

# Homework2

February 6, 2026

## 1 Homework 2

### 1.1 2)

Starting from the work in the pdf;

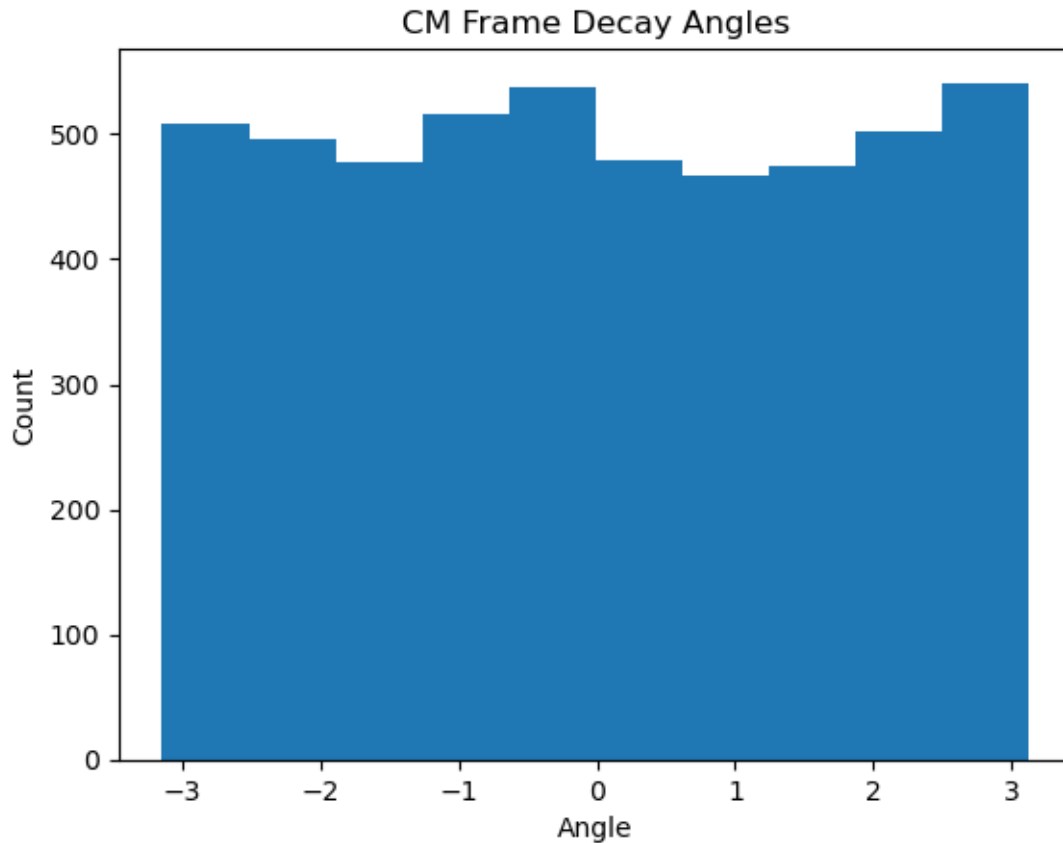
```
[70]: import matplotlib.pyplot as plt
import numpy as np
from math import cos, sin, tan
import math
import random
from scipy import integrate
```

```
[71]: pi_mass = 0.135
#theta_0 = math.pi/2
theta_0s = []
energies = [0.3, 1.0, 5.0, 50.0, 200.0]
for i in range(0,5000):
    #theta_0s.append(random.randrange(-math.pi, math.pi))
    rand_num = random.randrange(-180, 180)
    rand_num = rand_num * (math.pi/180.0)
    theta_0s.append(rand_num)

thetas = [[None] * len(theta_0s)] * len(energies)
```

```
[72]: plt.hist(theta_0s)
plt.title("CM Frame Decay Angles")
plt.xlabel("Angle")
plt.ylabel("Count")
```

```
[72]: Text(0, 0.5, 'Count')
```



```
[73]: for i, energy, in enumerate(energies):
        theta_energy = []
        for j, angle in enumerate(theta_0s):
            gamma = energies[i]/pi_mass
            beta = math.sqrt((gamma**2-1)/gamma**2)
            #print(beta)
            cos_theta_lab = (cos(theta_0s[j])+beta)/(1+cos(theta_0s[j])*beta)
            theta_lab = math.acos(cos_theta_lab)
            #thetas[i].append(theta_lab)
            theta_energy.append(theta_lab)

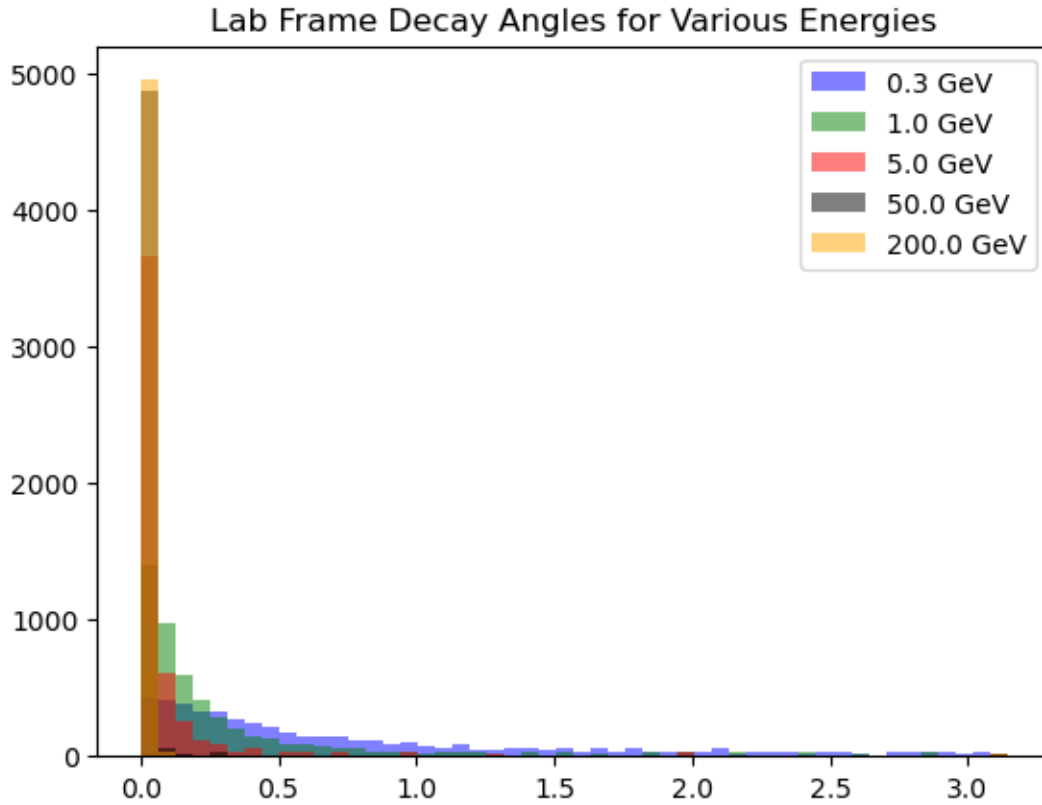
        thetas[i]= theta_energy
```

```
[74]: fig, ax = plt.subplots()
        facecolors = ["Blue", "Green", "Red", "Black", "Orange"]

        for i in range(len(facecolors)):
            ax.hist(thetas[i], bins=50, alpha=0.5, label=f"{energies[i]} GeV",
                    facecolor=facecolors[i], histtype='stepfilled')
```

```
ax.legend()
ax.set_title(r"Lab Frame Decay Angles for Various Energies")
```

[74]: Text(0.5, 1.0, 'Lab Frame Decay Angles for Various Energies')



As the  $\pi^0$  becomes more energetic, the resulting photons from decay will be increasingly colimated in the original direction of travel.

For the specific case  $\epsilon = 0.4789$  GeV we can figure out what fraction will be in the forward direction  $|\theta| \leq \pi/2$

```
[75]: epsilon = 0.4789
gamma = epsilon/pi_mass
beta = math.sqrt((gamma**2-1)/gamma**2)
counts = 0

for j in range(len(theta_0s)):
    lab_cos_theta = (cos(theta_0s[j])+beta)/(1+cos(theta_0s[j])*beta)
    theta_lab = math.acos(lab_cos_theta)
    if theta_lab <= math.pi/2:
        counts+=1
```

```
print(counts/len(theta_0s))
```

0.905

We can see that over %90 of the photons are emitted in the forward direction at only 0.4789 GeV.

## 1.2 5)

We can say that; from the Bethe-Bloch formula

```
[76]: #constants
me = 0.511
mp = 938.3
mu = 105.7
re = 2.82
K = 0.307
```

```
[77]: def bethe_bloche(p, mass, mat):
    Z, A, rho, I = mat
    K = 3.07075
    E = np.sqrt(p**2 + mass**2)
    beta = p/E

    dEdx = (K*(Z/A)*(1/beta**2)) * (0.5*np.log((2*me* beta**2 * gamma**2 / I)) -
    ↪beta**2)

    return dEdx * rho
    #return dEdx
```

```
[78]: # Listing in order Z, A, rho, I
materials = {
    "Al": (13, 26.98, 2.699, (166*10**(-6))),
    "Plastic": (0.538, 1, 1.06, (68.7*(10**(-6)))), # Has (Z/A) ratio listed, so
    ↪putting that in for Z and 1 for A
    "Cu": (29, 63.5, 8.96, (322 * (10**(-6)))),
    "Pb": (82, 207.2, 11.35, (823*(10**(-6)))),
    "H2O": (0.555, 1, 1, (79.7*(10**(-6))))
}
```

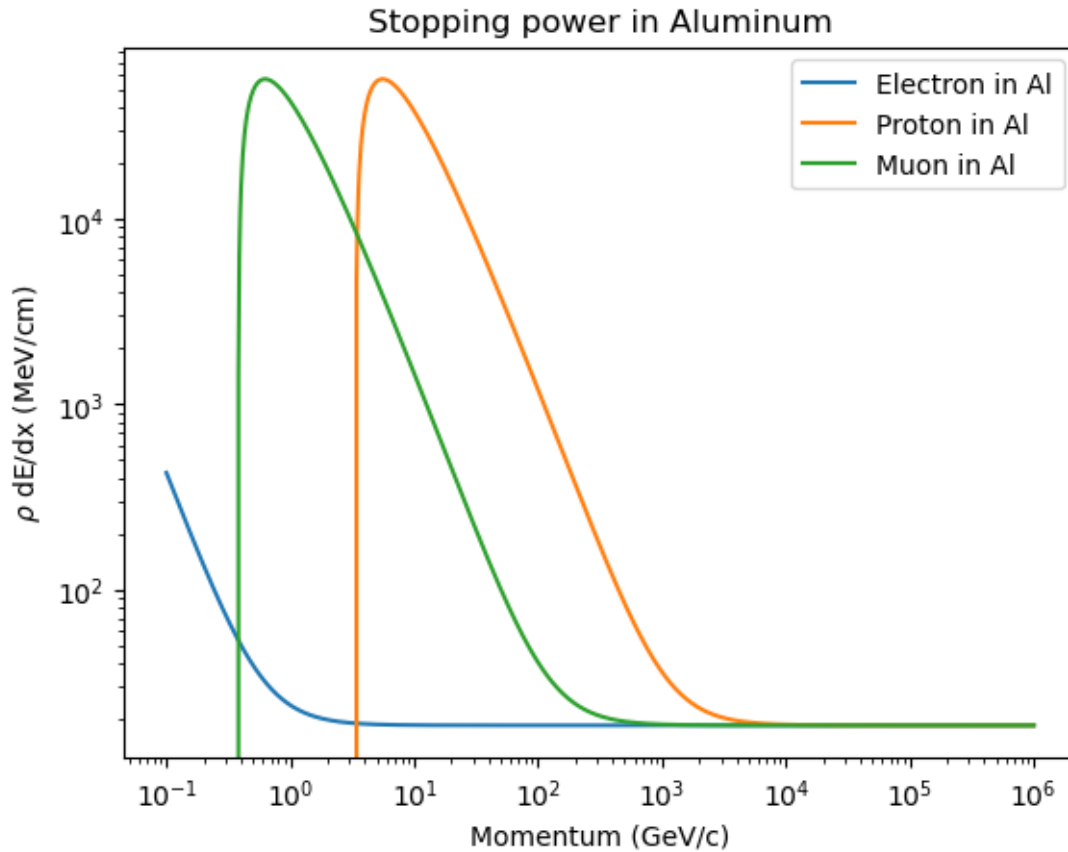
### 1.2.1 a)

```
[79]: p = np.logspace(-1, 6, 1000)
```

```
[106]: plt.figure()
for mass, label in [(me, "Electron"), (mp, "Proton"), (mu, "Muon")]:
    dEdx_Al = bethe_bloche(p, mass, materials["Al"])
    plt.loglog(p, dEdx_Al, label=f"{label} in Al")
```

```
plt.xlabel("Momentum (GeV/c)")
plt.ylabel(r"$\rho$ dE/dx (MeV/cm)")
plt.title("Stopping power in Aluminum")
plt.legend()
```

[106]: <matplotlib.legend.Legend at 0x79d9231cc650>

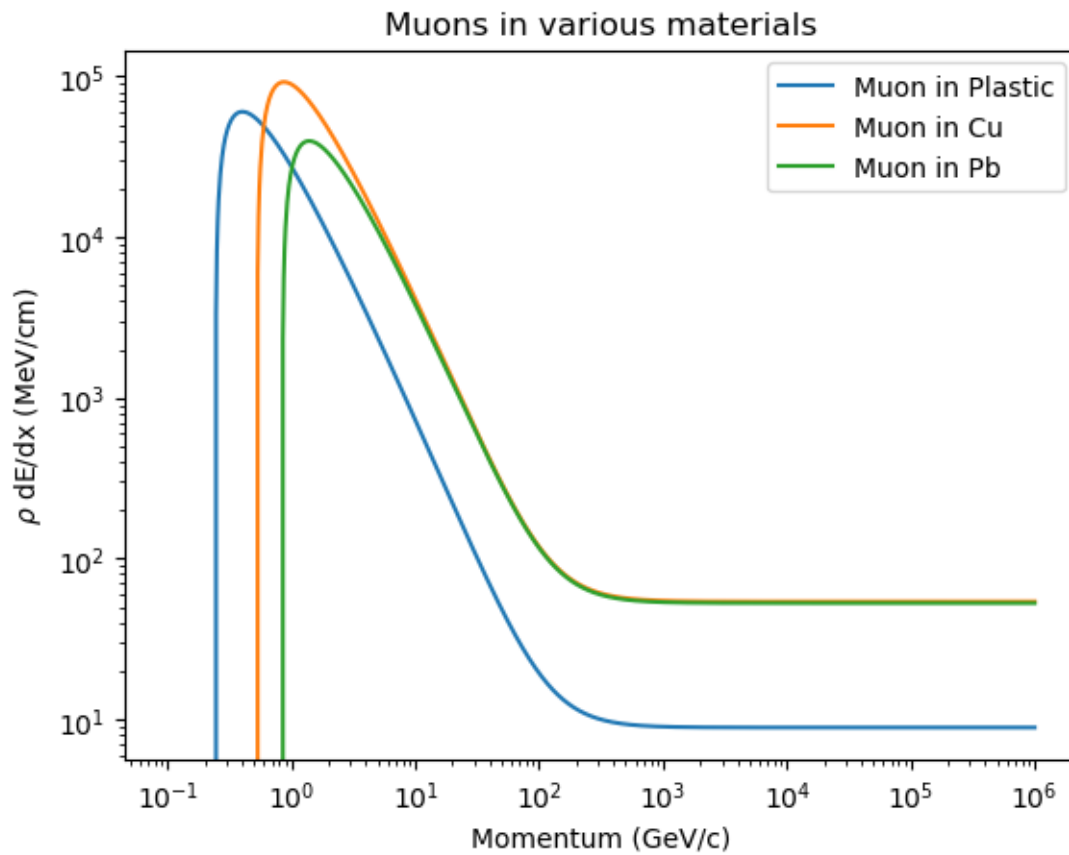


### 1.2.2 b)

```
[107]: plt.figure()
for material in ["Plastic", "Cu", "Pb"]:
    dEdx_Mu = bethe_bloche(p, mu, materials[material])
    plt.loglog(p, dEdx_Mu, label=f"Muon in {material}")

plt.xlabel("Momentum (GeV/c)")
plt.ylabel(r"$\rho$ dE/dx (MeV/cm)")
plt.legend()
plt.title("Muons in various materials")
```

[107]: Text(0.5, 1.0, 'Muons in various materials')



### 1.2.3 c)

Recall that the range is

$$R(E) = \int_E^0 dE / (dE/dx)$$

```
[82]: Es = np.sqrt(p**2 + mu**2)
      Ts = Es-mu

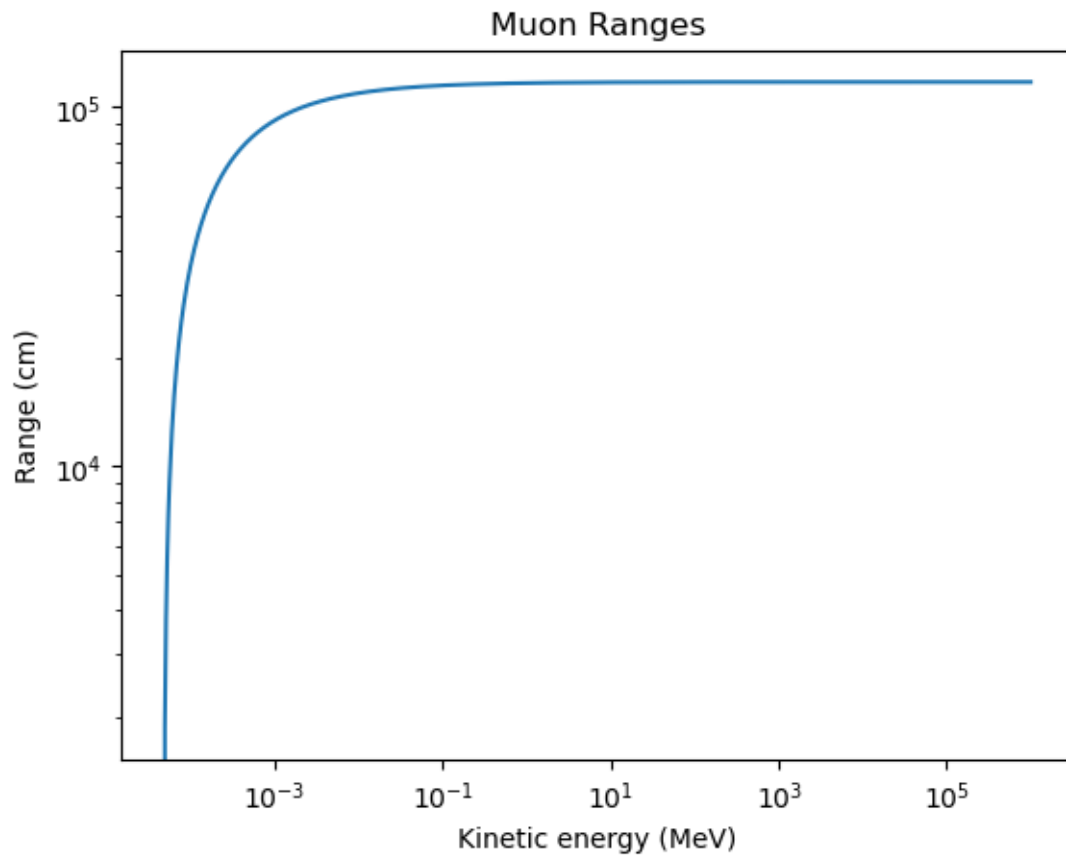
      dEdx_H2O = bethe_bloche(p, mu, materials["H2O"])
```

```
[ ]: R = -integrate.cumulative_trapezoid((1/dEdx_H2O[:-1]), Ts[:-1], initial=0)
      ↪ #negative to flip bounds of integration
```

```
[104]: plt.figure()
      plt.loglog(Ts, R)
      plt.xlabel("Kinetic energy (MeV)")
```

```
plt.ylabel("Range (cm)")  
plt.title("Muon Ranges")
```

```
[104]: Text(0.5, 1.0, 'Muon Ranges')
```



```
[ ]:
```