2.

d)

To convert to binary, we will use 10 bits, and essentially flip 0's to 1's from left to right such that we don't get any bigger sum than our target number. I unfortunately do not have the number, but it shouldn't have been that bad to calculate.

2)

Given $\hat{\pi} = \hat{p} + e\hat{A}$ and $\hat{\tilde{\pi}} = \hat{p} - e\hat{A}$

And $\hat{a} = \frac{\ell_B}{\sqrt{2}\hbar}\left(\pi_x - i\pi_y\right)$ and $\hat{b} = \frac{\ell_B}{\sqrt{2}\hbar}\left(\tilde{\pi}_x - i\tilde{\pi}_y\right)$

$\vec{A} = \frac{B}{2}[-y\vec{e}_x + x\vec{e}_y]$

So;

$$\pi_x = p_x - \frac{eB}{2}y$$

$$\pi_y = p_y + \frac{eB}{2}x$$

$$\tilde{\pi}_x = p_x + \frac{eB}{2}y$$

$$\tilde{\pi}_y = p_y - \frac{eB}{2}x$$

a)

$L_z = xp_y - yp_x$

Rewriting the π operators in terms of the more familiar position and momentum (really just adding things together), we get

$$p_x = \frac{\pi_x + \tilde{\pi}_x}{2}$$

$$x = \frac{\pi_y - \tilde{\pi}_y}{eB}$$

$$p_y = \frac{\pi_y + \tilde{\pi}_y}{2}$$

$$y = \frac{-\pi_x + \tilde{\pi}_x}{eB}$$

Which means that

$$
\begin{aligned}
L_z &= \left(\frac{\pi_y - \tilde{\pi}_y}{eB}\right)\left(\frac{\pi_y + \tilde{\pi}_y}{2}\right) - \left(\frac{-\pi_x + \tilde{\pi}_x}{eB}\right)\left(\frac{\pi_x + \tilde{\pi}_x}{2}\right) \\
&= \left(\frac{\pi_y^2 - \tilde{\pi}_y^2}{2eB}\right) - \left(\frac{-\pi_x^2 + \tilde{\pi}_x^2}{2eB}\right) \\
&= \frac{\pi_y^2 - \tilde{\pi}_y^2 + \pi_x^2 - \tilde{\pi}_x^2}{2eB} \\
&= \hbar\left(a^\dagger a - b^\dagger b\right)
\end{aligned}
$$

When we throw this at a state $|n, m\rangle$, we will get

$$
\hbar\left(a^\dagger a - b^\dagger b\right)|n, m\rangle = \hbar(n - m)|n, m\rangle
$$

and so the expectation value is simply

$$
\langle n, m|L_z|n, m\rangle = \hbar(n - m)
$$

b)

We can do a similar sort of trick to calculate K_z.

If $K_z = (\vec{r} \times \vec{\pi}))_z = x\pi_y - y\pi_x$, which has the same form as $L_z = (\vec{r} \times \vec{p})_z$. So then,

$$
\begin{aligned}
K_z &= x\pi_y - y\pi_x \\
&= \pi_y\left(\frac{\pi_y - \tilde{\pi}_y}{eB}\right) - \pi_x\left(\frac{-\pi_x + \tilde{\pi}_x}{eB}\right) \\
&= \frac{\pi_y^2 - \pi_y\tilde{\pi}_y + \pi_x^2 - \pi_x\tilde{\pi}_x}{eB} \\
&= 2\hbar\left(a^\dagger a\right) + \frac{-\pi_x\tilde{\pi}_x - \pi_y\tilde{\pi}_y}{eB} \\
&= \hbar\left(2a^\dagger a - ab - a^\dagger b^\dagger\right)
\end{aligned}
$$

If we throw this at a state, we get

$$
\hbar\left(2a^\dagger a - ab - a^\dagger b^\dagger\right)|n, m\rangle = \hbar\left(2n|n, m\rangle + \sqrt{nm}|n-1, m-1\rangle + \sqrt{(n+1)(m+1)}|n+1, m+1\rangle\right)
$$

The latter terms of which will be orthogonal with the state $|n, m\rangle$. The expectation value is thus

$$\langle n, m|K_z|n, m\rangle = 2\hbar n$$

c)

For coherent states α, β we can plug it into L_z and K_z and all that will really be different is the coefficients. Because of how the interaction between our operators and the coherent states are defined, we will get

$$\hbar \left(a^\dagger a - b^\dagger b\right) |\alpha\rangle|\beta\rangle = \hbar \left(|\alpha|^2|\alpha\rangle - |\beta|^2|\beta\rangle\right)$$

$$\langle \alpha, \beta|L_z|\alpha, \beta\rangle = \hbar \left(|\alpha|^2 - |\beta|^2\right)$$

and

$$\hbar \left(2a^\dagger a - ab - a^\dagger b^\dagger\right) |\alpha\rangle|\beta\rangle = \hbar \left(2|\alpha|^2|\alpha\rangle|\beta\rangle - 2\alpha\beta|\alpha\rangle|\beta\rangle\right)$$

$$\langle \alpha, \beta|K_z|\alpha, \beta\rangle = 2\hbar \left(|\alpha|^2 - \alpha\beta\right)$$

I'll be real, not sure of the physical significance of this result.