Communications

Comparing Spectra of a Series of Point Events Particularly for Heart Rate Variability Data

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Abstract—Different methods for spectral analysis of the heart rate signal—considered as series of point events—are used in studies on heart rate variability. This paper compares these methods, focusing on the two principal ones: the interval spectrum, i.e., the spectrum of the interval series, and the spectrum of counts, which is related to the representation of the event series as a series of spikes (delta functions). Both autospectra are estimated for experimental heart rate data and are shown to produce similar results. This similarity is proven analytically, and it is shown that for small variations in interval length, the ratio of these spectra is $P_I(f)/P_C(f) = [\sin(\pi f\bar{I})/(\pi f\bar{I})]^2$, with P_I and P_C the interval spectrum and the spectrum of counts, respectively, f the frequency, and $\bar{I}$ the mean interval length. It is concluded that both autospectra are equivalent for the considered heart rate data, but that, when relating the heart rate signal to other signals (e.g., respiration, blood pressure) by means of cross spectra, the technique to be used depends on the characteristics of the second signal.

I. Introduction

In the study of beat-to-beat fluctuations in heart rate, several authors have used spectral analysis methods [1], [4], [11]–[14], [16]–[18], [20]. For data from man [17], as well as from dog [1] and cat [4], three peaks in the spectrum are usually distinguished. One peak is due to respiration—for man, around 0.3 Hz. Often a peak at approximately 0.1 Hz is found, which seems related to the 10 s waves as seen in the blood pressure (Mayer waves [19]). A peak at still lower frequencies is attributed to properties of the thermoregulatory system.

The technique for spectral analysis of heart rate data is not straightforward, however, the successive heartbeats must be considered as a series of events, and different methods can be used for the estimation of spectra from such signals. In this paper, we present a survey and a comparison of the different spectra that can be defined. The emphasis is on data from heart rate variability studies. Characteristics of these data are: 1) the variation of interval lengths is much smaller than the mean length, and 2) the variation of lengths is more or less regular (e.g., due to respiratory influences).

In Section II, we discuss the three types of spectra that are used in heart rate variability studies. In Section III, we compare the interval spectrum and the spectrum of counts [6], which seem to us the most interesting ones. In Section III-A, both spectra are estimated and compared for the same sets of heart rate data, and in Section III-B, we prove analytically the similarity of the two kinds of spectra for this type of data. In the Conclusion, we discuss under what conditions each of the two spectra is most useful.

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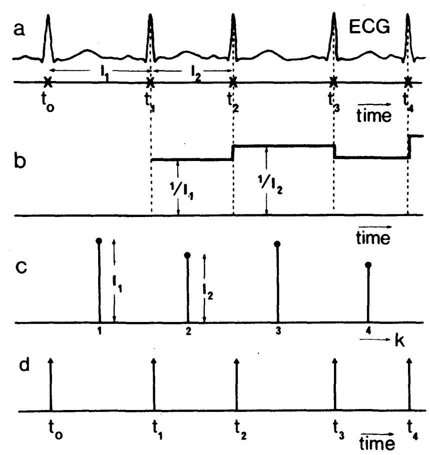

Fig. 1. (a) Event series representing *R*-waves of the ECG. (b) Heart rate signal derived from the event series in Fig. 1(a). (c) Interval series derived from Fig. 1(a). The discrete Fourier transform of this signal leads to an estimator for the *interval spectrum*. (d) In this figure, the events of Fig. 1(a) have been replaced by spikes (delta functions); the spectrum of this signal gives the *spectrum of counts*.

II. The Power Spectrum of a Series of Point Events

Standard Fourier analysis, i.e., the decomposition of a signal in sinusoids, is not possible for a series of point events. Hence, a power spectrum for such a series must be defined in a different way. In heart rate variability studies, the following three approaches are used to arrive at a useful concept for a spectrum of a point process.

1) A signal that is defined at all times is derived from the point event series [Fig. 1(a)]. Several possibilities exist [7], e.g., the transformation of the event series into a heart rate signal, Fig. 1(b). The spectrum of the signal is estimated by equidistant sampling followed by a discrete Fourier transform. A number of authors have used this approach [12], [13], [20]. A disadvantage of this method is that the signals deduced from the point event series are often not differentiable and sometimes not even continuous [cf. Fig. 1(b)]. This causes spurious contributions in the spectrum, in particular, in the higher frequencies. It is also a moot point that different procedures are used to derive a signal from the series without a clear preference. This calls for a more canonical definition of a spectrum.

In the literature on stochastic point processes, two different spectra are defined: the interval spectrum and the spectrum of counts [6], [9]. Both spectra are used in heart rate variability studies and will be discussed in the following.

2) The *interval spectrum* is the spectrum of the series of intervals spaced equidistantly [Fig. 1(c)]. Standard procedures for spectral estimation (e.g., computation of the periodogram) can be used. Several authors presented heart rate variability spectra in this way [1], [4], [14], [17], [18]. As the interval series is a function of interval number and not of time, the spectrum cannot be directly interpreted in terms of frequency. The relationship of this spectrum with frequency is taken up in Section III-B. Note that the interval spectrum of a fully regular process (all intervals equal) consists solely of a dc component.

3) The *spectrum of counts* is also used in the statistical analysis of series of events. This spectrum can be estimated by a straightforward calculation of the spectrum of the signal in Fig.

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1(d) where the events [Fig. 1(a)] have been replaced by delta functions [6]. Thus, the signal is described as $s(t) = \Sigma\delta(t - t_k)$ where t_k is the time of the kth occurrence of an event. For equal intervals $I = t_k - t_{k-1}$, the spectrum of counts consists of an infinite series of delta functions, spaced at distance $1/I$ along the frequency axis. Usually, one is only interested in frequencies much lower than the mean repetition frequency of the events, so only the low-frequency part of the spectrum needs to be considered. Two different approaches for the estimation of this part of the spectrum have been proposed.

- The signal is passed through an ideal low-pass filter with cutoff frequency f_{max}; this is equivalent to convolution of the signal $s(t)$ with the function $\sin(2\pi f_{max} t)/(\pi t)$ and amounts to replacing each delta function at time t_k by the function $\sin(2\pi f_{max}(t - t_k))/(\pi(t - t_k))$. The result is a continuous signal which was named the low-pass filtered event series (LPFES [11]). This signal is sampled, and the spectrum is calculated by a digital Fourier transform. An efficient algorithm was published by French and Holden [10]; see also [15]. Coenen *et al.* [5] described a hardware device to perform the convolution.
- The interesting (i.e., low-frequency) part of the spectrum $P_C(f)$ of the signal $s(t)$ can also be computed directly, using the estimator

$$P_C(f) = \frac{1}{N\bar{I}} \left[\left\{ \sum_{k=0}^{N-1} \cos(2\pi f t_k) \right\}^2 \Pi + \left\{ \sum_{k=0}^{N-1} \sin(2\pi f t_k) \right\}^2 \right] \tag{1}$$

with N the number of intervals in the period of observation and $\bar{I}$ the mean interval length [16].

As the spectrum of counts and the interval spectrum are the most common spectra in the study of point event series, we concentrate on these spectra in the following.

III. Comparison of the Interval Spectrum and the Spectrum of Counts of a Point Process

A. Experimental Comparison of the Spectra

For Fig. 2, we used 940 successive heart intervals, derived from the R-waves in the electrocardiogram of a healthy young person, breathing freely. The first 400 intervals are shown in Fig. 2(a). The mean interval length was 0.93 s. Similar data, when breathing at a fixed rate of 0.16 Hz, are shown in Fig. 3(a) (340 intervals with a mean of 0.94 s). From these data, we calculated the interval spectra [Figs. 2(b), 3(b)] and the spectra of counts [Figs. 2(c), 3(c), 3(d)] in the following way.

- The *interval spectrum* $P_I(f)$ was estimated by the periodogram using a fast Fourier transform. The intervals I_k were first normalized as $I'_k = (I_k - \bar{I})/\bar{I}$, $\bar{I}$ being the mean interval length. We added zeros to achieve 1024 data points (zero padding) and divided the spectral values by 0.875 to compensate for the 10 percent cosinus taper that was used [3]. The frequency axis was scaled by considering the intervals to be spaced at distances equal to the mean interval length $\bar{I}$ [17]. So frequency values in hertz were obtained and, as the effective sampling frequency is $1/\bar{I}$, the maximum frequency in the spectrum is $1/2\bar{I}$ (0.54 and 0.53 Hz for Figs. 2(b) and 3(b), respectively). This procedure will be justified in Section III-B.

No frequency smoothing was performed in Fig. 3(b), while a 27-point rectangular window was used in Fig. 2(b) (equivalent to a 25-point window for the unpadded data). $P_I(f)$ is the power spectrum; in all figures, we show the amplitude spectrum $[P(f)]^{1/2}$ to stress the higher harmonics.

- The *spectrum of counts* $P_C(f)$ was calculated using (1) which was modified to subtract the large dc component as well as the sidelobes, caused by the limited data length. In addition, we gave each spike an impulse content equal to $\bar{I}$, so the signal to be transformed becomes dimensionless: $s'(t) = \Sigma \bar{I} \cdot \delta(t - t_k) - N$.

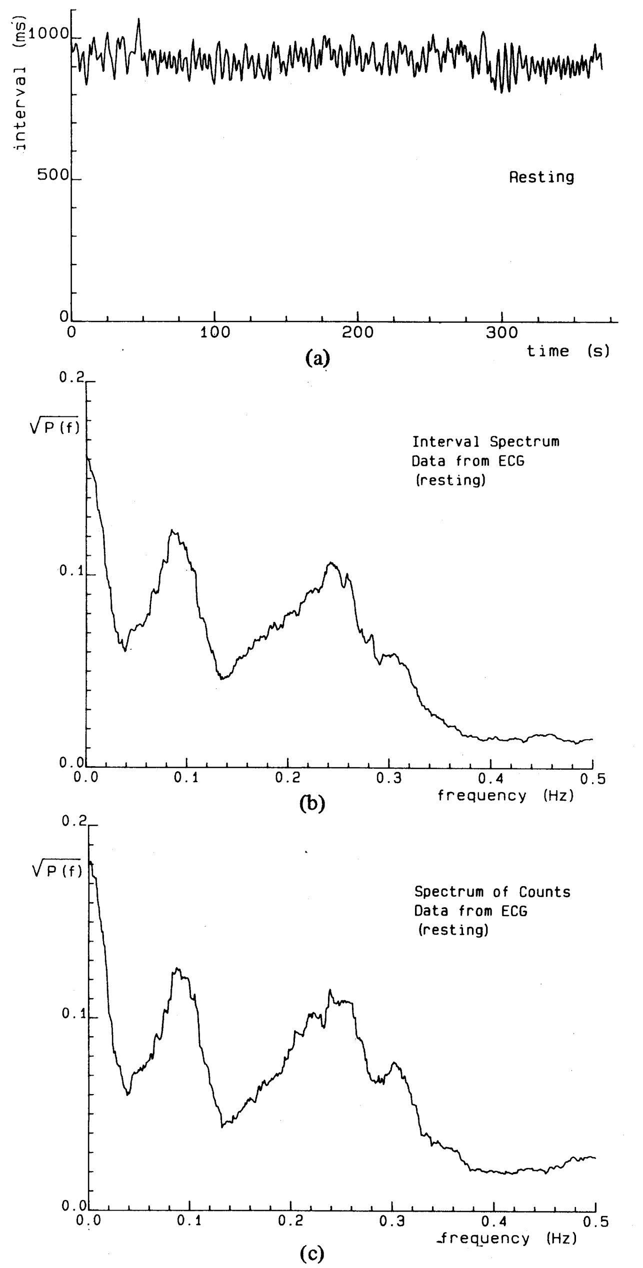

Fig. 2. (a) 400 heart intervals from a healthy young person, breathing freely. (b) Interval spectrum calculated from 940 intervals (Fig. 2(a) showing the first 400). The spectral values are smoothed (see text). All spectra presented are amplitude spectra, i.e., the square root of the power spectra. (c) Spectrum of counts in the range 0–0.5 Hz.

In Figs. 2(c) and 3(c), $[P_C(f)]^{1/2}$ is presented up to 0.5 Hz for frequencies at a distance 0.001 Hz apart. In Fig. 2(c), a 27-point rectangular window was used to smooth the spectrum. The width of this window is thus equal to the one used in Fig. 2(b). Whereas the interval spectrum—being a digital Fourier transform—is periodical and limited in frequency range, the spectrum of counts is not. This is shown in Fig. 3(d) where the spectrum of counts up to 2.5 Hz is presented at distances 0.005 Hz (no smoothing). The mean repetition frequency of the heart rate signal is apparent from the large contributions to the spectrum around 1.06 and 2.13 Hz (the mean interval length being 0.94 s).

It is striking that Fig. 2(b) and (c) are rather similar (cf. [14, Fig. 4]). In both cases, the spectrum consists of a low-frequency component, a peak around 0.1 Hz, and a peak in the region of the mean breathing frequency (0.25 Hz). For frequencies above 0.2 Hz, the spectrum of counts is somewhat larger than the other one. The spectra in Fig. 3(b) and (c) contain only contributions

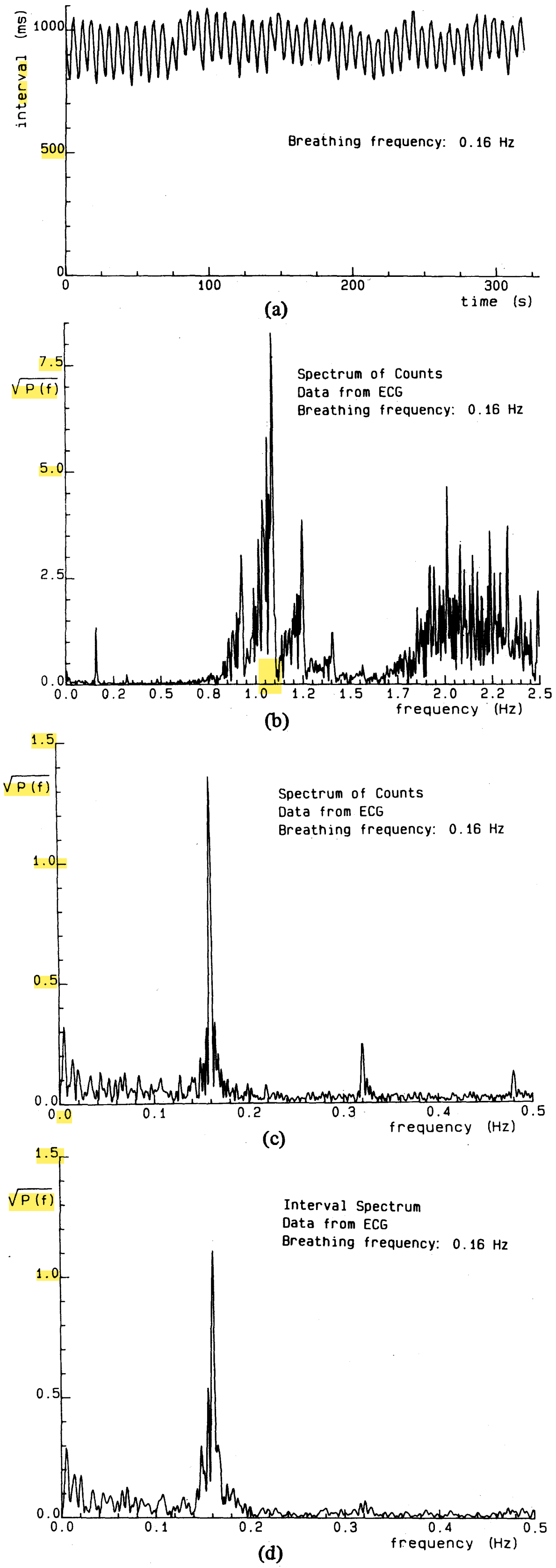

Fig. 3. (a) 340 heart intervals from a healthy young person, breathing at a fixed rate of 0.16 Hz. (b) Interval spectrum calculated from the data of Fig. 3(a). This spectrum is not smoothed. (c) Spectrum of counts in the range 0–0.5 Hz. (d) Spectrum of counts in the range 0–2.5 Hz. Note the large contributions around the mean repetition frequency (1.06 Hz) and twice this value (2.12 Hz).

at multiples of the fixed respiratory frequency (0.16 Hz). The higher harmonics are more pronounced in Fig. 3(c) than in Fig. 3(b). The peaks in the interval spectrum appear to be slightly wider than the ones in the spectrum of counts. In the next section, we explain why the spectra are so similar for these data.

B. Analytical Comparison of the Spectra

We compare, in the following, the raw estimators for the interval spectrum and the spectrum of counts. All summations are to be taken from $k = 0$ to $k = N - 1$.

The *interval spectrum*—or rather the spectrum of normalized intervals $I_k/\bar{I}$—is estimated by the periodogram $P_I(f') = (2/N\bar{I}) \cdot C_I(f') \cdot C_I^*(f')$ with $C_I(f') = \Sigma I_k \cdot \exp(-2\pi i f' k)$ for $f' = 0, 1/N, \cdots, 1/2$. We put $f = f'/\bar{I}$ ($f = 0, 1/N\bar{I}, \cdots, 1/2\bar{I}$), so $C_I(f) = \Sigma I_k \exp(-2\pi i f k \bar{I})$. The *spectrum of counts* is estimated as $P_C(f) = (2/N\bar{I}) \cdot C_C(f) \cdot C_C^*(f)$ with $C_C(f) = \Sigma \bar{I} \cdot \exp(-2\pi i f t_k)$.

From these expressions, the similarity of the spectra is not evident at first sight. The next analysis shows under which conditions the spectra are alike. We put $t_k = k \cdot \bar{I} + \delta_k$, so δ_k is the deviation from a regular train and $I_k = t_k - t_{k-1} = \bar{I} + \delta_k - \delta_{k-1}$. We assume that the deviations δ_k are sinusoidally modulated: $\delta_k = \delta \cdot \sin(2\pi k f_m \bar{I} + \phi)$, with f_m the modulation frequency ($f_m < 1/2\bar{I}$). This implies a sinusoidal modulation of the intervals as well: $I_k = \bar{I} + 2\delta \cdot \sin(\pi f_m \bar{I}) \cdot \cos(2\pi(k - 1/2) f_m \bar{I} + \phi)$. Using the complex representation for δ_k, we find for the *interval spectrum*

$$C_I(f) = \sum \bar{I} \cdot \exp(-2\pi i f k \bar{I}) + \delta \cdot \exp(i\phi) \cdot [1 - \exp(-2\pi i f_m \bar{I})] \sum \exp(2\pi i k (f_m - f)\bar{I}). \quad (2)$$

The first part of this expression (dc component) has amplitude N for $f = n/\bar{I}$ ($n = 0, 1, 2, \cdots$) and is of order one for all other frequencies. Its contribution will be neglected in the following, as it can easily be removed by subtracting the mean from the interval values. So $C_I(f) = N \cdot \delta \exp(i\phi) [1 - \exp(-2\pi i f_m \bar{I})]$ at $f = f_m$, while for $f \neq f_m$ and $N \to \infty$, $C_I(f)$ remains of order one. Thus, the spectrum contains a spike at frequency f_m as is to be expected.

Let us now make the additional assumption that $\delta_k << \bar{I}$, i.e., the event train is rather regular. We obtain for the *spectrum of counts*

$$C_C(f) \approx \bar{I} \sum \exp(-2\pi i f k \bar{I}) - 2\pi i f \bar{I} \delta \exp(i\phi) \cdot \sum \exp(2\pi i k (f_m - f)\bar{I}). \quad (3)$$

The first part of this expression is equal to the similar one in (2) and will be neglected as well. So $C_C(f) \approx -2\pi i f_m \delta \cdot \bar{I} N \cdot \exp(i\phi)$ at $f = f_m$, while for all other frequencies in our range of interest and $N \to \infty$, $C_C(f)$ remains of order one. Taking into account higher order terms in δ_k in (3), we find harmonics at frequencies $2f_m$, $3f_m$, etc.

The ratio of the power spectra at $f = f_m$ is $P_I(f_m)/P_C(f_m) = [\sin(\pi f_m \bar{I})/(\pi f_m \bar{I})]^2$, which for slow modulation, i.e., $f_m << 1/\bar{I}$, approximates the value one. Hence, for small sinusoidally modulated deviations δ_k, both types of spectra are similar; they have a peak at the modulation frequency f_m with relative amplitude $\delta \pi f_m \bar{I}$ (spectrum of counts) and $\delta \cdot \sin(\pi f_m \bar{I})$ (interval spectrum). In case the deviations cannot be described as sinusoids, a similar result holds due to the superposition principle which may be applied to (2) and (3).

The above analysis proves that for slowly modulated event series not too different from a regular pulse train, the interval spectrum and the spectrum of counts are equal to first order. In the calculation of the interval spectrum, the intervals must be considered to be spaced at distances equal to the mean interval length $\bar{I}$.

IV. Conclusions

In this paper, the interval spectrum and the spectrum of counts of heart rate variability data were compared. Both spectra lead to equivalent results (Figs. 2 and 3). We explained this likeness analytically. The spectra were shown to be similar for event series that are slowly and slightly modulated. The absolute amplitudes of the peaks in the spectra can be compared only if the spectra are correctly scaled, i.e., if the spectrum of counts is calculated as the Fourier transform of the dimensionless signal $\Sigma \bar{I} \cdot \delta(t - t_k)$ and if the interval spectrum is calculated as the digital Fourier transform of the numbers $I_k/\bar{I}$, $k = 0, 1, \cdots, N - 1$, which are considered to be spaced at distance $\bar{I}$.

The smaller value of the interval spectrum [Fig. 2(b)] as compared to the spectrum of counts [Fig. 2(c)] for $f > 0.2$ Hz can be explained by the theoretical ratio $\sin(\pi f \bar{I})/(\pi f \bar{I})$ between these spectra. This ratio has value 0.68 for $f = 0.5$ Hz, which corresponds roughly with the figures. A difference in the amplitude of the higher harmonics in the spectra from data with a fixed respiration rate (Fig. 3) is apparent. However, this difference does not lead to a preference for one of the spectra as harmonics may appear in both spectra in various ways.

For a sinusoidally modulated interval series, i.e., of the form $I_k = \bar{I} + \delta \cdot \sin(2\pi f_m k \bar{I})$, the interval spectrum consists of one harmonic and the spectrum of counts of many (Section III-B). On the other hand, physiological event series are often considered as originating from an integral pulse frequency modulation (IPFM) model [2], [11], and [16]. For a sinusoidally modulated input signal of an IPFM model, the interval spectrum of the generated event series consists of many harmonics [8], [14], whereas the spectrum of counts has a single peak at the frequency of the sinus (plus peaks and sidebands around multiples of the mean repetition frequency) [2]. Besides, a slightly modified IPFM model, e.g., with a refractory period [11], produces harmonics anyway. Finally, there is little reason to assume the respiratory influence on heart rate to be perfectly sinusoidal, so harmonics of unknown amplitude can be expected, as well in the spectrum of counts, as in the interval spectrum.

We conclude that it cannot be decided which spectrum is to be preferred, although the spectrum of counts has the advantage of being linked with the physiologically attractive IPFM model. In our opinion, two factors are of importance in deciding which spectrum to use in heart rate variability studies: ease of calculation, and the reason why the spectrum is needed.

As to ease of calculation, the interval spectrum has the advantage that it can be computed directly with standard fast Fourier techniques; the spectrum of counts requires special computer programs (see [10], [11], and [16]). The computation time of the two spectra need not be too different and will not be prohibitive. We computed for this paper the spectrum of counts in a straightforward but lengthy way, as we first of all wanted to stress the fundamental differences and similarities of the spectra. For the same reason, we computed the spectrum of counts at frequencies 0.001 Hz apart in Figs. 2(c) and 3(c), although probably only values at $1/(N+1)\bar{I}$ apart do contain independent information [9, p. 35]. Besides, if one computes $P_C(f)$ only for frequencies $f_k = k/(N+1)\bar{I}$ $(k = 1, 2, 3, \cdots)$, the contribution of the dc component is indiscernible, even when calculating the Fourier transform of the signal $s(t)$ instead of $s'(t)$ (Section III-A).

Both the interval spectrum and the spectrum of counts are useful if one is interested only in the presence and relative amplitudes of periodicities in the event series. However, a preference for one of the two spectra exists if one wants to study relationships between the event series and another signal. If the heartbeat event series is to be related to a continuous signal such as respiration or temperature, and one needs cross spectra or phase spectra, the spectrum of counts should be used as it is the transform of a function of time. The interval spectrum, being the transform of a signal which is essentially a function of interval number, is less applicable in this case. However, if one is interested in the relationship between heart interval and another variable that is defined on a beat-to-beat basis, e.g., systolic pressure, then the interval spectrum is the most logical choice.

In conclusion, the two described spectra are equally useful if periodicities in the heart rate series are sought. The resulting spectra are similar. A choice between the two spectra can be made on the grounds of ease of computation. Only if one needs to relate these periodicities to another signal can and must a definite choice for one of the spectra be made.

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