Aidan Kardan SDS 459 HW 2

1. $f(x|\alpha) = \frac{B^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-Bx)$, $\alpha > 0, B > 0$

$$\ell(\alpha, B) = \alpha \ln(B) - \ln\Gamma(\alpha) + (\alpha-1)\ln(x) - Bx$$

$$\frac{\delta \ell}{\delta \alpha} = \ln(B) - \frac{\delta}{\delta\alpha}\ln\Gamma(\alpha) + \ln(x) \qquad \frac{\delta^2 \ell}{\delta \alpha^2} = -\frac{\delta^2}{\delta\alpha^2}\ln\Gamma(\alpha)$$

$$\frac{\delta \ell}{\delta B} = \frac{\alpha}{B} - x \qquad \frac{\delta^2 \ell}{\delta B^2} = \frac{-\alpha}{B^2} \qquad \frac{\delta^2 \ell}{\delta\alpha\delta B} = 1/B = \frac{\delta^2 \ell}{\delta B \delta\alpha}$$

Hessian : $H(\alpha,B) = \begin{pmatrix} \frac{\delta^2 \ell}{\delta\alpha^2} & \frac{\delta^2 \ell}{\delta\alpha\delta B} \\ \frac{\delta^2 \ell}{\delta B\delta\alpha} & \frac{\delta^2 \ell}{\delta B^2} \end{pmatrix} = \begin{pmatrix} -\frac{\delta^2}{\delta\alpha^2}\ln\Gamma(\alpha) & 1/B \\ 1/B & \frac{-\alpha}{B^2} \end{pmatrix}$

Fisher: $I(\alpha,B) := -E[H(\alpha,B)] \Rightarrow -H(\alpha,B) = \begin{pmatrix} \frac{\delta^2}{\delta\alpha^2}\ln\Gamma(\alpha) & -1/B \\ -1/B & \frac{\alpha}{B^2} \end{pmatrix}$

$$\det(I(\alpha,B)) = \frac{\delta^2}{\delta\alpha^2}\ln\Gamma(\alpha) \cdot \frac{\alpha}{B^2} - (-1/B)(-1/B) = \frac{\alpha \frac{\delta^2}{\delta\alpha^2}\ln\Gamma(\alpha) - 1}{B^2}$$

$$\sqrt{\det(I(\alpha,B))} = \frac{1}{B}\sqrt{\alpha \frac{\delta^2}{\delta\alpha^2}\ln\Gamma(\alpha) - 1}$$

$\psi(\alpha)$: digamma: $\frac{\delta}{\delta\alpha}\ln[\Gamma(\alpha)]$

$\psi'(\alpha)$: trigamma: $\frac{\delta^2}{\delta\alpha^2}\ln[\Gamma(\alpha)]$

$$\psi'(\alpha) = \frac{\delta^2}{\delta\alpha^2}\ln[\Gamma(\alpha)]$$

Jeffrey's prior : $\pi_J(\alpha,B) \propto \sqrt{\det(I(\alpha,B))}$

$$\boxed{\pi_J(\alpha,B) \propto \frac{1}{B}\sqrt{\alpha\,\psi'(\alpha) - 1}}$$

2,3,4,5 done in separate pdf