$$\hat{\mathrm{Var}}(\hat{S}(t)) = \hat{S}(t)^2 \sum_{t_i \le t} \frac{d_i}{n_i(n_i-d_i)}$$

12/6/22

Usually, i is ~~sets~~ an index of nodes, but I'm just used to working with it.

First off, my τ's are in $\mathbb{N}$, so I think I can simplify this expression

$$\underset{\substack{V_\tau \\ \shortparallel}}{\hat{\mathrm{Var}}(\hat{S}(\tau))} = \hat{S}(\tau)^2 \sum_{i=1}^{\tau} \frac{d_i}{n_i(n_i-d_i)}$$

$$\le \sum_{i=1}^{\tau} \frac{d_i}{n_i(n_i-d_i)}$$

$$V_\tau \le \sum_{i=1}^{\tau} \frac{d_i}{n_i(n_i-d_i)}$$

$$\tau = 1: \quad V_1 \le \frac{d_1}{n_1(n_1-d_1)}$$

$n_{i+1} = n_i - d_i - c_i$ this is a random variable

$n_0 = \#$ messages injected, this is what I can control.

Note: Usually, the extinction times are limited to 1, but since some messages die immediately, it may be handy to define this.

There's a question of the correct interval type. For now, we'll focus on this problem; it's hard enough.

~~Probabilit~~

Probability extinction time exceeds t: $S(t)$

Probability censorship time exceeds t: $U(t)$

The time horizon T is a stopping time, ~~if M is the smallest number of let $\varepsilon>0$. We want~~ Fix ~~two~~ errors ε and confidence level α. We want $\hat{S}(t) \pm z_{\alpha/2}\sqrt{V_\tau}$ to be of length ε, so we need

$$z_{\alpha/2}\sqrt{V_\tau} < \varepsilon \Rightarrow \sqrt{V_\tau} < \frac{\varepsilon}{z_{\alpha/2}}$$

$$\Rightarrow V_\tau < \left(\frac{\varepsilon}{z_{\alpha/2}}\right)^2$$

A projected variance estimate is

$$S^2(t)\left[\frac{1}{rT}\int_0^{t^*} \frac{\lambda(u)\,du}{S(u)U(u)} + \frac{1}{r}\mathbb{1}[t>\tau]\int_\tau^t \frac{\lambda(u)\,du}{S(u)U(u)(T+t-u)}\right]$$

Uniform accrual of patients to the trial at rate r

T: The time horizon

$\theta(t)$: Loss to follow up at a rate t

$\lambda(t)$: hazard function.

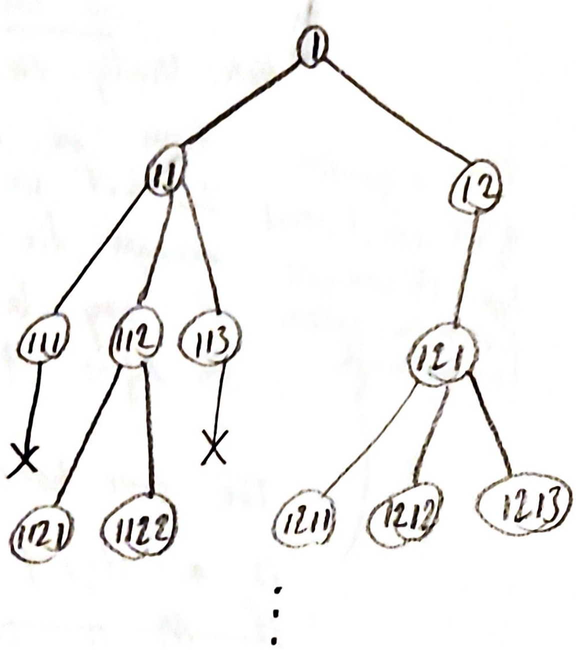
This is not satisfied by my problem, the loss to follow up happens once we've observed some number of extinctions M from each node. So ~~there is~~ θ depends on M.

We also don't need to involve integrals since the set of possible extinction times is discrete.

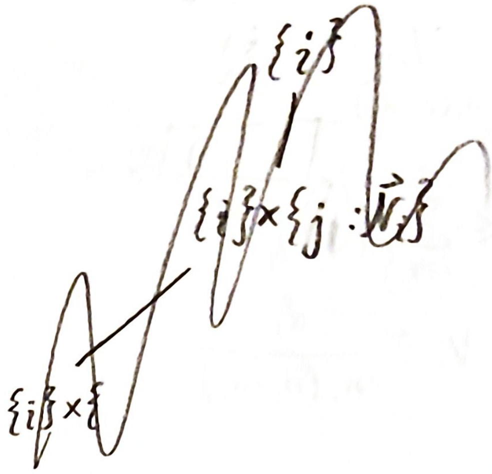
- Use Hermite normal forms to give an algorithm for the stationary distribution
- Use good notation for the kind of sets such as

$$\{x \in [I]_0^n : x_1 = 1, x_2 \text{ free}, x_3 = 1, x_4 \text{ free}, \text{etc.}\}$$

and a good labeling of a message tree - i.e.

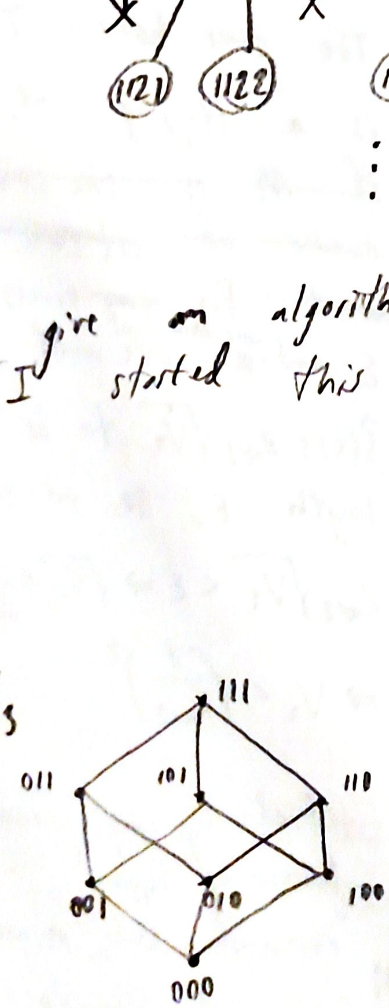


May be convenient to label by nodes



To give an algorithm expressing the probabilities of message survival (I started this way back in NB2, maybe I can find it)

X_3



for $x \in X_n$, let $[x\rangle$ be the set of all $y \in X_n$ s.t. $y \geq x$

now we can talk about sets like the example above with the characters

$$\bigcup_{h \in \bar{N}_i} [e_h\rangle$$

, where e_h is a ~~tr~~ standard basis vector in the h-th direction.

[All] you have to do is express these events.

$\tau=1$ means that a node in $\vec{N}_i$ was active, which happens w/p

$\sum\{\pi_x \text{ for } x \in \bigcup_{h\in\vec{N}_i}[e_h\rangle\}$

Let's extend the notation. If we pass a ~~subset of~~ neighborhood to $[\cdot\rangle$, then let

$$[\vec{N}_i\rangle = \bigcup_{h\in\vec{N}_i}[e_h\rangle$$

$$\mathbb{P}(\tau=1) = \sum_{x\in[\vec{N}_i\rangle} \pi_x$$

I want good error analysis in this work!

$$\mathbb{P}(\tau=2) = [1-\mathbb{P}(\tau=1)]\times\mathbb{P}(\text{all the children die}).$$

for every non-empty subset M_1 of $\vec{N}_i$:

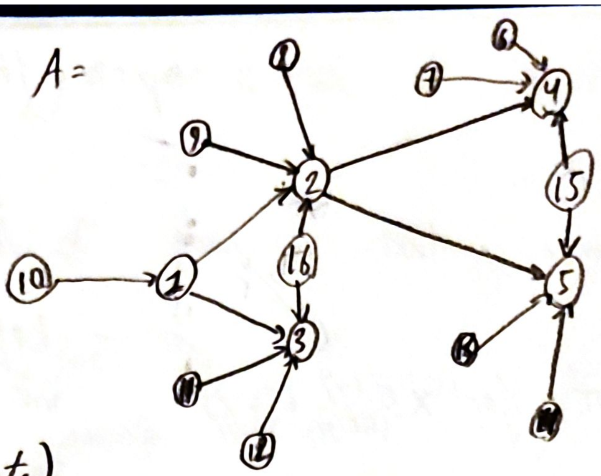
for each i in M_1:

$$\sum_{M_1\in\vec{N}_i} \mathbb{P}(\text{each } j\in M_1 \text{ has a collision \& no injection}) + \mathbb{P}(\text{each } j\in M_1 \text{ has no coll. but injection})$$

$+\mathbb{P}($

OMG

Once I know $\mathbb{P}(\tau=2)$, I can attempt induction example

A = 

We inject a message at 1

$\mathbb{P}(\tau=1)$

$= \mathbb{P}($node 10 was active in the previous state$)$

$= \sum_{x \in [\overleftarrow{N}_1\rangle} \pi_x$

$\mathbb{P}(\tau=2)$

$= \mathbb{P}(\tau \neq 1) \times \mathbb{P} \Big[$ a node in $\overleftarrow{N}_2 - \{1\}$ is active or

I have to be careful here: it $\mathbb{P}($no col at 2 $|$ 1 was occupied$)$

$\mathbb{P}(\tau \geq 1) = 1$

$\mathbb{P}(\tau \geq 2) = \mathbb{P}($one outgoing copy survives$)$

$= \mathbb{P}[($no collision at 2 & no injection at 2$)$ or $($no collision at 3 & no injection at 3$)]$

$= \mathbb{P}[$no coll 2$]\mathbb{P}[$no inj 2$] + \mathbb{P}[$no coll 3$]\mathbb{P}[$no inj 3$] - \mathbb{P}[$no coll 2 & no coll 3$]\mathbb{P}[$no inj 2 & no inj 3$]$

$\mathbb{P}(\overleftarrow{N}_2 - \{1\}$ not active$)\mathbb{P}[$no inj 2$] + \mathbb{P}(\overleftarrow{N}_3 - \{1\}$ not active$)\mathbb{P}[$no inj 3$]$

$- \mathbb{P}(\overleftarrow{N}_2 \cup \overleftarrow{N}_3 - \{1\}$ not active$)\mathbb{P}[$no inj 2 & 3$]$

So a key ingredient is a formula for the probability of no collision occuring at one set of nodes given that another set of nodes is known to be active.

formula for $\mathbb{P}[\text{no inj at } 2,3,\ldots,n]$ is much more complicated due to the ε modification to the chain. [5]

↳ If I go back to the old ~~formula~~ modification, the formula is simple again. Or yet a third modification.

$$\boxed{B = B_L \cup \{1\}, \quad b_t \sim \begin{cases} (1-\varepsilon)\,\text{unif}(B_L) & \text{on } B_L \\ \varepsilon\ \{1\} & \text{on } \{1\} \end{cases}}$$ This alphabet everywhere except the 0 state

$$\boxed{B = [1]_0^n \quad b_t \sim \begin{cases} (1-\varepsilon)\,\text{unif}(B_L) & \text{on } B_L \\ \varepsilon\ \text{unif}([1]_0^n - B_L) & \text{on } [1]_0^n - B_L \end{cases}}$$ at the 0 state

I believe we can use "convex combination" to simplify this notation.

This is good because the alphabet is the same for every state that could possibly lead to a collision; the last state could not have been $\bar{0}$ because a message was injected (except for the ~~very first state that could be~~ state right before the message was injected, that could be $\bar{0}$.