1 Bayes Classifiers

1.1 a)

Probabilities needed for joint Bayes classifier can be found by counting the number of occurances of each possible x1 x2 combination ([0,0] [0,1] [1,0] [1,1]). Then finding how many of each of these assosiate with each class. E.g.

Where

$$y = 0$$

$$P(y|x) = \frac{1}{4}\ ,\ x = [0, 0]$$

$$P(y|x) = \frac{1}{4}\ ,\ x = [0, 1]$$

$$P(y|x) = \frac{3}{3}\ ,\ x = [1, 0]$$

$$P(y|x) = \frac{3}{5}\ ,\ x = [1, 1]$$

Where

$$y = 1$$

$$P(y|x) = \frac{3}{4}\ ,\ x = [0, 0]$$

$$P(y|x) = \frac{3}{4}\ ,\ x = [0, 1]$$

$$P(y|x) = \frac{0}{3}\ ,\ x = [1, 0]$$

$$P(y|x) = \frac{2}{5}\ ,\ x = [1, 1]$$

Therefore, the complete class predictions on the test set looks as follows:

x1	x2	P(y=0—x)	P(y=1—x)	y-hat
0	1	25%	75%	1
1	0	100%	0%	0
1	1	60%	40%	0

1.2 (b)

To create a Naive Bayes classifier first the probabilities of each class will be needed:

$$P(y=0)=\frac{8}{16}$$

$$P(y=1)=\frac{8}{16}$$

Next, the probability that each x could be in each class:

	y=0	y=1
x1	6/8	2/8
x2	4/8	5/8

To classify the test set P(x—y) is calculated for each $x_1,\ x_2$ combination seen in the test set, on each class

$$P(x=[0,1]\mid y=0)=P(x_1\mid y)\cdot P(x_2\mid y)$$

$$=\left(1-\frac{6}{8}\right)\cdot\left(\frac{4}{8}\right)$$

$$=\frac{1}{8}$$

$$P(x=[0,1]\mid y=1)=P(x_1\mid y)\cdot P(x_2\mid y)$$

$$=\left(1-\frac{2}{8}\right)\cdot\left(\frac{5}{8}\right)$$

$$=\frac{15}{32}$$

$$P(x=[1,0]\mid y=0)=P(x_1\mid y)\cdot P(x_2\mid y)$$

$$=\left(\frac{6}{8}\right)\cdot\left(1-\frac{4}{8}\right)$$

$$=\frac{3}{8}$$

$$P(x=[1,0]\mid y=1)=P(x_1\mid y)\cdot P(x_2\mid y)$$

$$=\left(\frac{2}{8}\right)\cdot\left(1-\frac{5}{8}\right)$$

$$= \frac{3}{32}$$

$$P(x = [1,1] \mid y = 0) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{6}{8}\right) \cdot \left(\frac{4}{8}\right)$$

$$= \frac{3}{8}$$

$$P(x = [1,1] \mid y = 1) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{2}{8}\right) \cdot \left(\frac{5}{8}\right)$$

$$= \frac{5}{32}$$

Next the P(x) is calculated with the formula

$$P(x) = \sum_i P(x|y_i) \cdot P(y_i)$$

on all test data values of x.

$$P(x = [0,1]) = \left(\frac{1}{8} \cdot \frac{1}{2}\right) + \left(\frac{15}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{19}{64}$$

$$P(x = [1,0]) = \left(\frac{3}{8} \cdot \frac{1}{2}\right) + \left(\frac{3}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{15}{64}$$

$$P(x = [1,1]) = \left(\frac{3}{8} \cdot \frac{1}{2}\right) + \left(\frac{5}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{17}{64}$$

Finally, the probability of y given each x value can be found by using Bayes rule:

$$P(y|x) = \frac{P(x|y) \cdot P(y)}{P(x)}$$

On x=[0,1]:

$$P(y=0|x=[0,1]) = \frac{P(x|y) \cdot P(y)}{P(x)}$$

$$= \frac{(1/8) \cdot (1/2)}{19/64}$$

$$\approx 21\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 79\%$$

And on x=[1,0]:

$$P(y=0|x=[1,0]) = \frac{P(x|y) \cdot P(y)}{P(x)}$$

$$= \frac{(3/8) \cdot (1/2)}{15/64}$$

$$\approx 80\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 20\%$$

And on x=[1,1]:

$$P(y=0|x=[1,1]) = \frac{P(x|y) \cdot P(y)}{P(x)}$$

$$= \frac{(3/8) \cdot (1/2)}{17/64}$$

$$\approx 71\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 29\%$$

Therefore, the complete class predictions on the test set using Naive Bayes looks as follows:

x1	x2	P(y=0—x)	P(y=1—x)	y-hat
0	1	21%	79%	1
1	0	80%	20%	0
1	1	71%	29%	0