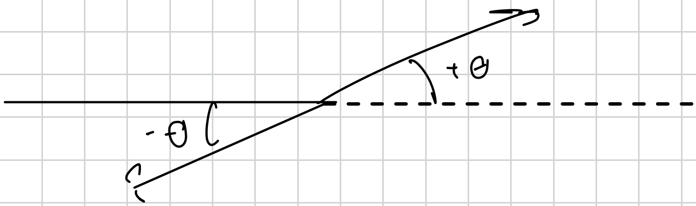
1) CM frame $\quad E'^2 - p'^2 = m^2$



$$p_1^{cm} = (E', p'\sin\theta', 0, p'\cos\theta')$$
$$p_2^{cm} = (E', -p'\sin\theta', 0, p'\cos\theta'$$

$$E = \gamma m \qquad p = m\beta\gamma$$

frame $\rightarrow +z \qquad V = \frac{p}{E+m}$ to remove p in cm frame

$$\gamma_{cm} = \frac{1}{\sqrt{1-V^2}} = \sqrt{\frac{E+m}{2m}}$$

$$E = \gamma_{cm}(E' + Vp_z')$$
$$p_z = \gamma_{cm}(p_z' + VE'$$
$$p_x = p_x'$$

$$p_{1x} = p' \sin\theta' \qquad p_{1z} = \gamma_{cm}(p'\cos\theta' + VE')$$

$$\tan\theta_1 = \frac{p_{1x}}{p_{1z}} = \frac{p'\sin\theta'}{\gamma_{cm}(p'\cos\theta' + VE')}$$

$$p_{2x} = p'\sin\theta \qquad p_{2z} = \frac{p'\sin\theta'}{\gamma_{cm}(VE' - p'\cos')}$$

$$\tan\theta_2 = \frac{-p'\sin\theta'}{\gamma_{cm}(VE' - p'\cos\theta')}$$

$$\tan\theta_1 \tan\theta_2 = \frac{-(p'\sin\theta)^2}{\gamma_{cm}^2((VE')^2 - (p'\cos\theta')^2)}$$

$$E'^2 - p'^2 = m^2 \qquad V^2 = \frac{E-m}{E+m}$$

$$p'^2\cos^2\theta' = p'^2(1 - \sin^2\theta') = (E'^2 - m^2)(1 - \sin^2\theta')$$

$$V^2E'^2 - (E'^2 - m^2) + (E'^2 - m^2)\sin^2\theta'$$

$$E'^2(\underbrace{V^2 - 1}_{-(1-V^2)} + \sin^2\theta') + m^2(1 - \sin^2\theta')$$

$$\tan\theta_1 \tan\theta_2 = \frac{(E'^2 - m^2)\sin^2\theta'}{\gamma^2_{cm}\left(E'^2(\sin^2\theta' - (1-V^2)) + m^2(1-\sin^2\theta')\right)}$$

$$= \frac{(E'^2 - m^2)\sin^2\theta'}{\gamma^2_{cm}\left[(E'^2 - m^2)\sin^2\theta' + m^2 - E'^2(1-V^2)\right)}$$

$$\gamma^2_{cm}(1-V^2) = 1 \rightarrow \gamma^2 E'^2(1-V^2) = E'^2$$

$$= \frac{(E'^2 - m^2)\sin^2\theta'}{\gamma^2_{cm}(E'^2 - m^2)\sin^2\theta' + \underbrace{\underbrace{\gamma^2 m^2}_{E'^2} - E'^2}_{=0}}$$

$$= \frac{1}{\gamma^2_{cm}} = \frac{E + m}{2m} \qquad \gamma = \frac{E}{m}$$

$$= \frac{1}{\gamma} \qquad \tan\theta_1 \tan\theta_2 = \frac{1}{\gamma}$$

2) $\pi^0 \to \gamma\gamma$ In CM $\theta = \pi$

$\theta = \pi$

$$\gamma = \frac{E}{m} = \frac{1}{\sqrt{1-\beta^2}}$$

$$(1-\beta^2) = \frac{1}{\gamma^2} \qquad \beta^2 = \frac{1-\gamma^2}{\gamma^2}$$

$$\beta = \sqrt{\frac{1-\gamma^2}{\gamma^2}}$$

$$\cos\theta = \frac{\cos\theta^* + \beta}{1+\cos\theta^* \beta}$$

3) n^0 $T_{max} \simeq T \dfrac{4mM_A}{(m+M_A)^2}$

np scattering

$M_n \simeq m \rightarrow T_{max} \simeq T$

from wikipedia $n = \frac{1}{\xi}\left[\ln E_0 - \ln E_1\right]$

$\xi = \ln \frac{E_0}{E} = 1 - \frac{(A-1)^2}{2A} \ln\left(\frac{A+1}{A-1}\right)$

for $A = 1$; $\xi = 1$

$n = \ln \frac{E_0}{E_f} = \ln \frac{2\times10^6}{0.025} \simeq 18.2$

on average takes 18.2 collisions

4) a)

$m^2 = p^\mu p_\mu$

$E = 250\ GeV$

$m^2 = E_1 E_2 - p_{1x} p_{2x} - p_{1y} p_{2y} - p_{1z} p_{2z}$

$= 53480.8$

$m = 231.3\ GeV$

Consistent w/ $\Lambda_c^{\pm}$

5)

$$\left\langle -\frac{dE}{dx}\right\rangle = K z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{2m_e c^2 \beta^2\gamma^2 W_{max}}{I^2}\right) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$