

$$P(A \& B) = P(A) \times P(B|A)$$

$$A = T \geq t(f), \quad B = T > t(f)$$

$$P(A \& B) = P(B) = S(t(f))$$

There are no failures during $(t(f-1), t(f))$ - so

$$P(A) = P(T > t(f-1)) = S(t_{f-1})$$

Putting it all together

$$S(t(f)) = S(t(f-1))P(T > t(f) | T \geq t(f-1))$$

1/17/2023

IV. The Log-Rank Test for Two Groups

- Evaluate if two KM curves are statistically equivalent, meaning no evidence to say they're different

The logrank test is a large-sample chi-square test that uses a test statistic that provides an overall comparison of the KM curves being compared.

- Observed vs. expected cell counts over outcome categories
- Categories defined by each of the ordered failure times over the entire dataset

The data on the next page comes from the leukemia example used before. Group 1: Treatment.
Group 2: placebo.

Expected cell counts: expected number at time f multiplied by the total number of failures at that time for both groups. So for group 1.

$$e_{if} = \left(\frac{n_{if}}{n_{if} + n_{2f}} \right) \times (m_{1f} + m_{2f})$$

e_{if} is the expected number of failures in group i at time f .

Example

Remission data: $n=42$

# failures			# in risk set	
$t_{(i)}$	m_{1f}	m_{2f}	n_{1f}	n_{2f}
1	0	2	21	21
2	0	2	21	19
3	0	1	21	17
4	0	2	21	16
5	0	2	21	14
6	3	0	21	12
7	1	0	17	12
8	0	4	16	12
10	1	0	15	8
11	0	2	13	8
12	0	2	12	6
13	1	0	12	4
15	0	1	11	4
16	1	0	11	3
17	0	1	10	3
22	1	1	7	2
23	1	1	6	1

Use a computer to calculate these statistics!

$$O_i - E_i = \sum_{i=1}^{\text{\#fail times}} (m_{if} - e_{if}) \quad i=1, 2$$

For the case of 2 groups, the logrank statistic is

$$\text{log-rank statistic} = \frac{(O_2 - E_2)^2}{\text{Var}(O_2 - E_2)} \quad \text{in this case, } i=2, \text{ but could be 1}$$

where

$$\text{Var}(O_i - E_i) = \sum_j \frac{n_{1f} n_{2f} (m_{1f} + m_{2f}) (n_{1f} + n_{2f} - m_{1f} - m_{2f})}{(n_{1f} + n_{2f})^2 (n_{1f} + n_{2f} - 1)} \quad i=1, 2$$

H_0 : no difference b/w survival curves.

Log-rank statistic $\sim \chi^2$ w/ 1 df under H_0 , so p-value comes from χ^2 test