MECH 579: Numerical Optimization
Department of Mechanical Engineering, McGill University

Assignment #4: Constrained Optimization and Model-Order Reduction
Due 21st. November, 2025

1. Minimize the Rosenbrock function defined below using `scipy.optimize`. The function is a non-convex function and has a global minimum at $(1, 1)$. The minimum is at the bottom of a narrow parabolic valley that is curved on the x-y plane. The problem statement is as follows

$$
\begin{aligned}
\text{minimize} \quad & f(x, y) = (1 - x)^2 + 100(y - x^2)^2 \\
\text{with respect to} \quad & x, y \in \mathbb{R}^n \\
\text{subject to} \quad & \hat{c}(x, y) = 1 - x^2 - y^2 \geq 0
\end{aligned}
$$

   (a) Write a `python` code to find the minimum of the function subject to the given constraint using the SLSQP (Sequential Least Squares Programming) method. Compute the gradient and provide it to the `scipy.optimize` library. Provide the following in a written report:

      (i) Convergence of the gradient (y-axis: log of the gradient, x-axis: iteration) and a comparison of the convergence.

      (ii) Contour plot of the path of the optimization algorithm.

      (iii) Discuss and compare the plots, as well as discuss the choice of parameters used in your results and their effect on the optimization.

2. Take the problem of the Brequet Range equation from Assignment 2. According to the aircraft manufacturer, the maximum altitude that the aircraft can fly at is $2 \cdot 10^4$ m. Furthermore, in the area of interest the maximum speed the aircraft can fly at is 540 km/hr.

   (a) Solve the same optimization problem as in Assignment 3 with the maximum constraints given above and appropriate minimum constraints using the `scipy.optimize`. Compute the gradient and provide it to the `scipy.optimize` library.

      (i) Convergence of the gradient of the Lagrangian (y-axis: log of the gradient, x-axis: iteration).

      (i) Convergence of the range and constraints as a function of design iterations.

      (iii) Contour plot of the path of the optimization algorithm.

3. Solve the unconstrained Brequet Range equation from Assignment 2 but use a Neural Network approach. You may use the provided code to optimize the Rosenbrock as a starting point. In your report, please provide the following:

(a) Train a Neural Network to compute the Brequet Range Equation. Define the initial sample set to be 50 data points (pairs of velocity and altitude) and decide on the number of epochs.

- Provide a plot of the convergence of the loss function as a number of epochs. (Note: Use 10000 as the maximum number of epochs and you may choose to increase it if necessary)
- Change the size of the initial sample set and compare the convergence of the loss function as a number of epochs.
- Change the Neural Network model itself. You may choose at least two different parameters: such as number of hidden layers, size of nodes per hidden layer, etc. Compare the convergence of the loss function between the cases.

(b) Build a Neural Network model to optimize the Brequet Range Equation.

- Provide a plot of the convergence of the objective function as a number of epochs. Increase the number of epochs and determine the final required number of epochs.
- Compare the convergence against the SQP approach with respect to time per iteration.

Reports must be handed in a PDF format. All plots must have both x- and y-axis labels, a legend clearly describing the various lines, and a title with a Figure number. Plots generated with MS Excel are not acceptable and assignments will not be graded.