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Human Kinetics
BRIEF REVIEW

The W' Balance Model: Mathematical and Methodological Considerations

Philip Friere Skiba and David C. Clarke

Since its publication in 2012, the W' balance model has become an important tool in the scientific armamentarium for understanding and predicting human physiology and performance during high-intensity intermittent exercise. Indeed, publications featuring the model are accumulating, and it has been adapted for popular use both in desktop computer software and on wrist-worn devices. Despite the model's intuitive appeal, it has achieved mixed results thus far, in part due to a lack of clarity in its basis and calculation. ***Purpose**:* This review examines the theoretical basis, assumptions, calculation methods, and the strengths and limitations of the integral and differential forms of the W' balance model. In particular, the authors emphasize that the formulations are based on distinct assumptions about the depletion and reconstitution of W' during intermittent exercise; understanding the distinctions between the 2 forms will enable practitioners to correctly implement the models and interpret their results. The authors then discuss foundational issues affecting the validity and utility of the model, followed by evaluating potential modifications and suggesting avenues for further research. ***Conclusions**:* The W' balance model has served as a valuable conceptual and computational tool. Improved versions may better predict performance and further advance the physiology of high-intensity intermittent exercise.

***Keywords**:* critical power, high-intensity interval training, oxygen consumption, anaerobic threshold

The power–duration relationship is of interest to athletes, coaches, and sport scientists because it provides insights into the athlete's physiology and performance capabilities. Traditionally, this relationship is measured using laboratory-based time-to-exhaustion trials of differing durations. In cycling, the availability of bike-mounted power meters enables the power–duration relationship to be computed as best-power-for-duration efforts, which may also be extracted from the maximal mean power profile.[1] Various mathematical models have been proposed to express the power–duration relationship in terms of physiologically interpretable parameters.[2] One such construct is the 2-parameter CP (critical power) model, which describes the capacity of an individual to sustain particular work rates as a function of time. In this way, the model summarizes the relationship between exercise intensity and duration for an individual. The model can be expressed in several algebraically equivalent ways,[3] but is perhaps most easily visualized in the following formulation (Figure 1):

$$P = \frac{W'}{t} + \mathrm{CP}, \tag{1}$$

where P is the power, and t is the duration for which that P was sustained.[4] The W' represents the finite work capacity that the athlete has available when exercising in excess of the model asymptote, or CP. The most salient model assumptions with respect to the present work are as follows[3,5]:

1. Power output is a function of 2 separate components (ie, CP and W').
2. CP represents an upper limit to the sustainable production of P.
3. Exercise in excess of CP is limited by the energy reserve represented by W'.
4. Exhaustion occurs when W' is fully depleted.

Among the existing power–duration models, the 2-parameter CP model is particularly attractive due to its mathematical simplicity. It is useful for modeling the power–duration relationship for maximal exercise lasting from approximately 2 and 30 minutes, that is, within the severe domain of exercise intensity.[3,5,6] Moreover, the 2-parameter CP model has been widely studied and implemented by coaches and athletes,[7,8] making it an important tool in the translation of laboratory science to practice. Indeed, popular sports training literature is replete with references to some level of effort above which fatigue rapidly ensues.[7,9,10] Athletes soon learn to respect this perceptual cue or consign themselves to premature exhaustion and suboptimal performance.[7]

The Need for an Intermittent Model

The CP model can serve as a tool for devising optimal pacing and tactical strategies in athletic competition and has been used to inform decisions in running, swimming, and kayaking.[3,11–13] While the 2-parameter CP model is useful for predicting continuous exercise performance in the severe-intensity domain, athletes generally execute training above CP as a series of intervals with defined parameters for work rate, as well as work and recovery durations.[9,14,15]

With this in mind, Morton and Billat[16] extended the 2-parameter CP model to intermittent exercise, which has been successfully applied to both running and cycle ergometer training.[16,17] While

Skiba is with the Dept of Sports Medicine, Advocate Lutheran General Hospital, Park Ridge, IL, USA, and the Dept of Sport and Health Sciences, College of Life and Environmental Sciences, St Luke's Campus, University of Exeter, Exeter, United Kingdom. Clarke is with the Dept of Biomedical Physiology and Kinesiology and the Sports Analytics Group, Simon Fraser University, Burnaby BC, Canada. Skiba (philip.skiba@aah.org) is corresponding author.

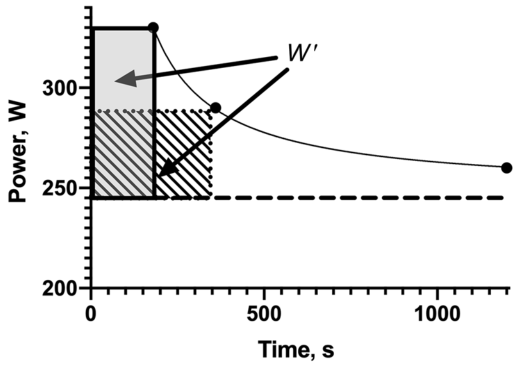

Figure 1 — The 2-parameter CP model. In this example, the model was fit to 3 constant power output efforts to exhaustion (black dots). The areas inside the hatched and gray boxes are equivalent and constant (the W'), representing the work that can be done for exercise above CP (dashed line). CP indicates upper limit to the sustainable production of power.

the model's assumptions are mathematically plausible (ie, linear discharge and recovery of the W'), the Morton–Billat[16] model oversimplifies a more complex system. In particular, the W' recovers curvilinearly after both exhaustive steady-state exercise[18–20] and during intermittent exercise.[21–24] This discrepancy is a concern to athletes and their advisors; without an accurate estimation of the recovery rate, it becomes challenging to accurately calculate the amount of W' remaining at any point in a workout or race simulation.

The W' balance (W'_{BAL}) model was formulated to devise a more practical and physiologically correct intermittent CP model.[21–24] From a practical standpoint, the model had to be applicable in the field without specialized equipment or protocols other than a power meter. From a physiological standpoint, the model had to replicate the curvilinear recovery of W'. Inspired by the impulse-response model and its mathematics, the first author (P.F.S.) proposed an integral form of the W'_{BAL} model ($W'_{\mathrm{BAL\text{-}INT}}$).[21] The derivation and subsequent validation of this model were empirical, and several experimental studies demonstrated satisfying performance of the model for analyzing and predicting intermittent exercise performance.[21–23] To improve the theoretical basis of the model, the second author (D.C.C.) derived the "differential form" of the W'_{BAL} model, which was inspired by chemical kinetics theory and the initial expressions for which were written as ordinary differential equations (ODEs), hence its abbreviation as $W'_{\mathrm{BAL\text{-}ODE}}$.[24] This version of the model offered more straightforward calculation by obviating the need for additional parameter fitting. Intriguingly, the 2 versions of the model provided different outputs in response to the same exercise protocols, but neither demonstrated unequivocal empirical superiority in terms of fitting or predicting data. Since then, both models have been increasingly studied and scrutinized, and their shortcomings have become evident. These shortcomings have pointed to paths forward for improved models of intermittent exercise.

The purpose of this narrative review is to explain in detail the W'_{BAL} models, including their assumptions and computations, and to critically review their strengths and limitations. As part of the latter discussion, we propose future directions for modeling of intermittent exercise, drawing in part upon literature featuring other models of dynamic exercise performance. We selectively cited the studies required to succinctly make our points rather than to comprehensively review all the relevant literature.

The W'_{BAL} Models

The $W'_{\mathrm{BAL\text{-}INT}}$ Model: Conceptualization, Assumptions, and Computation

The $W'_{\mathrm{BAL\text{-}INT}}$ model has been expressed as follows:

$$W'_{\mathrm{BAL\text{-}INT}}(t) = W' - \int_0^t W'_{\mathrm{EXP}} \cdot e^{\frac{-(t-u)}{\tau_{W'}}} \cdot \mathrm{d}u, \qquad (2)$$

where $W'_{\mathrm{BAL\text{-}INT}}(t)$ is the amount of W' remaining at any given time t, W' is the individual's known W' as determined from estimation of the 2-parameter CP model, W'_{EXP} represents the expended W', t and u represent time, and $\tau_{W'}$ is the time constant of the reconstitution of the W'. Typically, the W' quantities are expressed in units of joules and time in units of seconds. The equation expresses that the amount of W' remaining at any time t is equal to the difference between the known W' and the total sum of the W' expended before time t in the exercise session, each joule of which is being recharged exponentially. The rate of recovery is contingent on $\tau_{W'}$, which is calculated using the following function derived from group-averaged data[21]:

$$\tau_{W'} = 546 \cdot e^{(-0.01 \cdot D_{\mathrm{CP}})} + 316, \qquad (3)$$

where D_{CP} is the difference between CP and the (constant) P during the recovery bout.

Inspection of the $W'_{\mathrm{BAL\text{-}INT}}$ model demonstrates a first-order kinetic relationship with respect to the recovery of the W'. This form was selected because a more complex construct would present problems with parameter estimation due to the model being fit to a single data point: the time at which the subject reaches exhaustion.

Sreedhara et al[25] recently reported a problem with Equation 2. They performed a dimensional analysis of the equation and found an inequality of units: J on the left side of the equation, and $J - J \cdot s$ on the right. We now recognize that the definition of the model was mathematically imprecise: it was conceptualized as the *convolution* of the exponential function with $W'_{\mathrm{EXP}}(u)$, that is, the *function* of W'_{EXP} with respect to time. This conceptualization is analogous to that of the impulse-response model, which is clearly articulated in the appendix of Fitz-Clarke et al.[26] Sreedhara et al[25] assumed that W'_{EXP} was a constant with respect to time u, thus resulting in the inequality of units. *Therefore, to clarify that the model involves a convolution integral,* Equation 2 *ought to be written as follows*:

$$W'_{\mathrm{BAL\text{-}INT}}(t) = W'_o - \int_0^t \left[e^{\frac{-(t-u)}{\tau_{W'}}} \right] W'_{\mathrm{EXP}}(u)\mathrm{d}u, \qquad (4)$$

where

$$W'_{\mathrm{EXP}}(u) = \begin{cases} 0, P(u) \le \mathrm{CP} \\ \int (P(u) - \mathrm{CP})\mathrm{d}u,\ P(u) > \mathrm{CP} \end{cases} \qquad (5)$$

Critically, the model formulation as a convolution implies that some "recovery" of the W' is always going on, even when a net depletion of W' is observed.[23,27] This assumption represents the key distinction between the integral and differential forms of the model, and we discuss it in detail below. Equation 5 makes explicit the estimation of $W'_{\mathrm{EXP}}(u)$. We do not know $W'_{\mathrm{EXP}}(u)$ per se, but we know P and CP. We assume that the rate of change of $W'_{\mathrm{EXP}}(u)$, that is, $\frac{dW'_{EXP}(u)}{du}$, is equal to the difference between the P and CP, such that its integral gives $W'_{EXP}(u)$.

In practice, we deal with discrete digital data, such that the integrals are expressed as sums.[28]

$$W'_{BAL\text{-}INT,j} = W'_o - \sum_{i=1}^{j} e^{\frac{-(j-i)}{\tau_{W'}}} \cdot W'_{EXP,\,i} \cdot \Delta u_i, \qquad (6)$$

where

$$W'_{EXP,i} = \begin{cases} 0, \; P_i \le CP \\ (P_i - CP)\Delta u_i, \; P_i > CP \end{cases} \qquad (7)$$

and i = the ith segment of the total time subdivided into n segments, j = the segment for which $W'_{BAL\text{-}INT}$ is calculated, and P_i is a constant P exerted during the time Δu for segment i. Δu_i is commonly 1 second due to the typical 1 Hz sampling frequency of P measurement devices; however, any values for Δu_i can be specified.

To clarify the computation, we consider the following example. Note that we employ more significant figures than might be appropriate to demonstrate the small changes caused by the ongoing recovery assumption. Suppose an individual with a CP of 245 W and a W' of 14 kJ exercises at 300 W for a single second. The individual would thus expend 55 J of W' (Table 1; Figure 2). Assume that the individual continues for an additional second. Intuition tells us that they will have now expended a total sum of 110 J of W', 55 J for each second. However, the iterative evaluation developed by Skiba et al[21] indicates that the running sum is actually 109.85 J (Table 1). This lesser apparent expenditure results from the recovery of a tiny fraction of the W' in the time between the first and second seconds. That is, at the end of second 2, the sum of W' expended is 55 J plus the *remainder* of the 55 J expended in the first second (assuming $\tau_{W'}$ = 373.55 s, 54.85 J), for a running balance of 109.85 J. This yields a $W'_{BAL\text{-}INT}$ of 13,890.15 J. We arrive at this solution in the following way:

$$W'_{BAL\text{-}INT,2} = 14{,}000\,\text{J} - \left[55\,\text{W} \cdot e^{\frac{-(2-2)}{373.55}}(1\text{s}) + 55\,\text{W} \cdot e^{\frac{-(2-1)}{373.55}}(1)\right] \qquad (8)$$
$$= 13{,}890.15\,\text{J}$$

After 19 seconds, the total W'_{EXP} expended would be 1020.23 J, rather than 1045 J. The W' is indeed *depleting* with each second spent above CP. However, it is not depleting quite as quickly as might be *expected*. When P decreases below CP, the equation exhibits exponential recovery of W' with time constant $\tau_{W'}$ (Table 1; Figure 2).

$$\begin{aligned} W'_{BAL\text{-}INT,21} &= 14{,}000\,\text{J} \\ &- \left[\left(0 \cdot e^{\frac{-(21-21)}{373.55}}\right)(1\,\text{s}) + \left(0\,\text{W} \cdot e^{\frac{-(21-20)}{373.55}}\right)(1\,\text{s})\right. \\ &\left. + \ldots \left(55\,\text{W} \cdot e^{\frac{-(21-1)}{373.55}}\right)(1\,\text{s})\right] = 12{,}985.21\,\text{J} \end{aligned} \qquad (9)$$

The $W'_{BAL\text{-}INT}$ Model: Successes and Limitations

The initial success of the $W'_{BAL\text{-}INT}$ model was that the recovery time constants estimated from fitting the model to intermittent exercise protocols were similar to the time constant of an exponential function fitted to the recovery data obtained from constant work-rate exercise by Ferguson et al.[20,21] This success inspired its subsequent application to field data, modeling the performance of a cyclist in competition and providing a possible physiological rationale for suboptimal performance.[21] The $W'_{BAL\text{-}INT}$ model was then found to successfully differentiate between fatigued

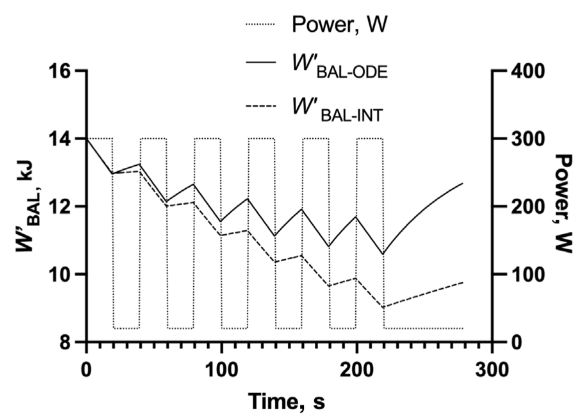

Figure 2 — Comparison between the behaviors of the $W'_{BAL\text{-}INT}$ and $W'_{BAL\text{-}ODE}$ models for the data in Table 1 (CP = 245 W; W' = 14,000 kJ). Note the faster recovery of the $W'_{BAL\text{-}ODE}$ model. ODE indicates ordinary differential equation; W'_{BAL}, W' balance; $W'_{BAL\text{-}INT}$, integral form of the W'_{BAL} model; $W'_{BAL\text{-}ODE}$, ODE form of W'_{BAL} model.

Table 1 Example Calculation of the $W'_{BAL\text{-}INT}$ and $W'_{BAL\text{-}ODE}$ Models for an Individual With CP = 245 W, W'_o = 14,000 J, and $\tau_{W'\text{-}INT}$ of 373.55 Seconds

Segment of time, *i* (*j*) (Δu = 1 s)	Power output, W	$W'_{BAL\text{-}INT}$ expended, J	$W'_{BAL\text{-}INT}$ remaining, J	$W'_{BAL\text{-}ODE}$ expended, J	$W'_{BAL\text{-}ODE}$ remaining, J
0	0	0	14,000	0	14,000
1	300	55	13,945	55	13,945
2	300	109.85	13,890.15	110	13,890
3	300	164.56	13,835.44	165	13,835
…	…	…	…	…	…
19	300	1020.23	12,979.77	1045	12955
20	20	1017.51	12,982.49	1028.34	12,971.66
21	20	1014.79	12,985.21	1011.94	12,988.06

Abbreviations: ODE, ordinary differential equation; W'_{BAL}, W' balance; $W'_{BAL\text{-}INT}$, integral form of the W'_{BAL} model; $W'_{BAL\text{-}ODE}$, ODE form of W'_{BAL} model. Note that the depletion of the $W'_{BAL\text{-}INT}$ occurs more slowly than is expected from the simple algebraic sum of the W' expended per time segment, as is calculated in the $W'_{BAL\text{-}ODE}$ model. These data may be used to check model outputs when implementing the $W'_{BAL\text{-}INT}$ and $W'_{BAL\text{-}ODE}$ models in software.

and nonfatigued states in athletes training and racing in the field.[23] In the time since its inception, the $W'_{\text{BAL-INT}}$ formulation has been applied by the first author (P.F.S.) to data from a range of sports with considerable success (eg, strategic planning in multisport racing; Figure 3).

Despite the successes of the model, it features several limitations. Biochemical and physiological evidence now exists for a relatively faster early recovery and slower later recovery than predicted by a simple exponential.[18,19,21,29] From a more practical standpoint, it is necessary to estimate the parameter, $\tau_{W'}$. The degree to which Equation 3[21] can be generalized is unknown, but the present evidence is not promising. For example, Bartram et al[30] reported $\tau_{W'}$ values considerably faster than those reported by Skiba et al[21] in a population of elite cyclists. In contrast, the first author (P.F.S.) has analyzed data from professional cyclists and triathletes and observed $\tau_{W'}$ values *slower* than those predicted by either Skiba et al[21] or Bartram et al.[30] In another study, Galbraith et al[31] attempted to model running by using the mean $\tau_{W'}$ value measured for cycling in the heavy domain,[21] concluding that the $W'_{\text{BAL-INT}}$ model was unable to accurately model intermittent running performance. They were more successful when they fit the equation to the runner's data.[31] At minimum, best practice requires estimating an *individualized* $\tau_{W'}$ for each athlete, in their *specific* mode of exercise.[22,23,27,30]

The method of calculation of the integral has important implications for model behavior. D_{CP} is typically set constant for a given protocol for convenience. However, D_{CP} varies dynamically during exercise, which implies that $\tau_{W'}$ should vary as well. Implementing this approach complicates the calculations. The resultant ongoing recovery during severe-intensity exercise departs from the assumption of Morton and Billat[16] and of the $W'_{\text{BAL-ODE}}$ model (described below), which states that W' only depletes when $P > \text{CP}$ and only recovers when $\text{CP} > P$. The assumption causes the $W'_{\text{BAL-INT}}$ model to produce an apparently slower and nonlinear depletion of W', which in turn can produce logically inconsistent behaviors.

For example, if the model is used to simulate a single continuous trial to exhaustion, it would predict a longer time to exhaustion than the 2-parameter critical power model with the same CP and W' values (Figure 4A), which is theoretically untenable. The magnitude of the error depends inversely on the power; higher power output during the work intervals will overwhelm the rate of recovery. Moreover, if we simulate the case of severe-intensity exercise to exhaustion followed immediately by exercise at CP, we find that the $W'_{\text{BAL-INT}}$ model predicts recovery when none should occur (Figure 4B). These extreme cases were not examined in the model validation studies, which involved successive short-duration efforts with unchanging work and recovery durations.[21,22] They were also not encountered in the training and racing data utilized for field validation studies.[23] Caution is therefore warranted when applying the model outside the conditions under which it was tested. Going forward, it will be necessary to evaluate the $W'_{\text{BAL-INT}}$ model utilizing a $\tau_{W'}$ that is continuously variable based upon D_{CP}. For now, best practice in the field is to use the $W'_{\text{BAL-INT}}$ model for situations in which work above CP is limited to short bursts (eg, time trial or triathlon racing).

The empirical success of the $W'_{\text{BAL-INT}}$ model despite its theoretical shortcomings motivates questions about the

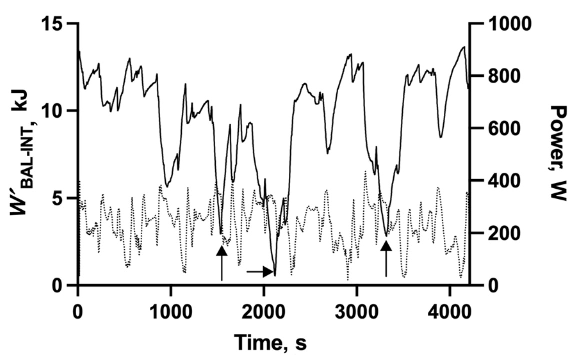

Figure 3 — Power-meter data for an elite multisport athlete (CP = 273 W and W' = 14 kJ) during a world-championship performance. The athlete was able to ride the race course multiple times in training, allowing W'_{BAL} modeling and strategic planning of race execution. The solid trace is indicative of the output of the $W'_{\text{BAL-INT}}$ model during the actual race, and the dashed trace indicates power output. Arrows point to instances of significant W' depletion. The athlete made 3 significant attacks, which resulted in a successful escape from the pack and facilitated a race win. The result of one attack was the near-complete depletion of the W' (middle arrow), which could have proved disastrous. W'_{BAL} indicates W' balance; $W'_{\text{BAL-INT}}$, integral form of the W'_{BAL} model.

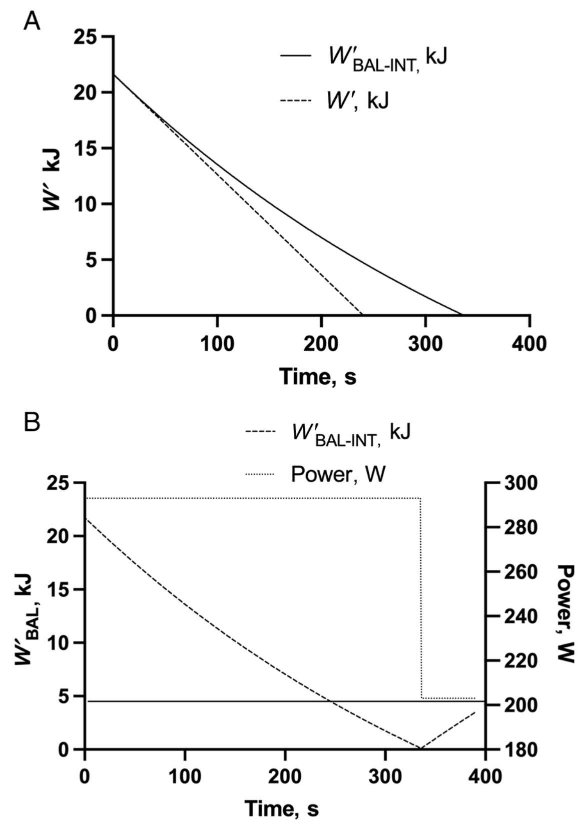

Figure 4 — Examples of extreme-case simulations that lead to the breakdown of the $W'_{\text{BAL-INT}}$ model. The simulated subject has a CP of 203 W and a W' of 21.7 kJ. (A) The comparison of the 2-parameter CP model (solid line) to the $W'_{\text{BAL-INT}}$ model (dashed line) during constant work rate exercise to exhaustion. Note the longer time required for depletion of the W' in the case of the $W'_{\text{BAL-INT}}$ model. (B) The case of a constant work rate trial to exhaustion, followed by riding at CP (solid line). Note the recovery of the $W'_{\text{BAL-INT}}$ model after approximately 330 seconds (dashed line). CP indicates upper limit to the sustainable production of power; W'_{BAL}, W' balance; $W'_{\text{BAL-INT}}$, integral form of the W'_{BAL} model.

physiology of severe-intensity exercise, particularly whether ongoing recovery can occur. Counterintuitively, evidence exists indicating that "microscopic" recovery of W' can occur during "macroscopic" depletion of W' under certain conditions.[32–34] First, Broxterman et al[32] demonstrated the sensitivity of $\tau_{W'}$ to duty cycle using handgrip exercise. In this modality, each muscle contraction takes some prescribed time, followed by a period where the muscle relaxes before the next contraction. The observations of Broxterman et al[32] were consistent with some recovery of W' *between* muscle contractions. Using a 50% duty cycle (ie, 1:1 work:rest by time), $\tau_{W'}$ was significantly longer than when using a 20% duty cycle (80:20 work:rest). In other words, $\tau_{W'}$ was observed to slow when the recovery time between contractions was reduced from 2.4 to 1.5 seconds. Broxterman et al[35] previously demonstrated increased blood flow and oxygen extraction during shorter duty cycles in this mode of exercise. It is possible that shorter duty cycles may permit greater recovery of some determinants of the W'. An analogous scenario exists during rhythmic whole-body exercise such as cycling and running, wherein the duty cycle is expressed as cadence. Vanhatalo et al[34] have observed that changes in cadence alter the apparent W': reduced cadence, that is, an increase in time between contractions, yields an increase in observed W'.

Second, recovery of metabolites has been observed during high-intensity exercise. During cycling at 80% of maximal oxygen consumption, quadriceps biopsies indicate a drop in [PCr] and adenosine triphosphate concentrations in essentially all fibers studied after 3 and 6 minutes of exercise.[33] However, by 20 minutes, some type I and type II fibers had recovered their [PCr] and adenosine triphosphate concentrations and were most likely not producing force. Assuming 80% of maximal oxygen consumption represents a work rate in the severe domain, this observation is intriguing. Further studies might consider whether these seemingly quiescent fibers can be "re-recruited" or "recycled" during intermittent exercise, and how such behavior may relate to intermittent exercise performance and the $W'_{BAL\text{-}INT}$ model.

In summary, the $W'_{BAL\text{-}INT}$ model represents an important conceptual and practical advance in sport science, as reflected by the expanding number of publications in which it has been studied and its implementation in athlete-monitoring devices and software. The $W'_{BAL\text{-}INT}$ form can yield significant physiological and performance insights, provided that the guidelines we describe are followed.[19,21–23] Nevertheless, the original model could have been more precisely defined mathematically, the computational complexities and need to fit $\tau_{W'}$ encumbers its use in practice, and it features a strong and debatable assumption about ongoing recovery during severe-intensity exercise that can lead to implausible behaviors under certain circumstances. As we discuss next, the $W'_{BAL\text{-}ODE}$ model addresses some of these issues, but it too features significant limitations for accurately modeling W' kinetics.

The $W'_{BAL\text{-}ODE}$ Model: Conceptualization, Assumptions, and Computation

The $W'_{BAL\text{-}ODE}$ model represents an attempt to derive a dynamic model of the W' from "first principles." Specifically, W' was conceptualized as a reactant in a vessel whose depletion produces breakdown product(s) that in turn contribute to the driving force for its repletion. ODE were then specified to model its rate of change as a function of power. As with the previous dynamic models of W', the model specifies that W' is depleted at a rate equal to the difference between CP and the P output when the power output P exceeds CP. Therefore, if 55 J are expended the first second and 55 J are expended the next, a total of 110 J is outstanding (Table 1). When P decreases below CP, the model specifies that the rate of W' recovery is proportional to 2 factors: (1) the amount of W' depleted relative to the known W' (W'_o), that is, the W' recovery rate slows as the W' approaches W'_o and (2) the difference in power between CP and P, which expresses the power available for recovery.[24] The model can be compactly expressed as a piecewise continuous differential equation in which the functions for depletion and recovery apply during the time segments defined by the magnitude of $P(u)$ relative to CP:

$$\frac{\mathrm{d}W'_{\text{BAL-ODE}}}{\mathrm{d}u} = \begin{cases} -(\mathrm{P}(u) - \mathrm{CP}), \ \mathrm{P}(u) \geq \mathrm{CP} \\ \left(1 - \frac{W'_{\text{BAL-ODE}}}{W'_{\mathrm{o}}}\right)(\mathrm{CP} - \mathrm{P}(u), \ \mathrm{CP} > \mathrm{P}(u). \end{cases} \tag{10}$$

It is convenient to further subdivide the time into periods for which $P(u)$ is constant. We then express the solutions to the equations as definite integrals from $u = t_a$ to $u = t_b$, where t_a and t_b are arbitrary times of interest that bound periods of constant P, as follows:

$$W'_{\text{BAL-ODE}}(t_b) = \begin{cases} W'_{\text{BAL-ODE}}(t_a) - (P - \mathrm{CP})(t_b - t_a), \ P \geq \mathrm{CP} \\ W'_o - (\mathrm{W}'_o - W'_{\text{BAL-ODE}}(t_a))\mathrm{e}^{-\left(\frac{\mathrm{CP-P}}{\mathrm{W}'_o}\right)(t_b - t_a)}, \ \mathrm{CP} > P. \end{cases} \tag{11}$$

Discrete versions of the continuous equations follow straightforwardly for the ith time period:

$$W'_{\text{BAL-ODE},i} = \begin{cases} W'_{\text{BAL-ODE},i-1} - (P_i - \mathrm{CP})\Delta u_i, \ P_i > \mathrm{CP} \\ W'_{\mathrm{o}} - (W'_{\mathrm{o}} - W'_{\text{BAL-ODE},i-1})\mathrm{e}^{-\left(\frac{\mathrm{CP}-P_i}{\mathrm{W}'_{\mathrm{o}}}\right)\Delta u_i}, \ \mathrm{CP} > P_i \end{cases} \tag{12}$$

These equations can be iteratively applied across each time segment of an entire intermittent exercise bout to estimate the time course of $W'_{\text{BAL-ODE}}$ and are straightforwardly implemented in spreadsheet software.

We demonstrate the calculations for depletion and recovery using the example numbers provided in Table 1 and Figure 2:

$$\begin{aligned} W'_{\text{BAL-ODE},3} &= 14{,}000\,\mathrm{J} \\ &- [(300 - 245\,\mathrm{W})(1\,\mathrm{s}) + (300 - 245\,\mathrm{W})(1\,\mathrm{s}) \\ &+ (300 - 245\,\mathrm{W})(1\,\mathrm{s})] = 13{,}835\,\mathrm{J}, \end{aligned} \tag{13}$$

$$\begin{aligned} W'_{\text{BAL-ODE},20} &= 14{,}000\,\mathrm{J} - \left[(14{,}000 - 12{,}955\,\mathrm{J})\mathrm{e}^{\frac{-(245-20\,\mathrm{W})}{14{,}000\,\mathrm{J}}(1\,\mathrm{s})}\right] \\ &= 12{,}971.66\,\mathrm{J} \end{aligned} \tag{14}$$

The $W'_{BAL\text{-}ODE}$ Model: Successes and Limitations

The $W'_{BAL\text{-}ODE}$ formulation features several theoretical and practical strengths relative to the $W'_{BAL\text{-}INT}$ model. First, its assumption of mutually exclusive depletion and recovery is more intuitive than the assumption of the $W'_{BAL\text{-}INT}$ model that invokes microscopic recovery simultaneously occurring with macroscopic depletion (Table 1).[24,27,30] Second, the $W'_{BAL\text{-}ODE}$ model is more straightforwardly computed than the $W'_{BAL\text{-}INT}$ model, although a computationally more efficient recursion equation approach has been implemented for the latter,[28,36] much like for the Banister impulse-response model.[5] Moreover, the $W'_{\mathrm{BAL\text{-}ODE}}$ also addresses (in principle) the issue of having to fit multiple $\tau_{W'}$ values for each individual, exercise modality, and exercise protocol. The apparent time constant for the $W'_{\mathrm{BAL\text{-}ODE}}$ is $\left(\frac{D_{\mathrm{CP}}}{W'_o}\right)^{-1}$ (ie, the time constant is the inverse of the rate constant, which is a feature of first-order rate equations). The recovery rate depends solely on the current power (known) and the CP and W'_o estimated from the original CP test, such that no formal parameter estimation is necessary. The $W'_{\mathrm{BAL\text{-}ODE}}$ model may therefore be more easily implemented than the $W'_{\mathrm{BAL\text{-}INT}}$ model.[24] Finally, although it is possible to obtain good results with the $W'_{\mathrm{BAL\text{-}INT}}$ model by assuming a constant recovery power calculated as the mean of all power values less than CP,[21–23] the $W'_{\mathrm{BAL\text{-}ODE}}$ model facilitates the calculation of a new D_{CP} every second, which should theoretically be superior.[24] Indeed, the first author (P.F.S.) has observed superior performance of the $W'_{\mathrm{BAL\text{-}ODE}}$ over the $W'_{\mathrm{BAL\text{-}INT}}$ formulation when analyzing training or racing data containing long continuous segments above CP (eg, the final climb of a mountain stage in a grand tour), presumably due to the continuously accumulating effects of the latter model's assumed recovery during exercise above CP.

The strengths and successes of the $W'_{\mathrm{BAL\text{-}ODE}}$ model are counterbalanced by several noteworthy limitations. First, the simplification of no time constant $\tau_{W'}$ implies that the $W'_{\mathrm{BAL\text{-}ODE}}$ model will be less flexible than the $W'_{\mathrm{BAL\text{-}INT}}$ model. For example, recovery kinetics depend on the power and duration of work and recovery alone.[19,22,32] However, reconstitution of the W' slows with repeated maximal exercise and varies more than expected based on the exercise intensity domain of recovery.[19,27,37] Without the individually customized $\tau_{W'}$ of the $W'_{\mathrm{BAL\text{-}INT}}$ model, the $W'_{\mathrm{BAL\text{-}ODE}}$ may be insensitive to these changes and yield unexpected results depending upon experimental conditions (Table 1; Figures 2 and 5A).

For example, applying the $W'_{\mathrm{BAL\text{-}ODE}}$ equation to the group average data of Ferguson et al[20] (CP = 213 and W' = 21.6 kJ, respectively) enables the comparison of the model-predicted W' remaining to the measured amount of W' remaining (Table 2). A simple exponential recovery fit to their data yields an apparent $\tau_{W'}$ of 336 seconds.[21] However, the $W'_{\mathrm{BAL\text{-}ODE}}$ model predictions imply a 3-fold faster recovery, with $\tau_{W'}$ equal to 112 seconds, that is $\left(\frac{D_{\mathrm{CP}}}{W'_o}\right)^{-1}$. Despite predicting more rapid recovery, the $W'_{\mathrm{BAL\text{-}ODE}}$ model may *also* predict excessively rapid exhaustion during intermittent exercise, in part because it assumes that depletion occurs without ongoing recovery. For example, we compared the $W'_{\mathrm{BAL\text{-}INT}}$ model to the $W'_{\mathrm{BAL\text{-}ODE}}$ model for a participant from Skiba et al[21] The participant performed work intervals in the severe domain for 60 seconds interspersed with 30-second recovery at 20 W. The $W'_{\mathrm{BAL\text{-}ODE}}$ model predicts exhaustion approximately 300 seconds sooner than the $W'_{\mathrm{BAL\text{-}INT}}$ model (Figure 5A). Thus, eschewing the slightly curvilinear depletion of the W' that occurs with the $W'_{\mathrm{BAL\text{-}INT}}$ model causes a potentially larger problem. Best practice requires running both models for a given scenario and using the one that proves more practically applicable to the situation and/or athlete in question.

Finally, given that the $W'_{\mathrm{BAL\text{-}ODE}}$ model is inspired by chemical reaction kinetics, it implicitly features certain assumptions that contradict known physiology. Chemical reaction kinetics models commonly (although by no means universally) assume free diffusion and homogeneous distribution of reactants throughout the reaction vessel.[38] This assumption oversimplifies the situation in the exercising limbs, the constituent muscles, and the motor units, which are spatially heterogeneous.[39–46] For example, diffusion in tissues is restricted due to membranes and tissue planes, and metabolites may be unequally distributed (eg, partitioned in discrete organelles). Muscle fiber types are differentially perfused[47,48] and feature unequal PCr depletion.[49,50] It is mathematically *possible* to account for spatially heterogeneous reactions but doing so would involve alternative, more complex model frameworks with

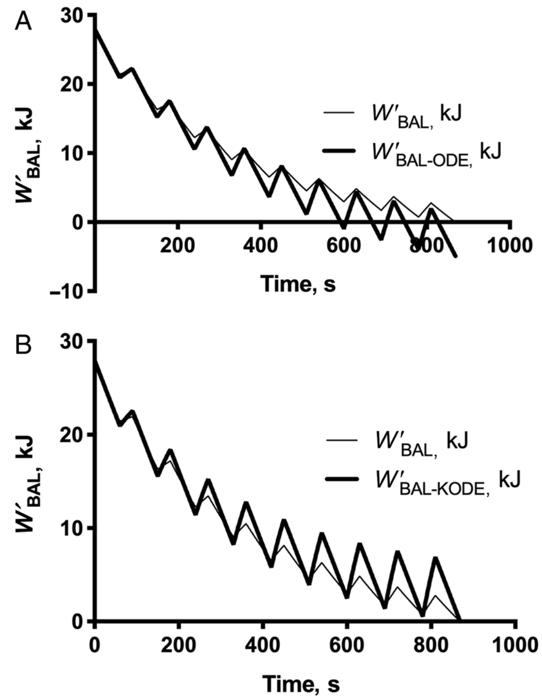

Figure 5 — Comparison of $W'_{\mathrm{BAL\text{-}INT}}$ model against the $W'_{\mathrm{BAL\text{-}ODE}}$ (A) and $W'_{\mathrm{BAL\text{-}KODE}}$ (B) forms. Subject performed a series of square wave intervals, with a 60-second work interval at 328 W, and a 30-second recovery interval at 20 W, until exhaustion. Note that both $W'_{\mathrm{BAL\text{-}INT}}$ and $W'_{\mathrm{BAL\text{-}ODE}}$ predict a similar W'_{BAL} at the end of each recovery interval, but that the $W'_{\mathrm{BAL\text{-}ODE}}$ model predicts a W'_{BAL} of 0 approximately 300 seconds early (A). $W'_{\mathrm{BAL\text{-}KODE}}$ obviates this problem, resulting in $W'_{\mathrm{BAL\text{-}INT}}$ and $W'_{\mathrm{BAL\text{-}ODE}}$ reaching 0 at the same time point (B). CP indicates the upper limit to the sustainable production of power; ODE, ordinary differential equation; W'_{BAL}, W' balance; $W'_{\mathrm{BAL\text{-}INT}}$, integral form of the W'_{BAL} model; $W'_{\mathrm{BAL\text{-}KODE}}$, $W'_{\mathrm{BAL\text{-}ODE}}$ model by introducing a constant K that increases the difference between CP and P; $W'_{\mathrm{BAL\text{-}ODE}}$, ODE form of W'_{BAL} model.

Table 2 Comparison Between Predicted and Measured W' Utilizing the Data Reported by Ferguson et al[20] and the $W'_{\mathrm{BAL\text{-}ODE}}$ Model

Time, s	W' predicted by $W'_{\mathrm{BAL\text{-}ODE}}$, kJ	W' actual, kJ
120	14.1	7.8
360	21	14.1
900	21.6	18.5

Abbreviations: ODE, ordinary differential equation; W'_{BAL}, W' balance; $W'_{\mathrm{BAL\text{-}ODE}}$, ODE form of W'_{BAL} model.

additional parameters requiring estimation. Any such framework would likely be overly speculative since little is known about the precise nature and localization of the determinants of the W'.

Foundational Issues and Future Directions

The relative strengths and weaknesses of both the $W'_{BAL\text{-}INT}$ and $W'_{BAL\text{-}ODE}$ in the practical and physiological senses are apparent. Yet, modifications to resolve these deficiencies will likely involve trade-offs that could render a given modified model even less suitable for purposes outside of the one for which it was designed. As Albert Einstein once opined, "As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality."[51]

For example, the models can be mathematically modified in a relatively straightforward manner, and such modifications have been proposed.[30] However, mathematical refinements to achieve a superior goodness of fit or predictive power neither necessarily reflect nor lead to enhanced understanding of the physiology. Conversely, modifying the model to better reflect physiological mechanisms will likely render the model too complex for routine use in sport science practice. With this caveat in mind, we discuss next the foundational issues that exist with the $W'_{BAL\text{-}INT}$ and $W'_{BAL\text{-}ODE}$ models that challenge their continued use and that will motivate future refinements, directions for which we propose.

W'_{BAL} Models for Enhancing Physiological Understanding: Uncertainties Regarding the CP and W' Inputs

One limitation of the W'_{BAL} models involves the error associated with estimates of W'_o, which propagates to the calculation of W'_{BAL}. Reported typical errors[52] for the W'_o range between 7% and 20%,[52–55] with one report estimating 46%,[53] depending upon the method of calculation used. The W'_{BAL} models depend strongly on the values of W'_o, both as the starting value but also for the time constant. Accordingly, a band of uncertainty surrounds the modeled W'_{BAL} values, such that exhaustion is unlikely to occur at exactly $W'_{BAL} = 0$ J.[23] Moreover, although task failure is assumed to correspond to W' depleting to 0, athletes can nevertheless continue exercising for more time if the power requirement is reduced to some *lower but still supra-CP value*.[56,57] Indeed, the first author (P.F.S.) has observed both model formulations oscillate around zero before the individual finally becomes exhausted (eg, Figure 5A). The W' is always observed through a slightly blurry lens. Practically speaking, the present authors have reported good performance of the 2-parameter CP model in the field for estimating CP and W'_o, whereby participants completed a minimum of 3 tests 2 to 20 minutes in duration.[23] The 3-minute all-out test can also be used[21,22]; however, the test is difficult to execute without a specialized ergometer, which represents a barrier to practitioners in the field.

Provided that CP and W'_o are accurately estimated, the W'_{BAL} models still assume that their values are constant both *within* and *between* exercise sessions. These assumptions are likely untrue. For example, the CP is sensitive to nutrition during exercise[58] and both the CP and W' have been found to decrease with altitude.[59] The CP and W' may be altered by prior exercise[58,60,61]: W' may be increased following prior exercise[62,63] and intermittent exercise has been found to functionally raise the CP,[64,65] or reciprocally to lower the apparent exercise intensity.[66–70] Discrepancies in the estimated versus actual CP and W'_o values will cause discrepant W'_{BAL} model predictions. Collectively, these observations imply that either the parameters of the W'_{BAL} models should be customized to the conditions in which the model is used or that more complex models will be required to accurately model W'_{BAL} under distinct circumstances, particularly in the field.

The W'_{BAL} models may be useful for studying possible changes in CP during intermittent exercise. For example, we return to the comparison of the $W'_{BAL\text{-}INT}$ model to the $W'_{BAL\text{-}ODE}$ model for a participant from Skiba et al.[21] We noted that the $W'_{BAL\text{-}ODE}$ model predicts exhaustion approximately 300 seconds sooner than the $W'_{BAL\text{-}INT}$ model (Figure 5A). However, suppose CP is elevated during intermittent exercise, perhaps due to the elevated baseline $\dot{V}O_2$ that follows a recovery bout prior to the start of the subsequent work bout.[18,22,64] This higher CP could be modeled in the $W'_{BAL\text{-}ODE}$ model by introducing a constant K that increases the difference between CP and P. (For clarity, this model is referred to as the $W'_{BAL\text{-}KODE}$ formulation.) The rate of W' recovery at some constant power output below CP would then be given by the following equation:

$$\frac{dW'_{BAL\text{-}ODE}}{dt} = \left(1 - \frac{W'_{BAL\text{-}ODE}}{W'_o}\right) K(CP - P),\ CP > P, \qquad (15)$$

the solution for which is

$$W'_{BAL\text{-}ODE}(t_b) = W'_o - \left(W'_o - W'_{BAL\text{-}ODE}(t_a)\right) e^{-K\left(\frac{CP-P}{W'_o}\right)(t_b - t_a)}. \quad (16)$$

In order for the $W'_{BAL\text{-}KODE}$ model to predict exhaustion at the same time as the $W'_{BAL\text{-}INT}$ model, the proposed constant K was set to 1.28 (Figure 5B). This value for K implies a 28% functional increase in CP during the intermittent exercise protocol. Notably, this amount is precisely the group average increase in CP reported by Soares-Caldeira et al[64] for intermittent exercise utilizing a recovery duration of 30 seconds. These data motivate additional study of the $W'_{BAL\text{-}KODE}$, $W'_{BAL\text{-}ODE}$, and $W'_{BAL\text{-}INT}$ model forms. The analysis reveals, for example, the possibility that the $W'_{BAL\text{-}INT}$ happens to work well with its assumption of ongoing recovery not because it is physiologically accurate but because it has the indirect effect of mimicking enhanced CP during intermittent exercise.

W'_{BAL} Models for Enhancing Physiological Understanding: Reconciliation With Mechanistic Power–Duration Models

By predicting the dynamics of fatigue-induced task failure during severe-intensity exercise, W'_{BAL} models reinforce the notion that fatigue is tied to a nebulous but predictable quantity known as W'. It stands to reason that better understanding the physiological determinants of W' ought to improve our understanding of fatigue physiology. Depletion of the W' is associated with demonstrable central fatigue,[71] an apparently "limiting" [PCr], pH, and $[P_i]$[72,73] as well as the attainment of maximal oxygen consumption.[3,73–76] Metabolic feedback from muscle metabolites (eg, $[H^+]$, $[P_i]$, $[K^+]$) underlie the sensations of fatigue that may contribute to reduced performance.[77] Recent data indicate that increased muscle carnosine (which enhances inorganic calcium release in type I myocytes, and functions as a inorganic calcium sensitizer in type I and II myocytes) correlates with faster recovery of the W'.[24] A question that arises is whether the model could be modified to better reflect some of these mechanisms and their dynamics?

Insight into this question comes from examining existing mechanistic models of the power–duration curve. Three main classes of these models have been studied: hydraulic "tank" models,[38,78–80] biochemical reaction kinetics models,[81] and motor-unit-based models.[82,83] The first 2 classes of models embody the energy systems paradigm of exercise physiology, which assumes that fatigue is chiefly caused by depletion of metabolic energy from the 3 energy systems (adenosine triphosphate/PCr, glycolysis, and oxidative phosphorylation). In hydraulic models, the muscle energy systems are conceptualized as liquids stored within tanks or vessels, with the liquid volumes representing energy and the flows representing power.[38,78] Biochemical reaction kinetics models are based on chemical reaction kinetics and explicitly represent the biochemical reactions of central metabolism. Motor-unit-based models focus on the neuromuscular basis of performance, featuring phenomena such as motor-unit recruitment, rate coding, and fatigability.

Hydraulic models are appealing because their mathematics are based on well-established fluid dynamics principles, they are relatively easy to visualize in thought experiments, and they are relatively parsimonious. Biochemical reaction kinetics models are appealing because they feature tangible biochemical processes operating within cells, such that they can be used to understand how these processes interact to produce observed emergent behaviors such as $\dot{V}O_2$.[81] Motor-unit models are appealing because their main output is force or power, and they more realistically represent physiology at the muscle level.

Despite their benefits, each model type features important conceptual and practical limitations for modeling exercise performance. From a conceptual standpoint, the first 2 model types focus primarily on *metabolic* energy and power, whereas physical performance is dictated by *mechanical* power. Metabolic power is converted to mechanical power through processes that are far less than 100% efficient, and efficiency during severe-intensity exercise is dynamic.[84] Only the most recent iterations of these models represent efficiency and fatigue, but they do so in a phenomenological manner.[79,81] These models are therefore restricted in their ability to understand the molecular basis of fatigue. In contrast, motor-unit models focus on mechanical outputs, but their functions and parameters are also phenomenological and agnostic to the molecular mechanisms causing the reduced force production.[82,83] A fourth class of models unifies muscle activation with force output, but these models are typically applied to data from single muscle fibers or isolated muscle preparations.[85,86] Collectively, these mechanistic modeling frameworks are valuable for advancing understanding of muscle fatigue and performance physiology, but no single framework is currently uniquely suited for understanding W' kinetics. However, we envisage that integrated versions of these models promise to represent the multifactorial nature of the W'. Such a model could be simulated in response to diverse intermittent exercise protocols, and its outputs studied to predict the primary physiological factors that contribute to exhaustion at different work rates. Experiments could then be performed to implement the exercise protocols, model the W' kinetics, and determine the associations between the W' kinetics and measurements of the predicted factors.

Regarding performance prediction in the field, mechanistic models are clearly unsuitable at the present time. The models are large and feature numerous adjustable parameters whose values would be inestimable from power data alone. This is a significant shortcoming as practitioners in the field typically have limited access to laboratory equipment. The W'_{BAL} models were conceived not only as an instrument to interrogate the underlying physiology, but to be a practically useful tool to athletes and their advisors. It may therefore be more reasonable to modify the existing W'_{BAL} models to better reflect the underlying physiology, as opposed to using mechanistic models. Practitioners would then be free to deploy the appropriately modified model for any particular scenario (eg, a cyclist training or racing at altitude[59]).

A Happy Medium? Toward a 2-Component W'_{BAL} Model

Strong experimental evidence was recently reported that supports the need for a multicomponent model. Specifically, Caen et al,[18] evaluated a biexponential equation for fitting W' recovery time course data. The participants cycled at a power predicted to result in exhaustion in 4 minutes (P4), identical to the work power prescribed by Skiba et al.[21,22] On each occasion, the cyclists were assigned a different recovery duration: 0.5, 1, 2, 3, 4, 5, 10, or 15 minutes, before starting a second effort to exhaustion at P4. Caen et al[18] observed a clearly superior fit of the biexponential equation compared with a monoexponential equation, particularly when the amplitude parameter values were allowed to vary. This result strongly points to a multicomponent model being needed to adequately fit and predict W' recovery kinetics in exercising humans.

Multicomponent formulations are possible for each of the W'_{BAL} model forms but none have been formally studied to date. As an example, we discuss a previously proposed multicomponent version of the $W'_{BAL\text{-}INT}$ model ($W'_{BAL\text{-}MULTI}$), which Skiba et al[21] proposed to address the limitations of a single component model. The model was originally expressed as the following equation:

$$W'_{BAL\text{-}MULTI}(t) = W' - \int_0^t \left[k_1 \cdot W'_{EXP} \cdot e^{\frac{-(t-u)}{\tau_1}} + k_2 \cdot W'_{EXP} \cdot e^{\frac{-(t-u)}{\tau_2}}\right] \cdot du, \tag{17}$$

where k_1 and k_2 are gain terms, and τ_1 and τ_2 are the time constants for the 2 different components. To clarify that the integral term represents a convolution, we restate the equation using the form and notation from Equation 4:

$$W'_{BAL\text{-}MULTI}(t) = W'_o - \int_0^t \left[k_1 \cdot e^{\frac{-(t-u)}{\tau_1}} + k_2 \cdot e^{\frac{-(t-u)}{\tau_2}}\right] W'_{EXP}(u) du, \tag{18}$$

where $W'_{EXP}(u)$ is the function expressed in Equation 5.

Like its single component counterpart, the $W'_{BAL\text{-}MULTI}$ bears similarity to the Banister impulse-response model.[5,87,88] It also resembles exponential models governing $\dot{V}O_2$ kinetics.[89] The central assumptions of the $W'_{BAL\text{-}MULTI}$ model are as follows:

1. The W' may be apportioned to 2 separate compartments.
2. The terms reflecting the compartments have different time constants to reflect different rates of W' repletion.
3. The absolute contribution to the W' of the 2 compartments is different, reflected by fractional gain terms.
4. The sum of W' in both compartments is equal to the known W'.

The components of the $W'_{BAL\text{-}MULTI}$ were notionally conceptualized to represent the type I and type II muscle fiber pools.[21,22] As part of their investigation, Caen et al[18] collected muscle biopsies to investigate the mechanistic underpinnings of their results. However, they did not assess the association of individual participants' amplitude parameter values or time constants to fiber distribution, which may have shed light on the extent to which the $W'_{BAL\text{-}MULTI}$ components could relate to muscle fiber types.[18] They did, however, assess the association between fiber types and the CP and W' estimates, observing no statistically significant relationship. Further studies will therefore be required to more definitively assess the relationship of quantities within the W'_{BAL} models to muscle fiber physiology.[90,91] Future studies should also consider alternative hypotheses regarding the multiple components. For example, the presence of a high level of P_i in fatiguing muscle fibers causes calcium phosphate precipitation in the sarcoplasmic reticulum, potentially reducing both the amount of calcium available to initiate contraction and the driving force for calcium out of the sarcoplasmic reticulum.[92–95] This calcium phosphate precipitate solubilizes quickly ($t_{1/2} = 10$ s),[95] compatible with the τ_1 reported by Caen et al[18] This mechanism, potentially connected to muscle carnosine,[24] may be worthy of investigation as one contributor to the rapid early phase of W' recovery.

The potential benefits of the $W'_{BAL\text{-}MULTI}$ are evident: the model structure allows for 2 primary dynamic processes driving the recovery of W', which allows for a much improved goodness of fit to observed time course. Furthermore, the number of adjustable parameters is still within reason, such that the model may be estimable from a logistically feasible set of intermittent exercise protocols. However, this model does not address the limitations regarding the uncertainties in CP and W'_o discussed above. It is also susceptible to all of the same shortcomings and caveats as the $W'_{BAL\text{-}INT}$ model. Thus, multicomponent versions of the other forms of the W'_{BAL} model ought to be formulated and studied as well.

Practical Applications

Provided that best practices are followed and calculations performed with care, the dynamic state of the W' predicted with reasonable fidelity in the laboratory and in the field.[22,23] This provides important insight and actionable intelligence for experimental design and for training program construction. For example, it is possible to develop a theoretically optimal interval session or pacing strategy for an athlete. It is also possible to provide post hoc analysis of training or racing data, in order to optimize performance.[21,23] The first author (P.F.S.) has used these models to advise athletes in elite training and competition for more than a decade, with favorable results (Figure 3). However, it is important for practitioners to understand the limitations we have discussed. The W'_{BAL} models should not be misinterpreted as any sort of "final word" on ability or exercise tolerance. They exist at the intersection of biochemistry, physiology and performance, and as such will require continual revision and refinement.

Conclusions

The W'_{BAL} models represent important conceptual and practical tools for understanding and predicting intermittent exercise in the severe-intensity domain. Two primary formulations of the model exist, each based on distinct assumptions regarding W' recovery dynamics. It is important for exercise physiology researchers and sport scientists to understand the basis and limitations of whichever formulation they choose. Despite their limitations, the W'_{BAL} models represent an important tool to understand athlete performance, and will continue to evolve through the efforts of curious scientists, coaches and athletes.

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