1 Bayes Classifiers

1.1 a)

A joint Bayes classifier can be found by counting how many times each possible (x1, x2) combination occurs, then finding how many of each of these associate with the y class

Where

$$x = [0, 0]$$

$$P(y|x) = \frac{1}{4} \ , \ y = 0$$

$$P(y|x) = \frac{3}{4} \ , \ y = 1$$

Where

$$x = [0, 1]$$

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Where

$$x = [1, 0]$$

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Where

$$x = [1, 1]$$

$$P(y|x) = \frac{3}{5} \ , \ y = 0$$

$$P(y|x) = \frac{2}{5} \ , \ y = 1$$

So the complete class table is:

x1	x2	P(y=0|x)	P(y=1|x)	y-hat
0	1	25%	75%	1
1	0	100%	0%	0
1	1	60%	40%	0

1.2 (b)

For a Naive Bayes classified the probabilities of each class, which are equal, will be needed

$$P(y = 0) = \frac{8}{16}$$

$$P(y = 1) = \frac{8}{16}$$

Next, the probability of each x for the two classes is needed

	y=0	y=1
x1	6/8	2/8
x2	4/8	5/8

Next, calculate P(x|y) for each ()$x_1,\ x_2$) combination for each class, seen in the test set

$$P(x = [0,1] \mid y = 0) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(1 - \frac{6}{8}\right) \cdot \left(\frac{4}{8}\right)$$

$$= \frac{1}{8}$$

$$P(x = [0,1] \mid y = 1) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(1 - \frac{2}{8}\right) \cdot \left(\frac{5}{8}\right)$$

$$= \frac{15}{32}$$

$$P(x = [1,0] \mid y = 0) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{6}{8}\right) \cdot \left(1 - \frac{4}{8}\right)$$

$$= \frac{3}{8}$$

$$P(x = [1,0] \mid y = 1) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{2}{8}\right) \cdot \left(1 - \frac{5}{8}\right)$$

$$= \frac{3}{32}$$

$$P(x = [1,1] \mid y = 0) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{6}{8}\right) \cdot \left(\frac{4}{8}\right)$$

$$= \frac{3}{8}$$

$$P(x = [1,1] \mid y = 1) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{2}{8}\right) \cdot \left(\frac{5}{8}\right)$$

$$= \frac{5}{32}$$

Calculate P(x) for each combination of ()$x_1,\ x_2$) with the following formula

$$P(x) = \sum_i P(x|y_i) \cdot P(y_i)$$

$$P(x = [0,1]) = \left(\frac{1}{8} \cdot \frac{1}{2}\right) + \left(\frac{15}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{19}{64}$$

$$P(x = [1,0]) = \left(\frac{3}{8} \cdot \frac{1}{2}\right) + \left(\frac{3}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{15}{64}$$

$$P(x = [1,1]) = \left(\frac{3}{8} \cdot \frac{1}{2}\right) + \left(\frac{5}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{17}{64}$$

Then use Bayes Rule to find the probability of y given each x value.

$$P(y|x) = \frac{P(x|y) \cdot P(y)}{P(x)}$$

x=[0,1]:

$$P(y=0|x=[0,1]) = \frac{P(x|y)\cdot P(y)}{P(x)}$$

$$= \frac{(1/8)\cdot(1/2)}{19/64}$$

$$\approx 21\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 79\%$$

x=[1,0]:

$$P(y=0|x=[1,0]) = \frac{P(x|y)\cdot P(y)}{P(x)}$$

$$= \frac{(3/8)\cdot(1/2)}{15/64}$$

$$\approx 80\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 20\%$$

x=[1,1]:

$$P(y=0|x=[1,1]) = \frac{P(x|y)\cdot P(y)}{P(x)}$$

$$= \frac{(3/8)\cdot(1/2)}{17/64}$$

$$\approx 71\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 29\%$$

The complete predictions on the test set using Naive Bayes is then

x1	x2	P(y=0|x)	P(y=1|x)	y-hat
0	1	21%	79%	1
1	0	80%	20%	0
1	1	71%	29%	0