CAB420 - Assignment 2

Contents

Part A: SVMs and Bayes Classifiers

1)

```
clear
close all
load data_ps3_2.mat;
```

set 1 - linear

```
svm_test(@Klinear, 1, 1000, set1_train, set1_test);
title('Set 1 - Linear');
hold off;
```

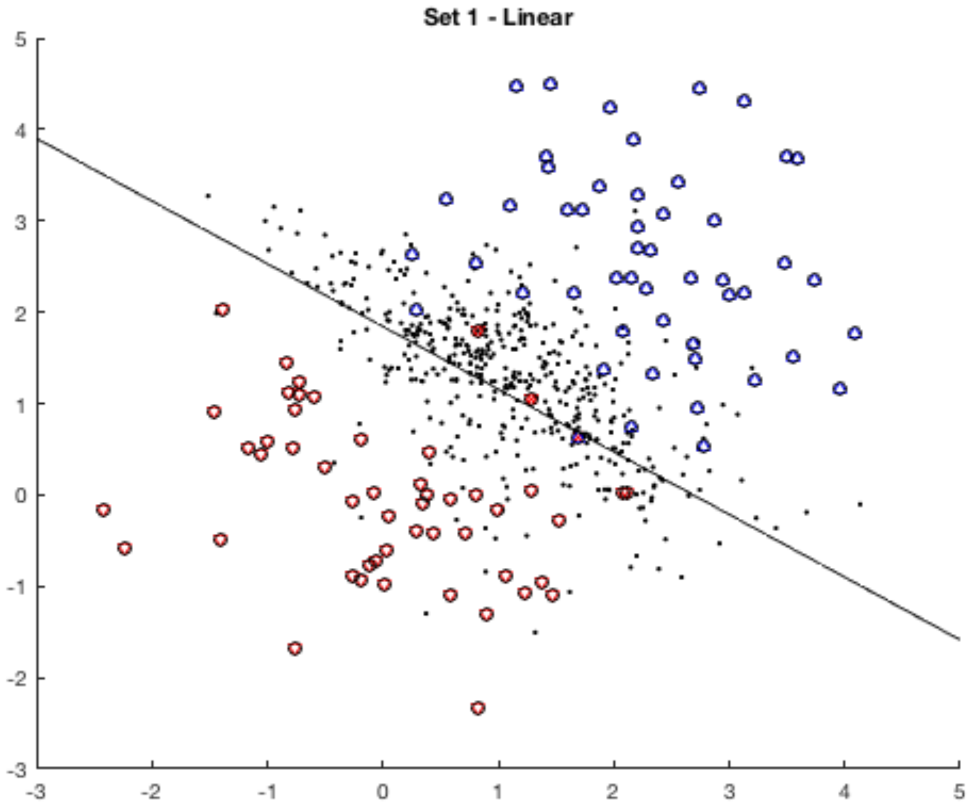
set 2 - polynomial

```
svm_test(@Kpoly, 2, 1000, set2_train, set2_test);
title('Set 2 - Polynomial');
hold off;
```

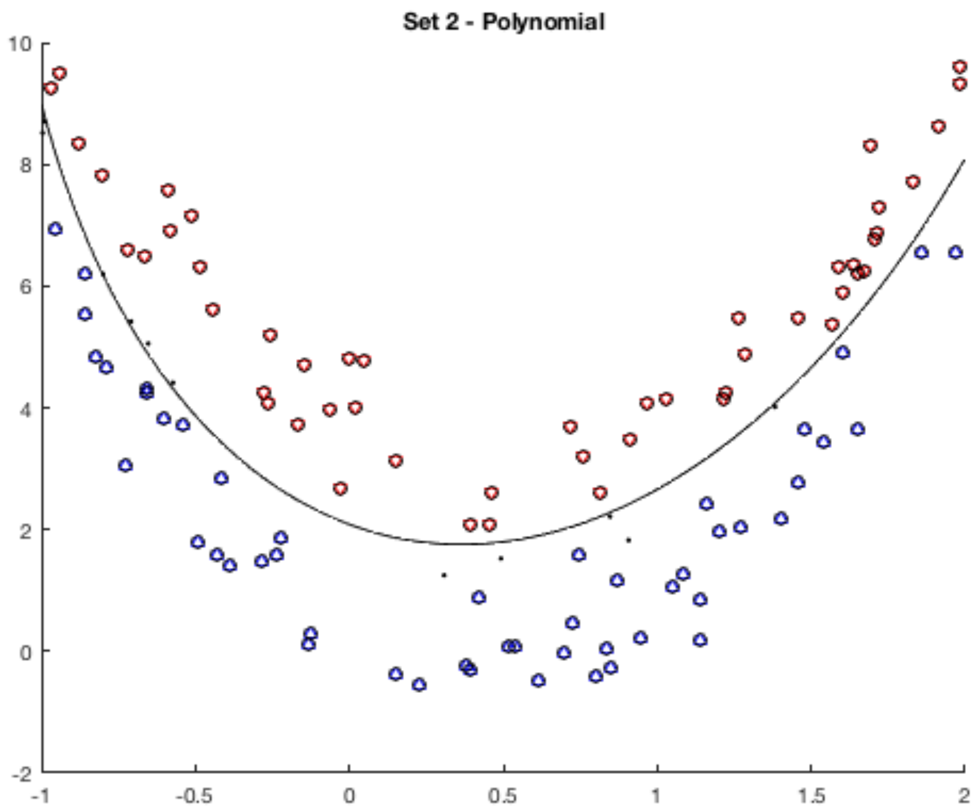
set 3 - gaussian

```
svm_test(@Kgaussian, 1, 1000, set3_train, set3_test);
title('Set 3 - Gaussian');
hold off;
```

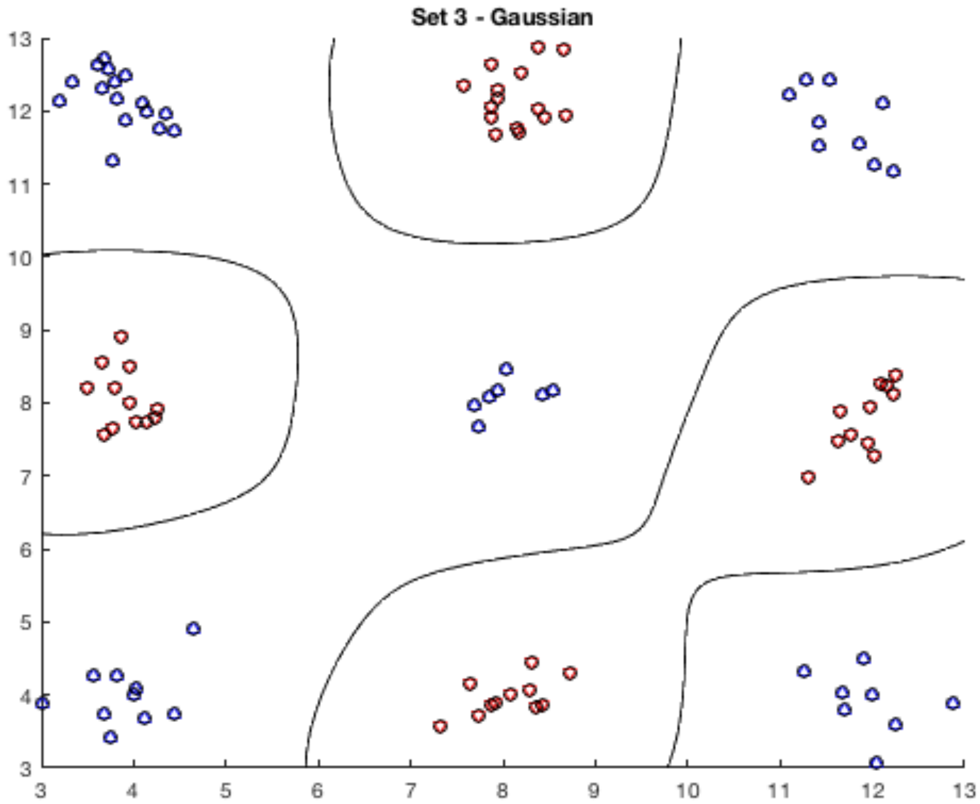
```
The interior-point-convex algorithm does not accept an initial point.
Ignoring X0.

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

WARNING: 3 training examples were misclassified!!!
TEST RESULTS: 0.0446 of test examples were misclassified.
The interior-point-convex algorithm does not accept an initial point.
Ignoring X0.

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

TEST RESULTS: 0.011 of test examples were misclassified.
The interior-point-convex algorithm does not accept an initial point.
Ignoring X0.

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

TEST RESULTS: 0 of test examples were misclassified.
```



Set 1 - Linear



Set 2 - Polynomial



2)

```
% set 4 - all of them
linear_error = svm_test2(@Klinear, 1, 1000, set4_train, set4_test);
poly_error = svm_test2(@Kpoly, 2, 1000, set4_train, set4_test);
gauss_error = svm_test2(@Kgaussian, 1.5, 1000, set4_train, set4_test);
```

The guassian method has the best results, with only 0.085 test examples being miscalsified. Polynomial was next best with 0.12, and linear was the worst with 0.1375

```
The interior-point-convex algorithm does not accept an initial point.
Ignoring X0.

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

TEST RESULTS: 0.1375 of test examples were misclassified.
The interior-point-convex algorithm does not accept an initial point.
Ignoring X0.

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

TEST RESULTS: 0.12 of test examples were misclassified.
The interior-point-convex algorithm does not accept an initial point.
```

```
Ignoring X0.

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

TEST RESULTS: 0.085 of test examples were misclassified.
```

1 Bayes Classifiers

1.1 a)

A joint Bayes classifier can be found by counting how many times each possible (x1, x2) combination occurs, then finding how many of each of these associate with the y class

Where

$$x = [0, 0]$$

$$P(y|x) = \frac{1}{4}\ ,\ y = 0$$

$$P(y|x) = \frac{3}{4}\ ,\ y = 1$$

Where

$$x = [0, 1]$$

$$P(y|x) = \frac{1}{4}\ ,\ y = 0$$

$$P(y|x) = \frac{3}{4}\ ,\ y = 1$$

Where

$$x = [1, 0]$$

$$P(y|x) = \frac{3}{3}\ ,\ y = 0$$

$$P(y|x) = \frac{0}{3}\ ,\ y = 1$$

Where

$$x = [1, 1]$$

$$P(y|x) = \frac{3}{5}\ ,\ y = 0$$

$$P(y|x) = \frac{2}{5}\ ,\ y = 1$$

So the complete class table is:

x1	x2	P(y=0|x)	P(y=1|x)	y-hat
0	1	25%	75%	1
1	0	100%	0%	0
1	1	60%	40%	0

1.2 (b)

For a Naive Bayes classified the probabilities of each class, which are equal, will be needed

$$P(y = 0) = \frac{8}{16}$$

$$P(y = 1) = \frac{8}{16}$$

Next, the probability of each x for the two classes is needed

	y=0	y=1
x1	6/8	2/8
x2	4/8	5/8

Next, calculate P(x|y) for each ()$x_1,\ x_2$) combination for each class, seen in the test set

$$P(x = [0, 1] \mid y = 0) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(1 - \frac{6}{8}\right) \cdot \left(\frac{4}{8}\right)$$

$$= \frac{1}{8}$$

$$P(x = [0, 1] \mid y = 1) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(1 - \frac{2}{8}\right) \cdot \left(\frac{5}{8}\right)$$

$$= \frac{15}{32}$$

$$P(x = [1, 0] \mid y = 0) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{6}{8}\right) \cdot \left(1 - \frac{4}{8}\right)$$

$$= \frac{3}{8}$$

$$P(x = [1, 0] \mid y = 1) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{2}{8}\right) \cdot \left(1 - \frac{5}{8}\right)$$

$$= \frac{3}{32}$$

$$P(x = [1,1] \mid y = 0) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{6}{8}\right) \cdot \left(\frac{4}{8}\right)$$

$$= \frac{3}{8}$$

$$P(x = [1,1] \mid y = 1) = P(x_1 \mid y) \cdot P(x_2 \mid y)$$

$$= \left(\frac{2}{8}\right) \cdot \left(\frac{5}{8}\right)$$

$$= \frac{5}{32}$$

Calculate P(x) for each combination of ()$x_1,\ x_2$) with the following formula

$$P(x) = \sum_i P(x|y_i) \cdot P(y_i)$$

$$P(x = [0,1]) = \left(\frac{1}{8} \cdot \frac{1}{2}\right) + \left(\frac{15}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{19}{64}$$

$$P(x = [1,0]) = \left(\frac{3}{8} \cdot \frac{1}{2}\right) + \left(\frac{3}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{15}{64}$$

$$P(x = [1,1]) = \left(\frac{3}{8} \cdot \frac{1}{2}\right) + \left(\frac{5}{32} \cdot \frac{1}{2}\right)$$

$$= \frac{17}{64}$$

Then use Bayes Rule to find the probability of y given each x value.

$$P(y|x) = \frac{P(x|y) \cdot P(y)}{P(x)}$$

x=[0,1]:

$$P(y=0|x=[0,1]) = \frac{P(x|y)\cdot P(y)}{P(x)}$$

$$= \frac{(1/8)\cdot(1/2)}{19/64}$$

$$\approx 21\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 79\%$$

x=[1,0]:

$$P(y=0|x=[1,0]) = \frac{P(x|y)\cdot P(y)}{P(x)}$$

$$= \frac{(3/8)\cdot(1/2)}{15/64}$$

$$\approx 80\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 20\%$$

x=[1,1]:

$$P(y=0|x=[1,1]) = \frac{P(x|y)\cdot P(y)}{P(x)}$$

$$= \frac{(3/8)\cdot(1/2)}{17/64}$$

$$\approx 71\%$$

Therefore:

$$P(y=1|x=[0,1]) \approx 29\%$$

The complete predictions on the test set using Naive Bayes is then

x1	x2	P(y=0|x)	P(y=1|x)	y-hat
0	1	21%	79%	1
1	0	80%	20%	0
1	1	71%	29%	0

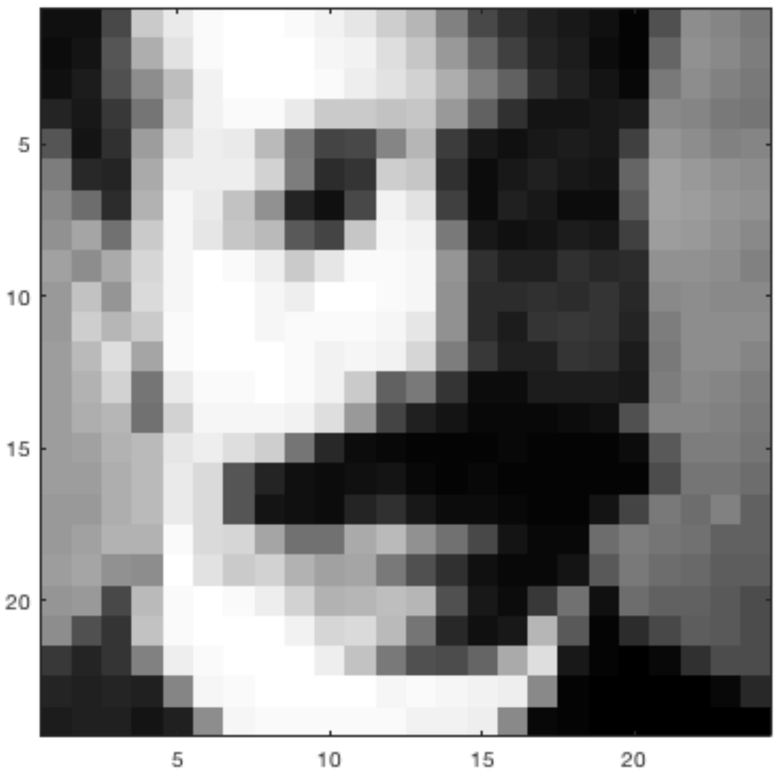
Part B: PCA & Clustering

clear and load stuff

```
clear
close all
X = load('data/faces.txt'); % load face dataset
```

understand the data

```
i=2;
img = reshape(X(i,:),[24 24]); % convert vectorized datum to 24x24 image patch
imagesc(img); axis square; colormap gray; % display an image patch
```



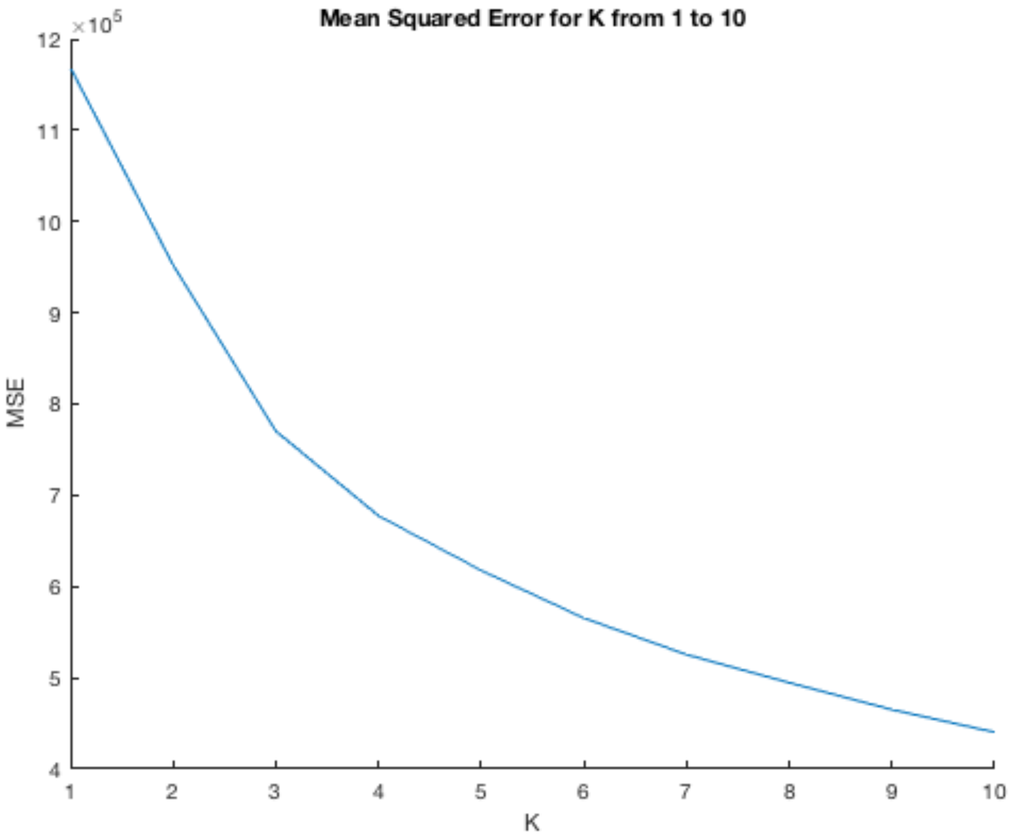
a)

```
[m, n] = size(X);
```

```
% subtract the mean of the face images to make the data sero-mean
mu = mean(X);
X0 = bsxfun(@minus, X, mu);

% take the SVD of the data
[U, S, V] = svd(X0);
W=U*S;
```

b)

```
K = 1:10;
meansquarederr = zeros(size(K));
for i=1:length(K)
    X0_hat = W(:, 1:K(i))*V(:, 1:K(i))';
    meansquarederr(i) = sum(mean((X0-X0_hat).^2));
end

figure();
hold on;
plot(meansquarederr);
xlabel('K');
ylabel('MSE');
title('Mean Squared Error for K from 1 to 10');
hold off;
```



c)

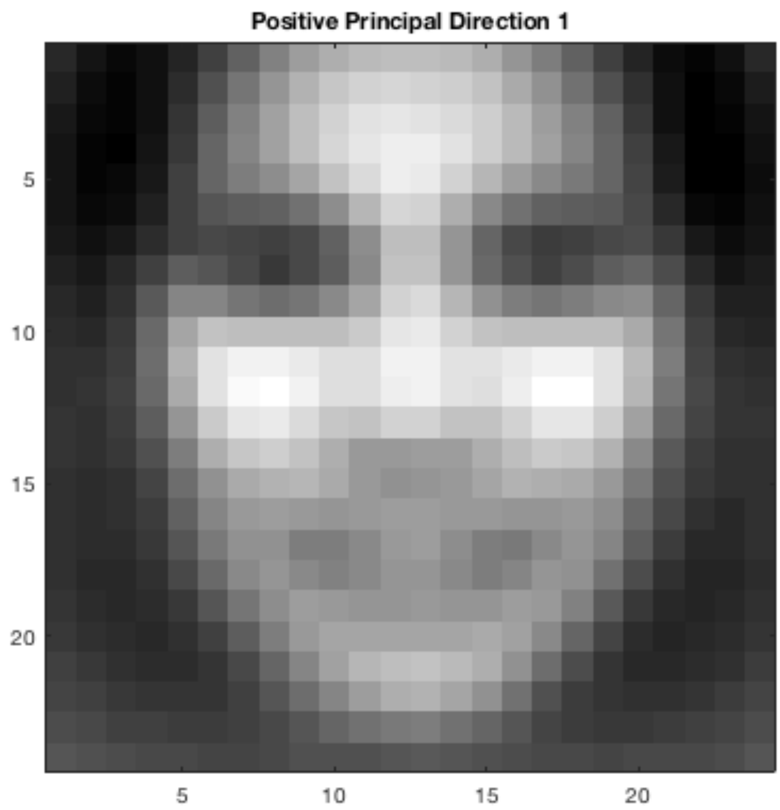
```
positive_principals = {};
negative_principals = {};
```

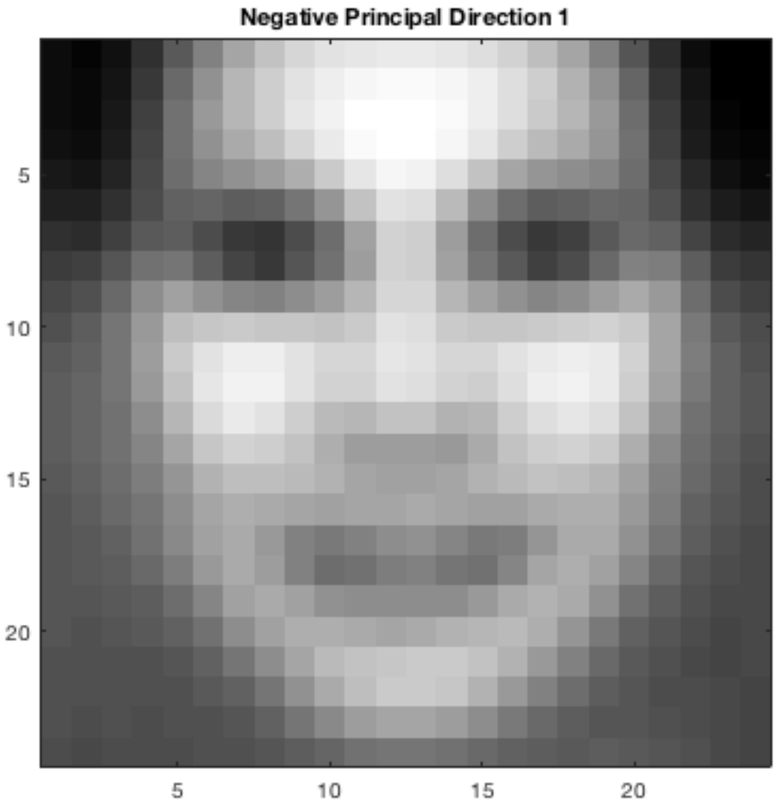
Compute the principal directions

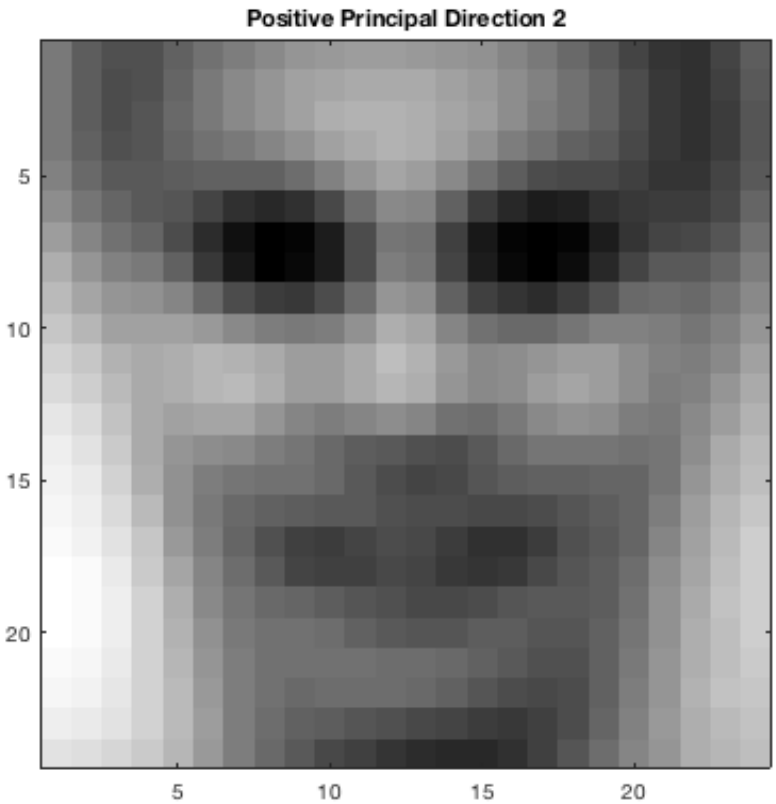
```
for j=1:10
    alpha = 2*median(abs(W(:, j))); % scale factor
    positive_principals{j} = mu + alpha*(V(:, j)');
    negative_principals{j} = mu - alpha*(V(:, j)');
end
```

Reshape them and view them as images

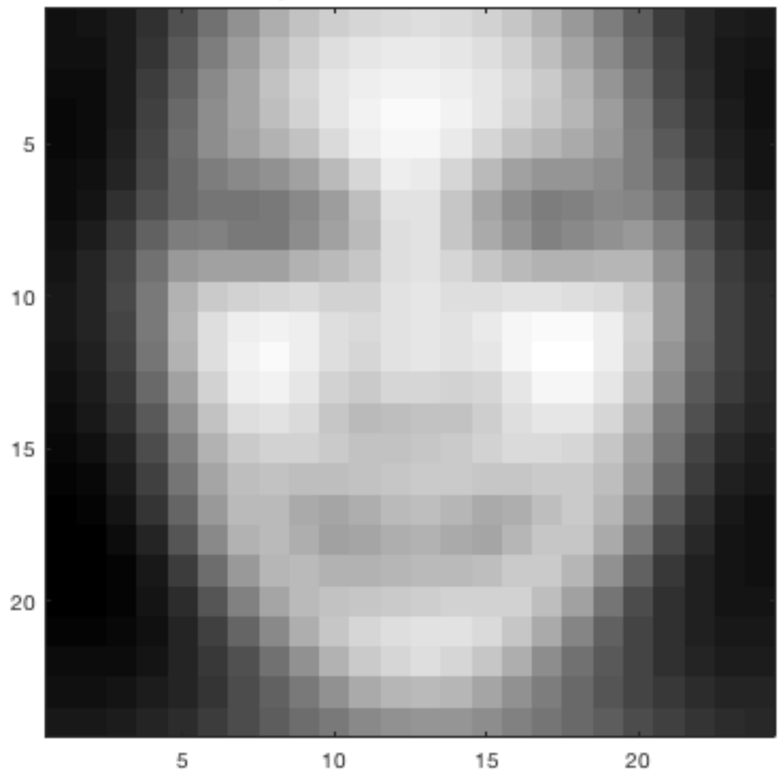
```
for i=1:3
    img = reshape(positive_principals{i}, [24, 24]);
    figure('name', sprintf('Positive Principal Direction %d', i));
    imagesc(img);
    title(sprintf('Positive Principal Direction %d', i));
    axis square;
    colormap gray;

    img = reshape(negative_principals{i}, [24, 24]);
    figure('name', sprintf('Negative Principal Direction %d', i));
    imagesc(img);
    title(sprintf('Negative Principal Direction %d', i))
    axis square;
    colormap gray;
end
```



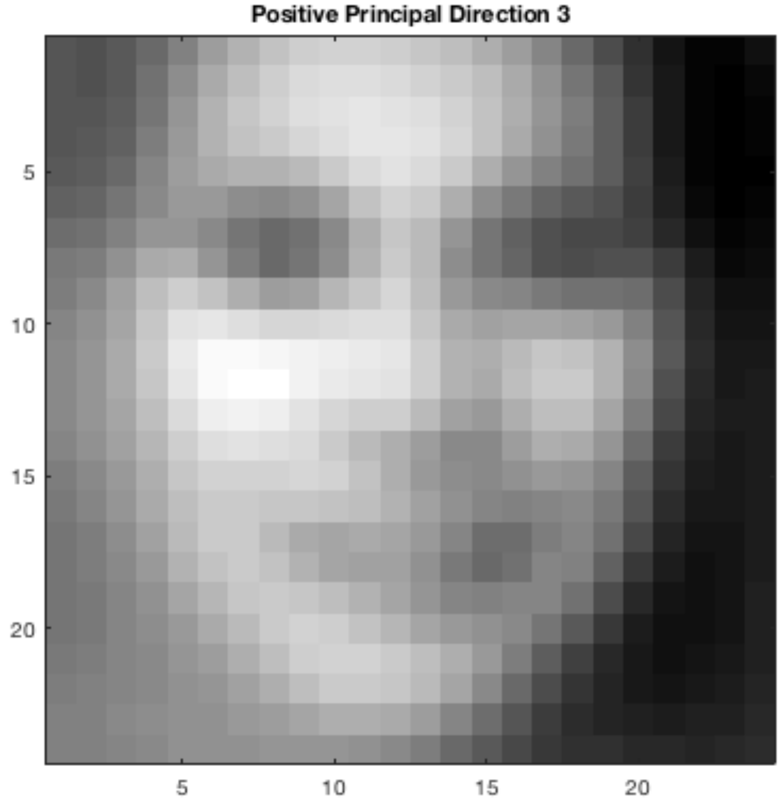

Negative Principal Direction 1
5
10
15
20
5
10
15
20

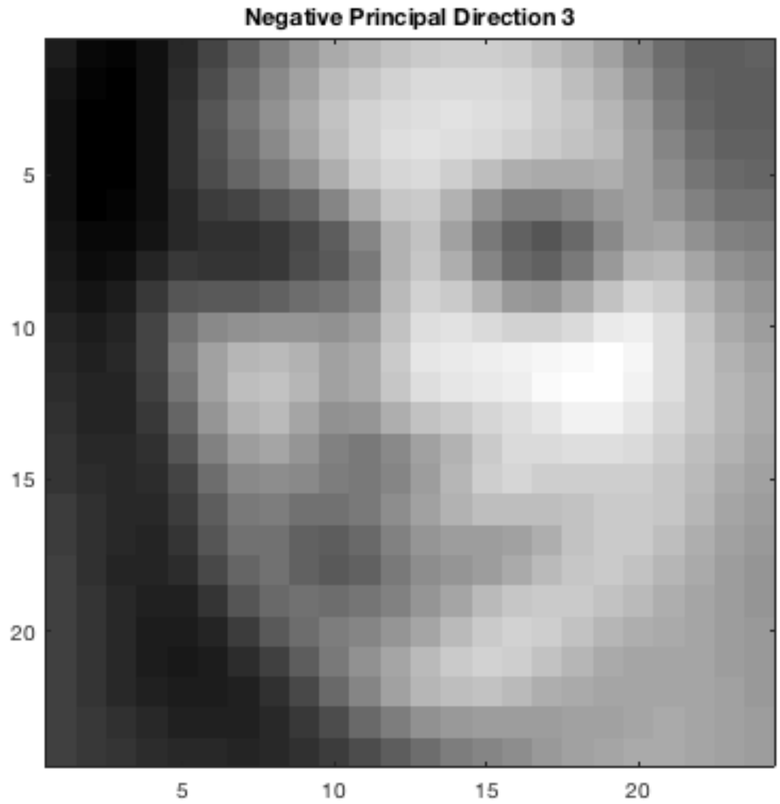

Positive Principal Direction 2
5
10
15
20
5
10
15
20

Negative Principal Direction 2



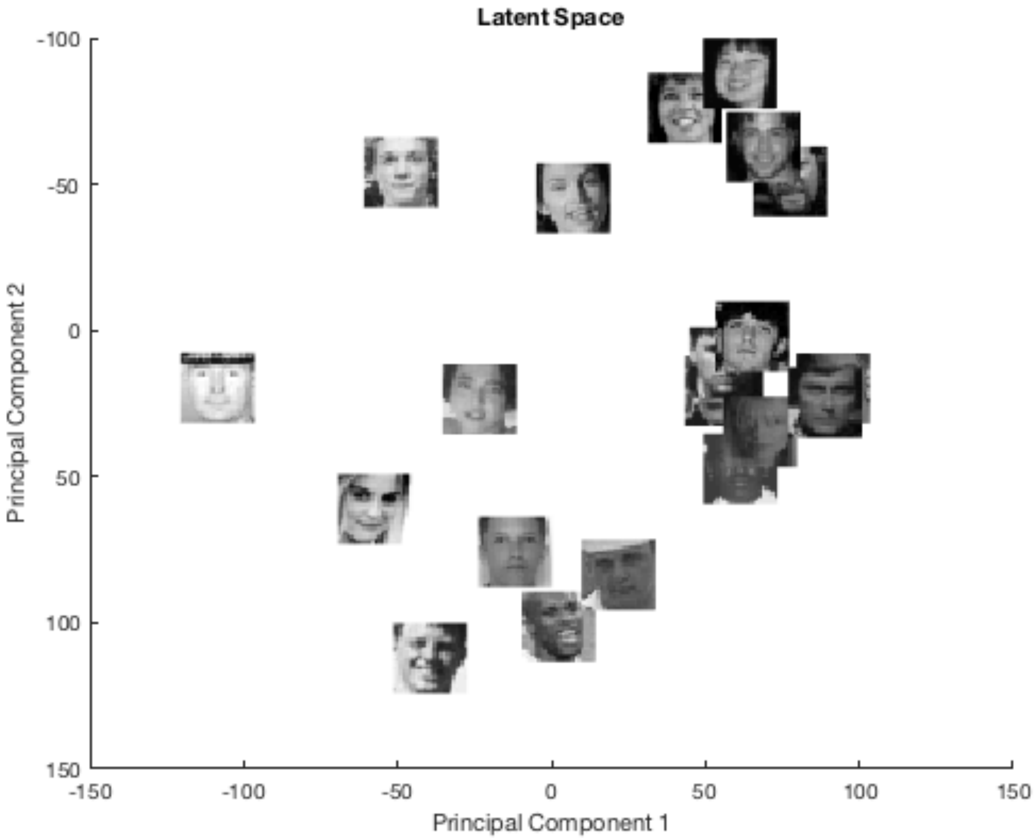
Positive Principal Direction 3





d

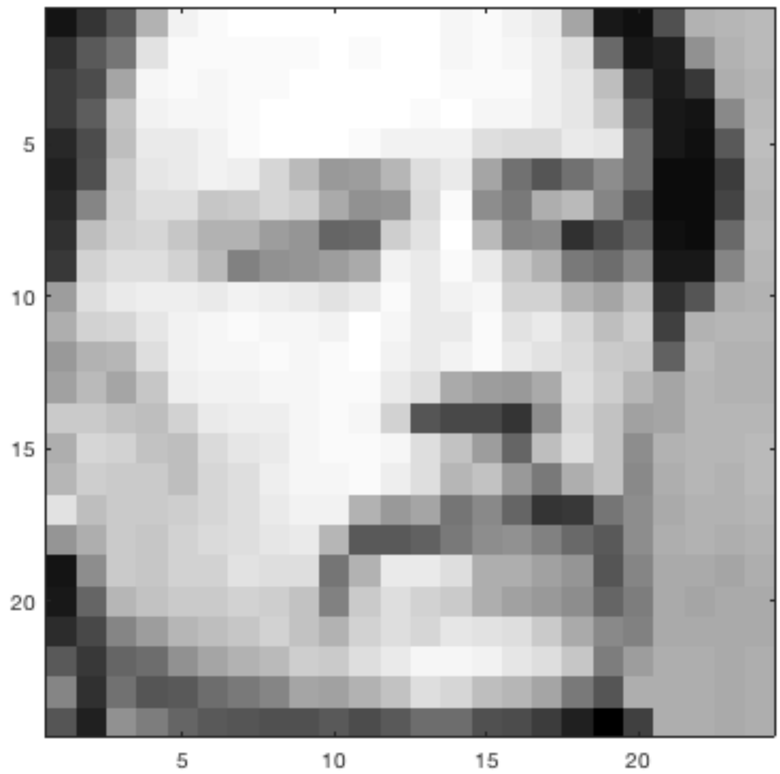
```
idx = randperm(576, 20); % Generate random numbers for the index

figure; hold on; axis ij; colormap(gray);
title('Latent Space')
xlabel('Principal Component 1');
ylabel('Principal Component 2');

range = max(W(idx, 1:2)) - min(W(idx, 1:2)); % find range of coordinates to be plotted
scale = [200 200]./range; % want 24x24 to be visible

for i=idx
    imagesc(W(i,1)*scale(1),W(i,2)*scale(2), reshape(X(i,:), 24, 24));
end
```



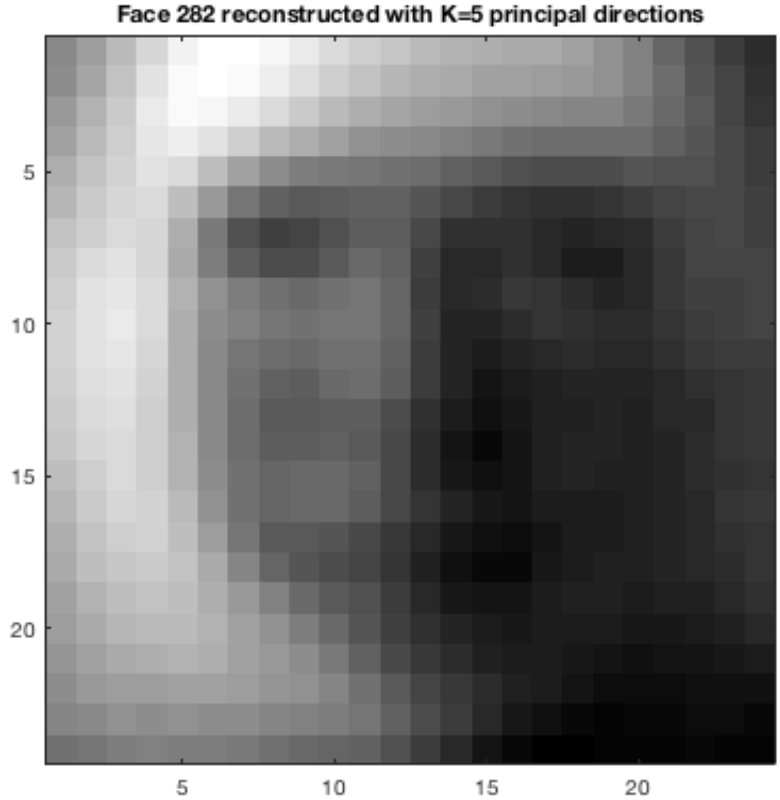
e

```
K = [5, 10, 50]; % the number of principal directions
idx = randperm(576, 2); % pick two random faces

for f=1:length(idx)% for every face
    figure;
    imagesc(reshape(X(idx(f),:), [24, 24]));
    axis square;
    colormap gray;
    title(sprintf('Face %d', idx(f)));

    for i=1:length(K) % for every K value get a face estimation
        figure;
        imagesc(reshape(W(idx(f), 1:K(i))*V(1:576, 1:K(i))', 24, 24));
        axis square;
        colormap gray;
        title(sprintf('Face %d reconstructed with K=%d principal directions', idx(f), K(i))
);
    end
end
```

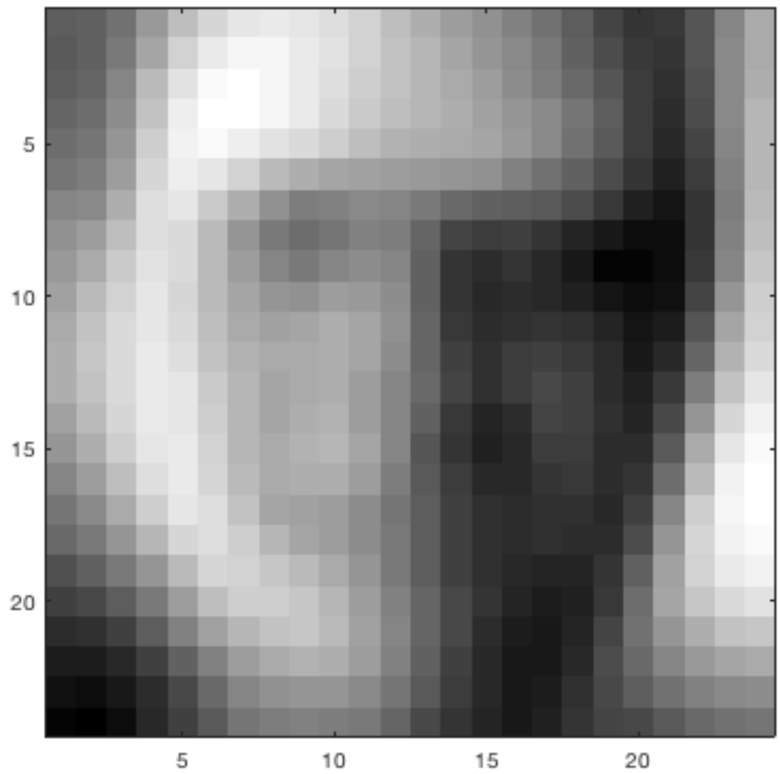
Face 282



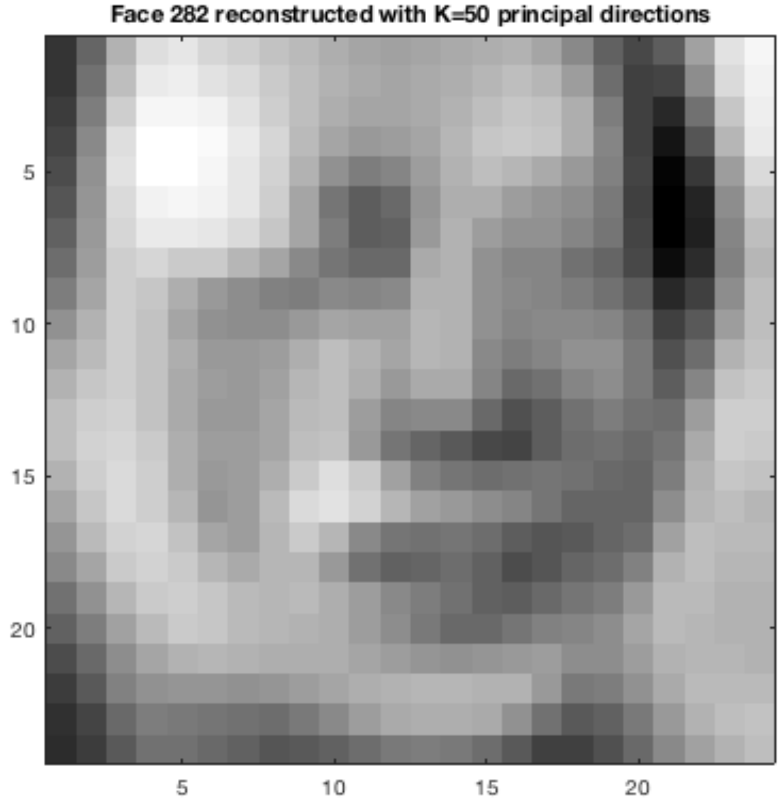
Face 282 reconstructed with K=5 principal directions

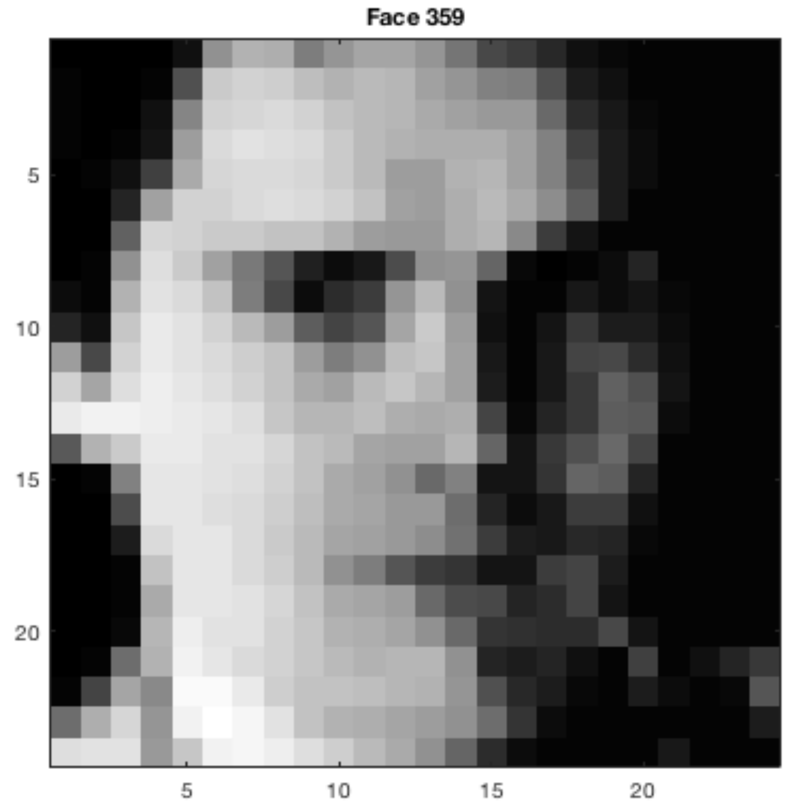


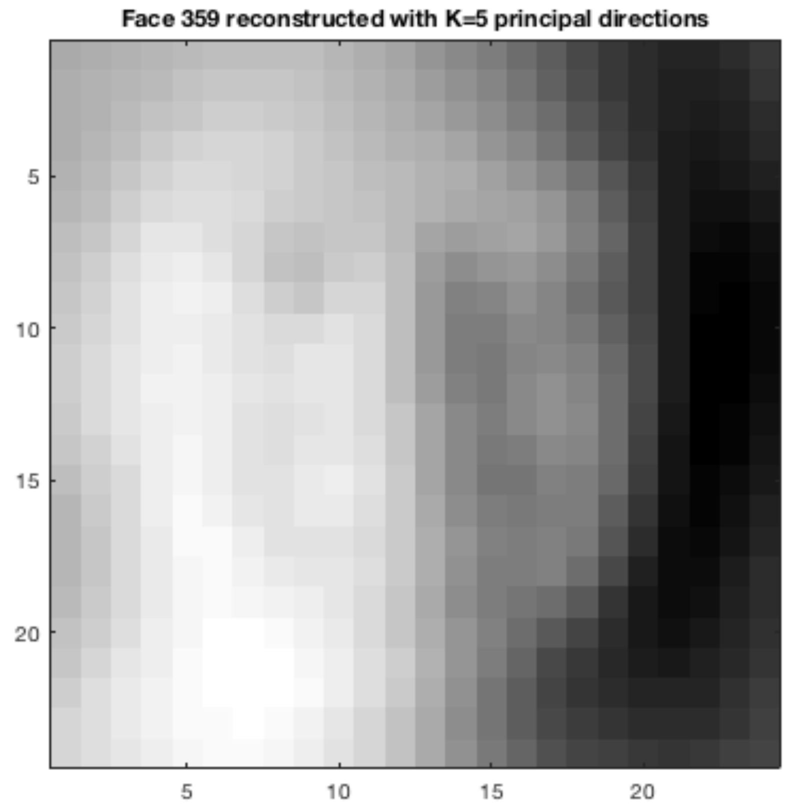
Face 282 reconstructed with K=10 principal directions

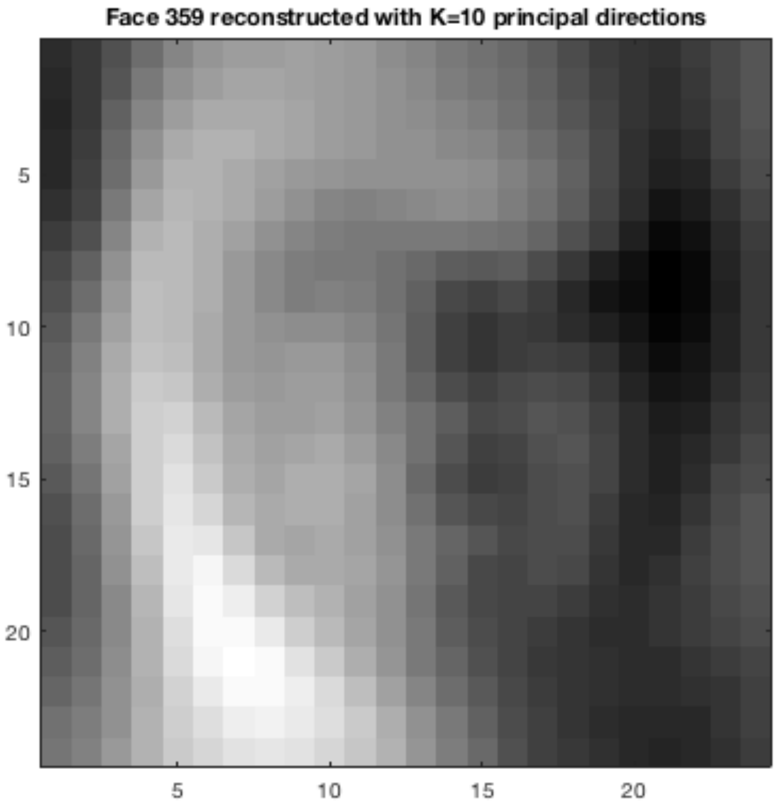


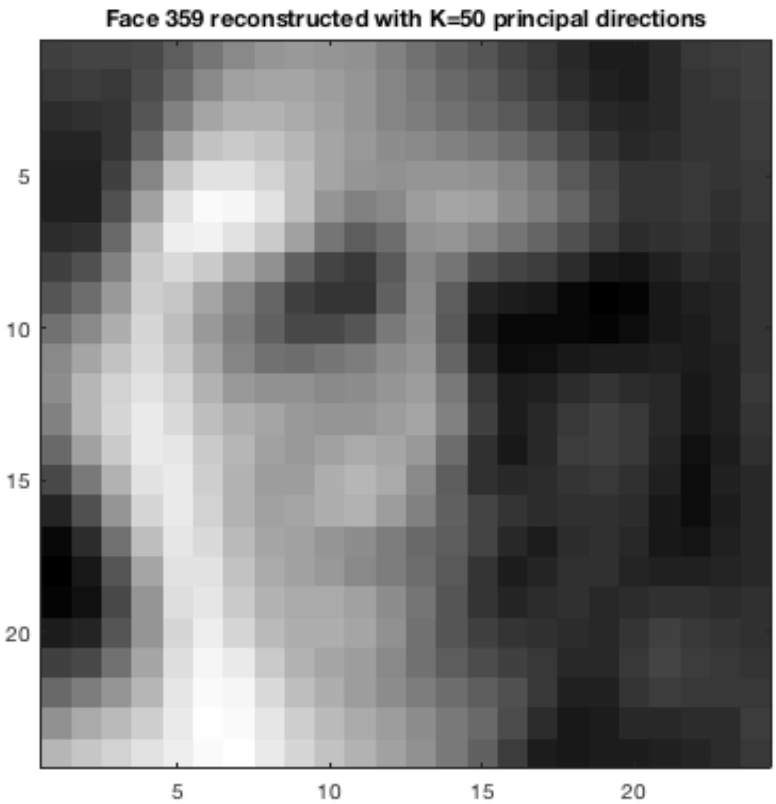
Face 282 reconstructed with K=50 principal directions




Face 359
5
10
15
20
5
10
15
20


Face 359 reconstructed with K=5 principal directions
5
10
15
20
5
10
15
20



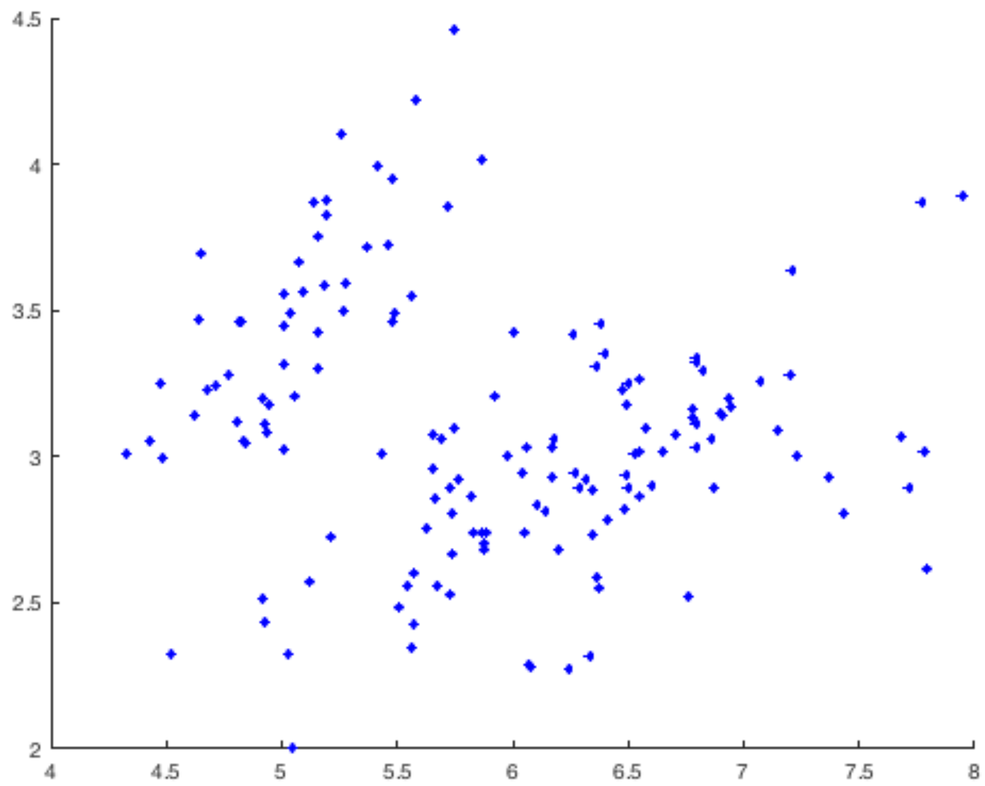


Clustering

a)

Clear everything out

```
close all
clear

% load the iris data restricted to the first two features
load('iris.txt');
iris = iris(:,1:2);

% plot the data to see the clustering
scatter(iris(:,1), iris(:,2), 15, 'b*');
```

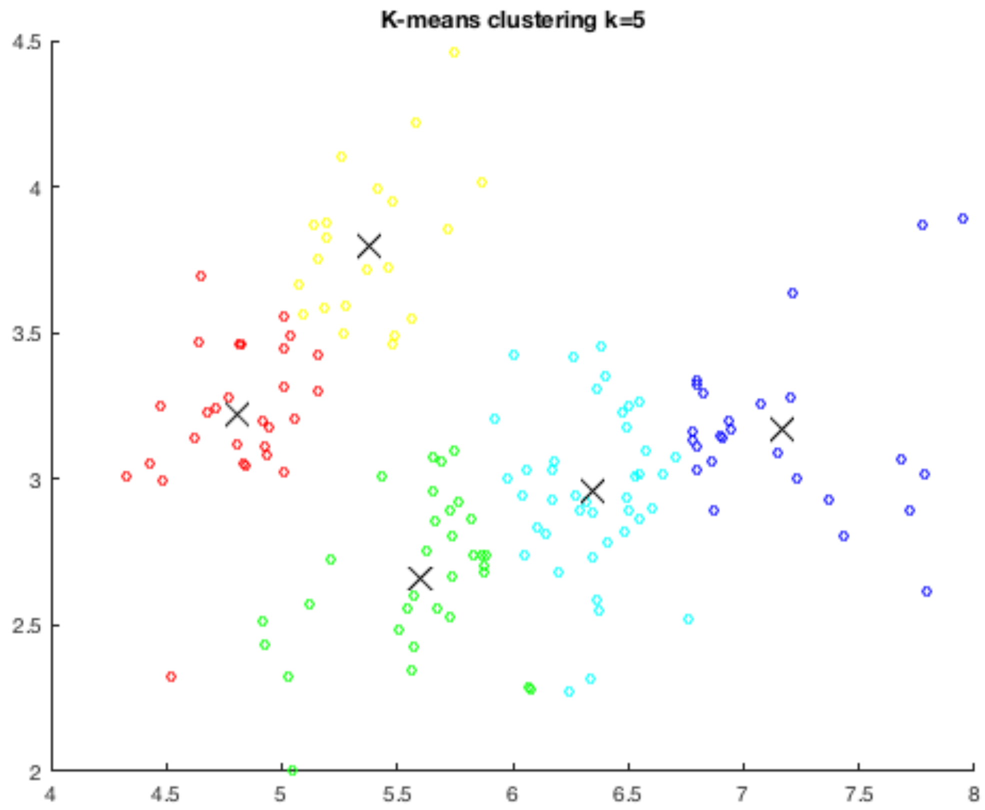


b) run k-means on the data

K=5

```
K1=5;
initial_k5 = [
    4.68    3.22;
    5.48    3.95;
    4.52    2.32;
    6.18    3.06;
    7.2     3.2;
];

centroids = initial_k5;
for i = 1:100
    idx = findClosestCentroids(iris, centroids);
    centroids = computeCentroids(iris, idx, K1);
end

% plot
figure; hold on;
plotDataPoints(iris, idx, K1);
```

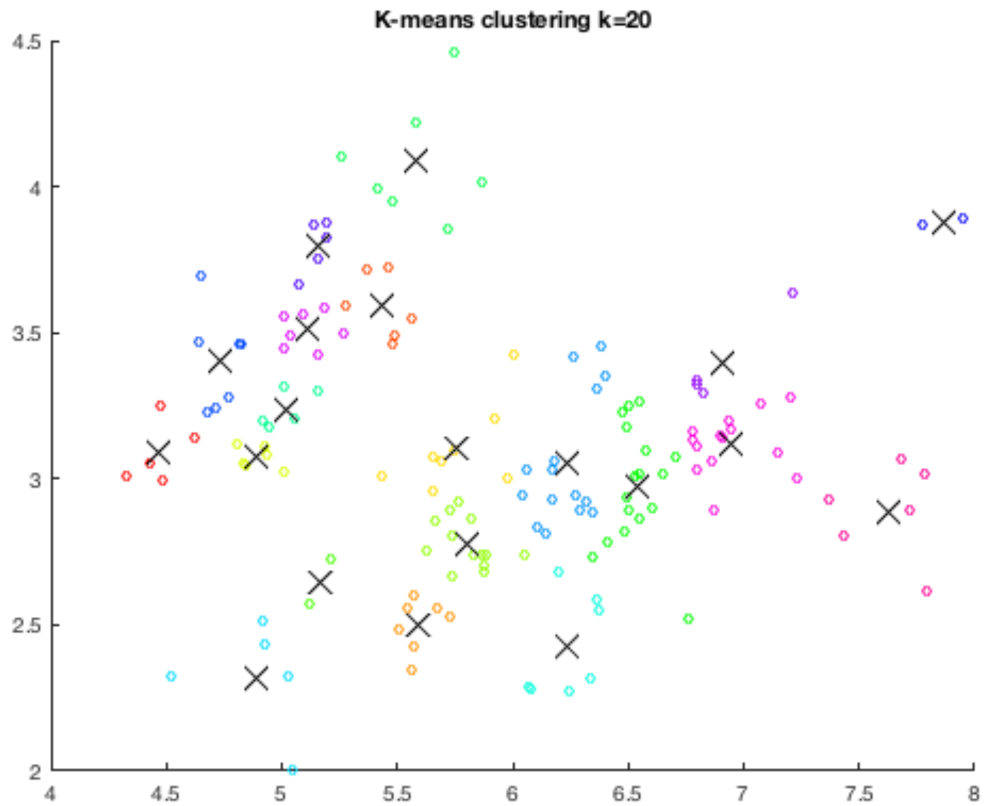
```
plot(centroids(:,1), centroids(:,2), 'kx', 'MarkerSize', 15);
title('K-means clustering k=5');
```



K=20

```
K2=20;
initial_k20 = [
    4.41          3.23;
    5.37          3.24;
    5.69          2.22;
    5.69          3.08;
    4.84          2.86;
    5.83          2.91;
    5.28          2.36;
    6.84          2.70;
    5.47          4.02;
    5.21          3.17;
    5.94          2.33;
    4.97          2.08;
    6.09          2.83;
    4.47          3.41;
    7.42          3.45;
    5.06          3.73;
    7.07          3.50;
    4.94          3.55;
    7.03          2.87;
    7.62          2.90;
];

centroids = initial_k20;
for i = 1:10
    idx = findClosestCentroids(iris, centroids);
```

```
    centroids = computeCentroids(iris, idx, K2);
end

% plot
figure; hold on;
plotDataPoints(iris, idx, K2);
plot(centroids(:,1), centroids(:,2), 'kx', 'MarkerSize', 15);
title('K-means clustering k=20');
```

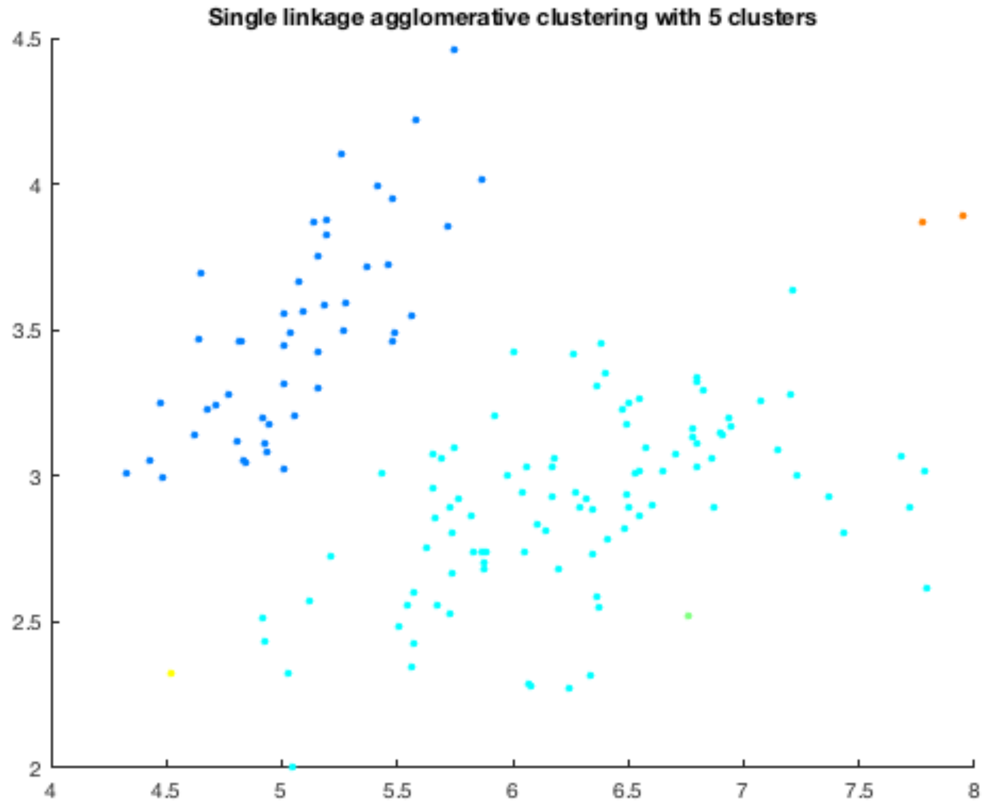


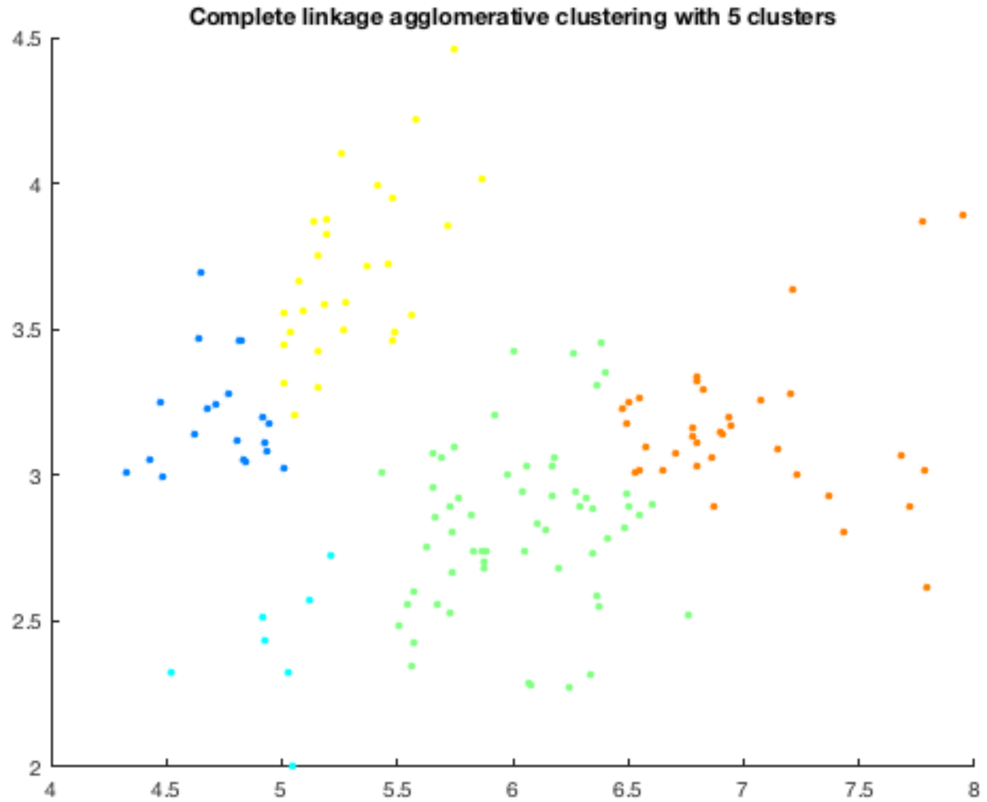
c) agglomerative clustering

```
sLink = linkage(iris, 'single');
cLink = linkage(iris, 'complete');
```

5 clusters

```
colors5 = jet(5);
clust = cluster(sLink, 'maxclust', 5);
figure;
scatter(iris(:,1), iris(:,2), 15, colors5(clust,:), 'filled');
title('Single linkage agglomerative clustering with 5 clusters');

clust = cluster(cLink, 'maxclust', 5);
figure;
scatter(iris(:,1), iris(:,2), 15, colors5(clust,:), 'filled');
title('Complete linkage agglomerative clustering with 5 clusters');
```

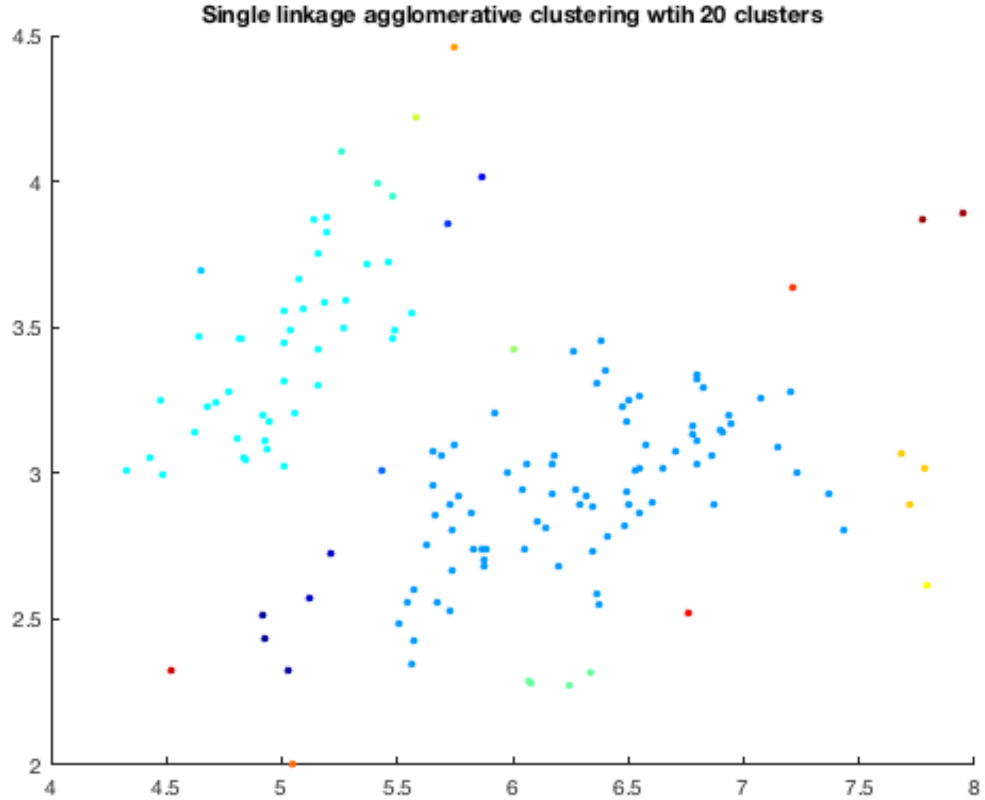


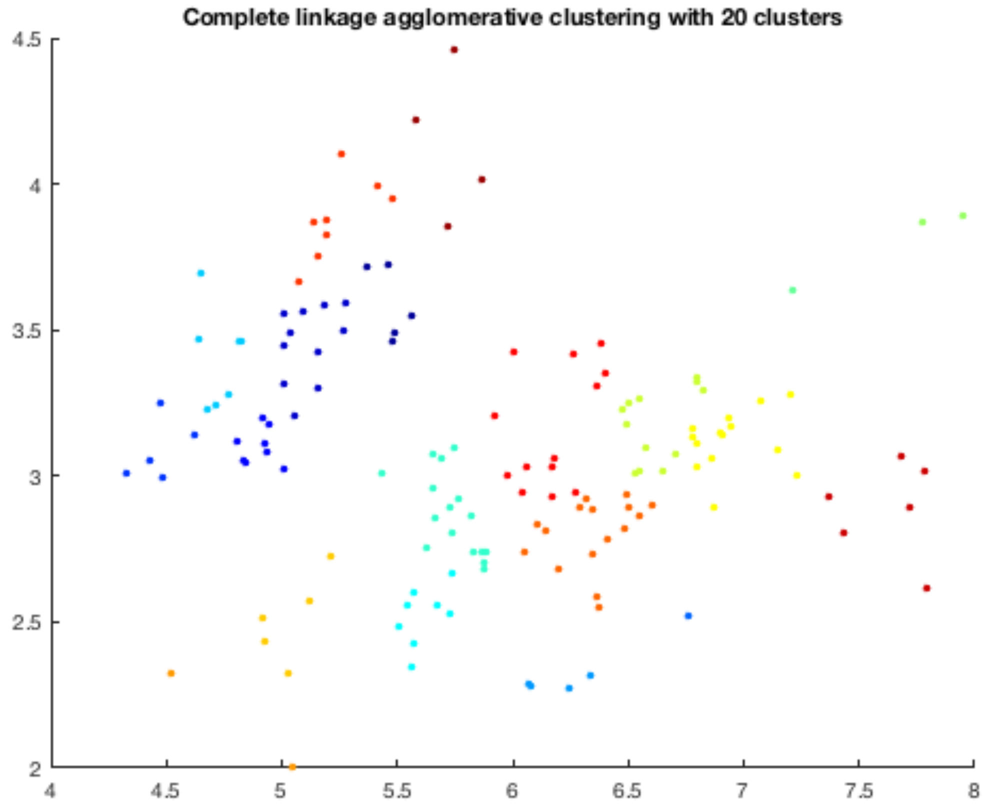


20 clusters

```
colors20 = jet(20);
clust = cluster(sLink, 'maxclust', 20);
figure;
```

```
scatter(iris(:,1), iris(:,2), 15, colors20(clust,:), 'filled');
title('Single linkage agglomerative clustering wtih 20 clusters');

clust = cluster(cLink, 'maxclust', 20);
figure;
scatter(iris(:,1), iris(:,2), 15, colors20(clust,:), 'filled');
title('Complete linkage agglomerative clustering with 20 clusters');
```





Single linkage with 5 clusters gives poor results, forming only two clusters with more than one point. Complete linkage gives much better results, with 5 relativley equal groups positioned in a logical manner. K-means clustering is slightly better as the groups have a more similar number of elements in them.

Single linkage with 20 clusters also give poor results, with many clusters having few points, and only two main groups. Complete linkage gives better reuslts, with more evenly distributed clusters, but is messy. This is probably too many clusters for this dataset. This looks to be similarly effective to k-means clustering.

d) EM Gaussian

```
clear;
% Load data
load('iris.txt');
iris = [iris(:,1), iris(:,2)];

% set the colormaps for the different number of clusters
colors5 = jet(5);
colors20 = jet(20);
```

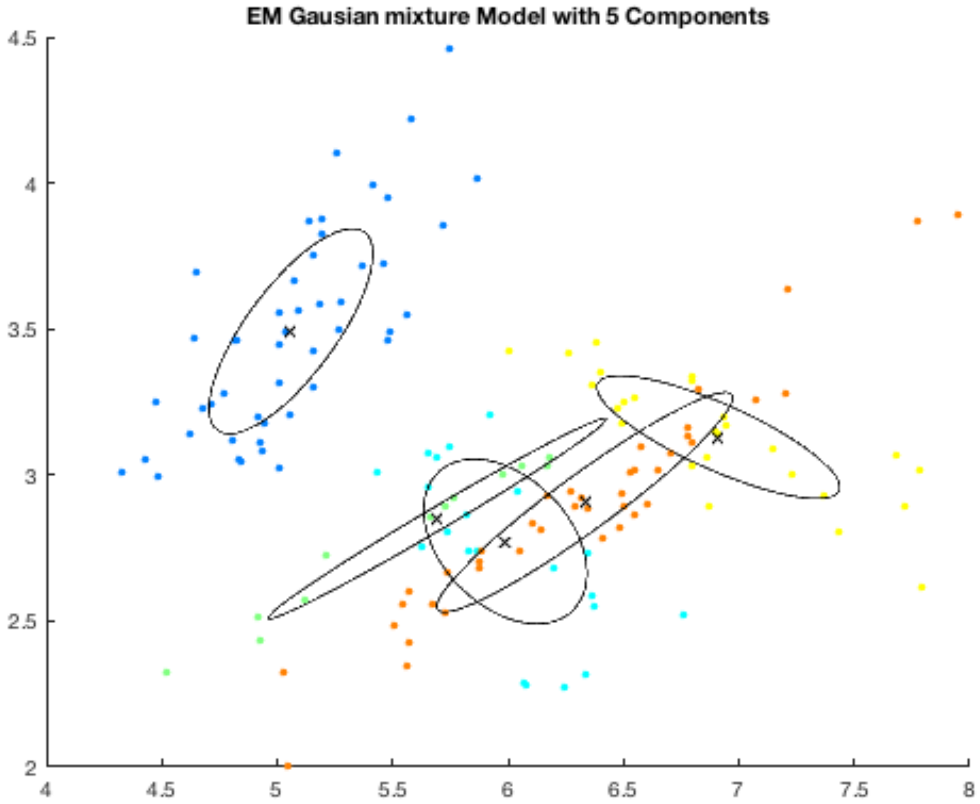
5 components

can try changing the initial clusters to get better results

```
K = 5;
initial_clusters = [
    4.68    3.22;
    5.48    3.95;
    4.52    2.32;
    6.18    3.06;
    7.2     3.2;
];
```

```
% run EM gaussian mixture model
[assign, clusters, ~, ~] = emCluster(iris, 5, initial_clusters);

figure; hold on;
scatter(iris(:,1), iris(:,2), 15, colors5(assign,:), 'filled');
for i = 1:K
    plotGauss2D(clusters.mu(i,:), clusters.Sig(:,:,i), 'k', 'linewidth', 1);
end
title('EM Gausian mixture Model with 5 Components');
```

```
Warning: emclust:iter :: stopped after reaching maximum number of iterations
```

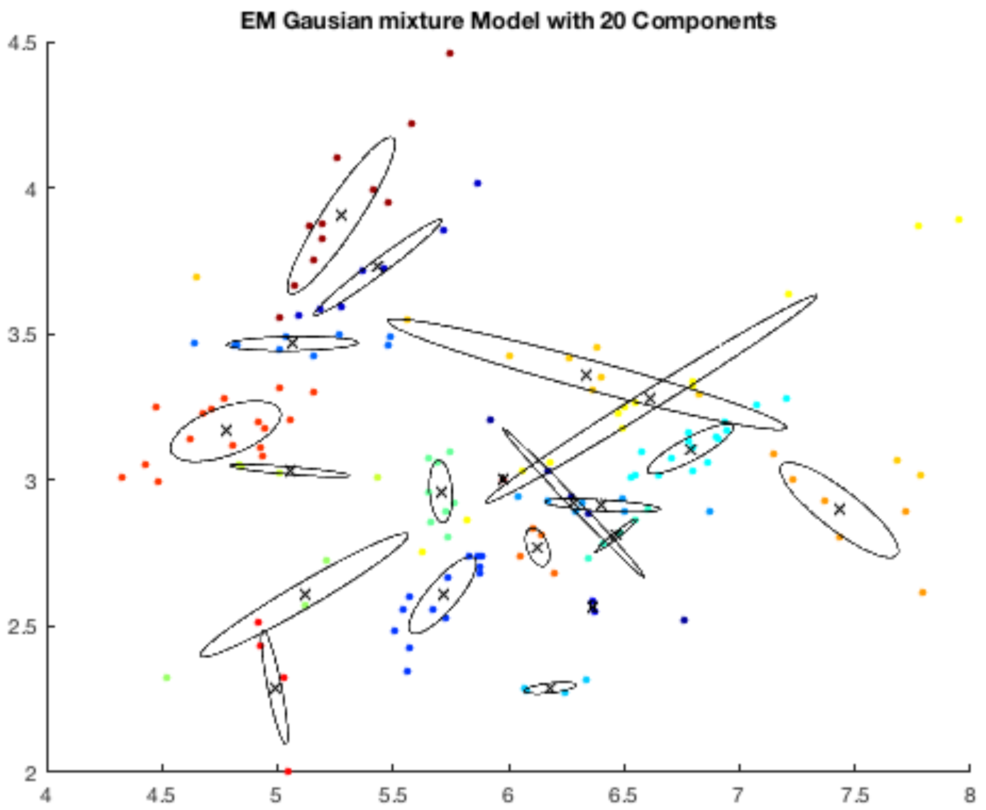


20 components

```
K = 20;
initial_clusters = [
    6.4913    2.9333;
    4.4814    2.9917;
    6.2679    2.9460;
    6.8588    3.0600;
    4.7738    3.2812;
    4.9284    2.4325;
    5.7432    3.0969;
    7.7151    2.8911;
    5.8724    2.7003;
    6.4796    2.8164;
    6.3604    2.5873;
    5.4780    3.4583;
    7.9528    3.8910;
```

```
    6.4738    3.2295;
    6.4996    3.2526;
    6.1055    2.8347;
    4.3266    3.0099;
    4.9465    3.1763;
    5.1536    3.3008;
    4.8142    3.4620;
];

% run EM gaussian mixture model
[assign, clusters, ~, ~] = emCluster(iris, 20, initial_clusters);

% Plot the results
figure; hold on;
scatter(iris(:,1), iris(:,2), 15, colors20(assign,:), 'filled');
for i = 1:K
    plotGauss2D(clusters.mu(i,:), clusters.Sig(:,:,i), 'k', 'linewidth', 1);
end
title('EM Gausian mixture Model with 20 Components');
```

```
Warning: emclust:iter :: stopped after reaching maximum number of iterations
```



The EM gaussian mixture model with 5 components doesnt give great results, it is worse than the complete linkage and the k-means models. The groups are overlapping each other and dont make much sense as groupings.

The EM gaussian mixutre model with 20 components is once again very messy, and also has overlapping groups. This is worse than the k-means and complete linkage clustering algorithms

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