7.3: Basis Functions

Polynomials & step functions are special examples of the basis function approach. Have

$$b_1(X), \ldots b_K(X)$$

and fit the linear model

$$y_i = \beta_0 + \sum_{j=1}^{K} \beta_j b_j(x_i) + \varepsilon_i \quad (7.7)$$

All the tools of chapter 3 are available in this setting.

Other basis functions include:
- Wavelets
- Fourier series
- Regression splines

2/22/23
Day 16

7.4: Regression Splines

7.4.1: Piecewise Polynomials

Piecewise polynomial regression involves fitting separate low-degree polynomials over different regions of X. For example, piecewise cubics. The points where the coefficients change are called knots.

Use least squares within each region.
K knots $\Rightarrow$ $K+1$ polynomials

7.4.2: Constraints and Splines

Can have constraints of C^2 continuity.

A cubic spline with K knots uses $4+K$ degrees of freedom.

7.4.3: The Spline Basis Representation

A cubic spline with K knots can be modeled as
$y_i = \beta_0 + \beta_1 b_1(x_i) + \cdots + \beta_{K+3} b_{K+3} + \varepsilon_i$
for function b_j $j \in [K+3]$. Then fit with least squares.

Direct representation: start off with $1, x, x^2, x^3$, and then add one truncated power basis function per knot ξ, where such a function is:

$$h(x, \xi) = (x - \xi)_+^3 = (x-\xi)^3 \mathbb{1}_{(\xi, \infty]}(x)$$

Least squares on $X, X^2, X^3, h(X, \xi_1), \ldots, h(X, \xi_K)$

Natural splines are required to be linear in the boundary region.