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4. a) $f(y|\theta,\phi) = \exp\left(\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\right)$: General Form

$f(y) = \lambda e^{-\lambda y}$, $\lambda, y > 0 \Rightarrow \exp(\log\lambda - \lambda y) = \exp(-\lambda y + \log(\lambda))$

$y\theta = -\lambda y \Rightarrow \theta = -\lambda$

$b(\theta) = -\log(\lambda) \Rightarrow -\log(-\theta)$

There is no dispersion term $\Rightarrow \phi = 1 \Rightarrow a(\phi) = 1$

$c(y,\phi) = 0$

$$\boxed{\begin{array}{l} \theta = -\lambda \\ b(\theta) = -\log(-\theta) \\ a(\phi) = 1 \\ \phi = 1 \\ c(y,\phi) = 0 \end{array}}$$

b) $\eta = g(\mu) = \theta$, $\theta = -\lambda$ $E[Y] = \mu = \frac{1}{\lambda} \Rightarrow \frac{-1}{\theta} = \theta = \frac{-1}{\mu}$

$\eta = g(\mu) = -\frac{1}{\mu}$: canonical link function : $g(\mu) = -\frac{1}{\mu}$

Variance function: $Var(Y) = \frac{1}{\lambda^2} = \mu^2 \Rightarrow V(Y) = \mu^2$

c) the canonical link $-\frac{1}{\mu}$ is not defined at 0 and will have trouble with numerical stability if values of μ are small. This can cause difficulties in model convergence and interpretation, especially when dealing with data that has low # of observations w/ small or near 0 means.

d) In GLMs, nested models comparisons are conducted using likelihood ratio tests which follow a χ^2 distribution. F-test should primarily be used for classical linear regression

e) $D = 2\sum_i [y_i(\log(y_i/\hat{\mu}_i) - 1 + \hat{\mu}_i]$ $f(y|\mu) = \frac{1}{\mu} e^{-y/\mu}$

$$\boxed{D = 2\sum_i \left(\frac{y_i - \hat{\mu}_i}{\hat{\mu}_i} - \log\frac{y_i}{\hat{\mu}_i}\right)}$$

5.

b) $\eta = \log(\mu)$

$$\frac{d\eta}{d\mu} = \frac{1}{\mu} \quad , \text{ variance : } V(\mu) = \mu$$

$$w_i = \frac{1}{(d\eta/d\mu)^2 V(\mu)} = \frac{1}{(1/\mu)^2 \mu} = \mu_i$$

$$z_i = \eta + (y-\mu)\frac{d\eta}{d\mu} = \log(\mu_i) + (y_i - \mu_i)\frac{1}{\mu_i}$$

$\eta = \log(\mu)$ $\quad V(\mu) = \mu$

$$z_i = \log(\mu_i) + \frac{y_i - \mu_i}{\mu_i}$$

$\frac{d\eta}{d\mu} = 1/\mu$ $\quad w_i = \mu_i$