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PHY f105M: Lab for PHY f317K
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Lab 5: Reproducible Experiments

Note that the instructions have been appended to the end of the document.

Objective

The objective of this experiment was to test Hooke's law

$$|F| = k|x|,$$

on a set up of hanging wires, where $|x|$ is the length of the wire, $|F|$ is the magnitude of the force exerted on the wire, and k is the spring constant.

Hypothesis

We predicted that the model would be accurate for small values of $|F|$, meaning that k would be constant, but that one it $|F|$ was large enough, k would decrease, indicating that the wire was being stretched by the force of the hanging mass.

Materials

Unlike previous labs that had two parts, this lab has two experiments happening in one part. With that being said, the materials used in this lab were the hanging string set up (prepared by the LI), painter's tape, one 200 g weight, four 500 g weights, six 1000 g weights, a digital scale, a calculator, and note taking materials.

Methods

Of the four strings present, we chose to test the nylon string and the guitar string immediately to the left of it.

Instead of testing the model as written above, we instead calculated k as

$$k = \frac{|\Delta F|}{|\Delta x|},$$

which was an acceptable change because, theoretically, this gives the same value for k.

For each experiment, we had to modify the set up to help us take measurements before we could conduct trials. To do this, we visually estimated the distance between the wire being tested and the meterstick, tore off a piece of tape at least twice that length, stuck the halfway point of the tape to the wire at a point below the hole through which the wire passed, but above the knot (to keep the tape straight), stuck the rest of the tape to itself by folding it in half, and then tearing it at the end not stuck to the string so that there was enough tape to wrap around the meterstick, giving good visual estimates for the value of $|\Delta x|$.

Without attaching a weight the the wires, they had slack, which made it difficult to measure their baseline length. To get around this, we massed a small weight, recorded the measurement, and then hung it from the wire, and then measured the baseline length for that configuration. For the nylon string, the small weight was one labeled 200 g, while for the guitar string, the small weight was one labeled 500g. To do this, one person would place their fingers on the hanging weight to prevent sideways movement, making sure not to apply any vertical force, and then they wold gently pull the tape to the meterstick and measure the meterstick reading of the top of the tape. The other person would stand back and visually assess if the tape was level, and only when both people agreed that the tape was level would the length measurement be taken.

Now, for each trial, a person would take the mass of the next weight to be added (see Table 1 for the weight labels) with the digital scale, the cumulative mass would be recorded, the mass would be attached to the string, x was measured, F was calculated, and then $|\Delta x|$, $|\Delta F|$, and k were calculated.

Table 1 lists the label on the weight added to each of the string for each trial.

Table 1: Labeled Mass Added to Each Wire

Trial	Labeled Mass Added to Nylon Wire (g)	Labeled Mass Added to Guitar Wire (g)
0	200	500
1	500	1000
2	500	1000
3	500	1000
4	500	1000
5	1000	1000
6	1000	1000

Data

Table 2 contains all of the data and measurements for the nylon string, while Table 3 contains that for the guitar string. Note that the statistical weights fit better in the analysis section, and so are included there. Note that $F = mg$ and that, as instructed, g was taken to be 981 $\frac{cm}{s^2}$ without error.

Table 2: Raw Data for Nylon String

Trial	m (g)	x (cm)
0	200.15 +/- 0.01	10.0 +/- 0.05
1	700. 94 +/- 0.01	9.8 +/- 0.05
2	1201.23 +/- 0.01	9.6 +/- 0.05
3	1701.45 +/- 0.01	9.4 +/- 0.05
4	2200.83 +/- 0.01	9.2 +/- 0.05
5	3200.18 +/- 0.01	8.5 +/- 0.05
6	4199.72 +/- 0.01	7.7 +/- 0.05

Table 3: Calculated Values for Nylon String

Trial	F ($\frac{g \cdot cm}{s^2}$)	$\lvert\Delta x\rvert$ (cm)	$\lvert\Delta F\rvert$ ($\frac{g \cdot cm}{s^2}$)	k ($\frac{g}{s^2}$)
0	196347.15 +/- 9.81	NA	NA	NA
1	687622.14 +/- 9.81	0.20 +/- 0.05	491274.99 +/- 9.81	2456374.95 +/- 614093.74
2	1178406.63 +/- 9.81	0.20 +/- 0.05	490784.49 +/- 9.81	2453922.45 +/- 613408.61
3	1669122.45 +/- 9.81	0.20 +/- 0.05	490715.82 +/- 9.81	2453579.10 +/- 613394.77
4	2159014.23 +/- 9.81	0.20 +/- 0.05	489891.78 +/- 9.81	2449458.90 +/- 612364.73
5	3139376.58 +/- 9.81	0.70 +/- 0.05	980362.35 +/- 9.81	1400517.64 +/- 100036.98
6	4119925.32 +/- 9.81	0.80 +/- 0.05	980548.74 +/- 9.81	1225686.64 +/- 76605.37

Note that commas have been omitted in an effort to keep all of the data on one line as much as possible.

Table 4: Raw Data for Guitar String

Trial	m (g)	x (cm)
0	500.78 +/- 0.01	6.00 +/- 0.05
1	1501.31 +/- 0.01	5.90 +/- 0.05
2	2500.66 +/- 0.01	5.40 +/- 0.05
3	3500.69 +/- 0.01	4.90 +/- 0.05
4	4500.22 +/- 0.01	4.20 +/- 0.05
5	5499.68 +/- 0.01	3.50 +/- 0.05
6	6499.51 +/- 0.01	2.60 +/- 0.05

Table 5: Calculated Values for Guitar String

Trial	F ($\frac{g \cdot cm}{s^2}$)	$\|\Delta x\|$ (cm)	$\|\Delta F\|$ ($\frac{g \cdot cm}{s^2}$)	k ($\frac{g}{s^2}$)
0	491265.18 +/- 9.81	NA	NA	NA
1	1472785.11 +/- 9.81	0.10 +/- 0.05	981519.93 +/- 9.81	9815199.30 +/- 4907599.65
2	2453147.46 +/- 9.81	0.50 +/- 0.05	980362.35 +/- 9.81	1960724.70 +/- 196072.47
3	3434176.89 +/- 9.81	0.50 +/- 0.05	981029.43 +/- 9.81	1962058.86 +/- 196205.89
4	4414715.82 +/- 9.81	0.70 +/- 0.05	980538.93 +/- 9.81	1400769.90 +/- 100054.99
5	5395186.08 +/- 9.81	0.70 +/- 0.05	980470.26 +/- 9.81	1400671.80 +/- 100047.99
6	6376019.31 +/- 9.81	0.90 +/- 0.05	980833.23 +/- 9.81	1089814.70 +/- 60545.26

Note that commas have been omitted in an effort to keep all of data on one line as much as possible

Even though we weren't interested in the error of F, we did need to report it. $F(m) = mg$, so the formula for that error was

$$\delta F = \sqrt{(\frac{dF}{dm} \times \delta m)^2} = \frac{dF}{dm} \times 0.01 = 0.01g = 9.81(\frac{g \cdot cm}{s^2}).$$

The units are unusual, but they helped us avoid issues of numerical instability that will be discussed in the analysis section.

We were instructed not to use propagation of error in the calculation of $|\Delta x|$ or $|\Delta F|$, but we did use it for the calculation of *k*. The formula for that is also provided in the analysis section.

Analysis/Discussion

First, we will obtain the expression for δk. Since *g* is constant and assumed to be without error, $k = k(|\Delta m|, |\Delta x|)$. We will show the values of $|\Delta m|$ in a table below. Let $u = |\Delta x|$, $v = |\Delta m|$. We will be differentiating with respect to *u* and *v*, but since neither of these quantities are ever 0 in our data, this is a valid operation. Now,

$$\delta k = \sqrt{(\frac{\partial k}{\partial u} \times \delta u)^2 + (\frac{\partial k}{\partial v} \times \delta v)^2} = \sqrt{(\frac{981v}{u^2} \times 0.05)^2 + (\frac{981}{u} \times 0.01)^2}$$

$$\sqrt{(\frac{49.05v}{u^2})^2 + (\frac{9.81}{u})^2}\ \frac{g}{s^2}.$$

Using this formula, we obtain Table 6. Note that we have rounded to 2 decimal places for this table, but in our calculations for the weighted averages, we use all of the decimal points, to avoid rounding error.

Table 6: Values of δk (in $\frac{g}{s^2}$)

Trial	1	2	3	4	5	6
Nylon String	614,093.74	613,408.61	613,394.77	612,364.73	100,036.98	76,605.37
Guitar String	4,907,599.65	196,072.47	196,205.89	100,054.99	100,047.99	60,545.26

Regarding the numerical instability mentioned earlier, we originally tried to use kilograms and meters; however, because the displacements were so small, dividing by the displacement in meters led to unacceptable variation in the values for *k*. This being the case, we decided to use grams and centimeters.

Now, we had to calculate the weighted average of *k*, call it $\overline{k}_{weighted}$, as well as the error in this quantity, call it $\delta\overline{k}_{weighted}$, where these quantities are given by

$$\overline{k}_{weighted} = \frac{\sum_{i=1}^{N} k_i w_i}{\sum_{i=1}^{N} w_i} \text{ and } \delta\overline{k}_{weighted} = \frac{1}{\sqrt{N-1}} \sqrt{\frac{\sum_{i=1}^{N} (k_i - \overline{k}_{weighted})^2 w_i}{\sum_{i=1}^{N} w_i}},$$

with N being the number of trials and $w_i = \frac{1}{(\delta k_i)^2}$, where i is an index over the trials.

Just so that it is clear, $N = 6$.

The calculation of these quantities is lengthy and cumbersome, but it is dobale to plug everything in. Doing this, we produced Tables 8 and 9. Again, we have rounded here to two decimal points here, but carried all decimal points through the calculation.

After many attempts that resulted in incorrect calculations, it was realized that the matrix functionality of the TI-84 once again helped us tremendously, as with this, we could easily compare the data to be calculated on to the data in the tables, and we could make all of the summations more compact by expressing them as dot products.

Table 8: Values of w_i (in $\frac{s^4}{g^2}$)

Trial	Nylon String	Guitar String
1	2.65×10^{-12}	4.15×10^{-14}
2	2.66×10^{-12}	2.60×10^{-11}
3	2.66×10^{-12}	2.60×10^{-11}
4	2.67×10^{-12}	9.99×10^{-11}
5	1.00×10^{-10}	1.00×10^{-11}
6	1.70×10^{-10}	2.73×10^{-10}

Table 9 contains the values of $\overline{k}_{weighted}$ and $\delta\overline{k}_{weighted}$.

Table 9: Weighted Mean Spring Constant, and Error (in

String	Weighted Mean Spring Constant ($\frac{g}{s^2}$)	Error in Weighted Mean Spring Constant ($\frac{g}{s^2}$)
Nylon String	1,334,326.65	98,942.76
Guitar String	1,295,279.06	123,207.29

Finally, we have included a graph of the spring constant versus the force applied and the displacement.

Figure 1

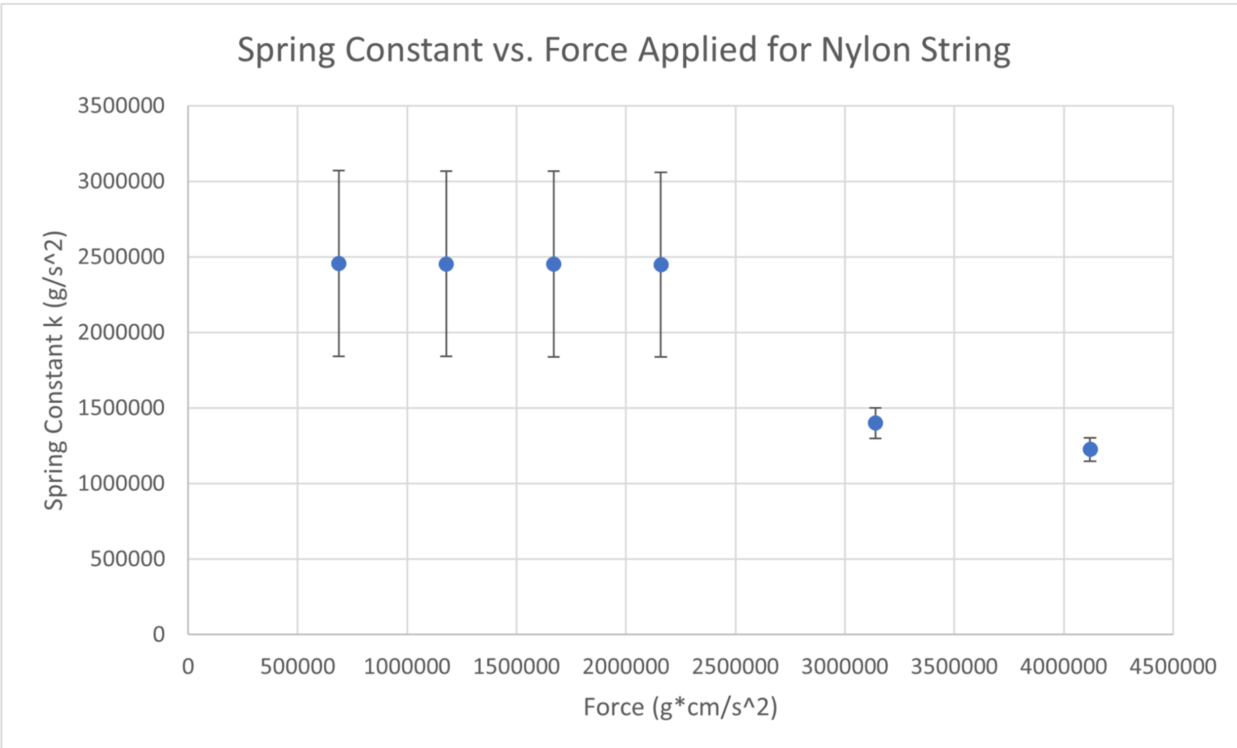


Figure 2

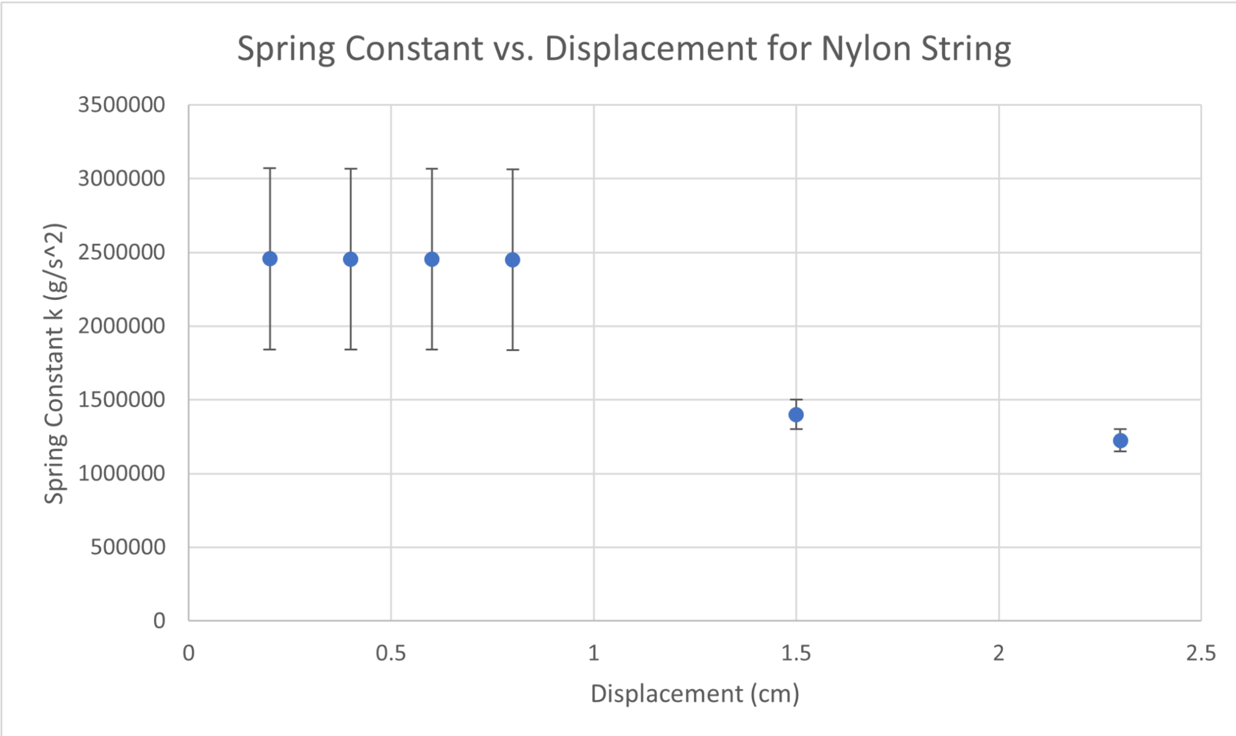


Figure 3

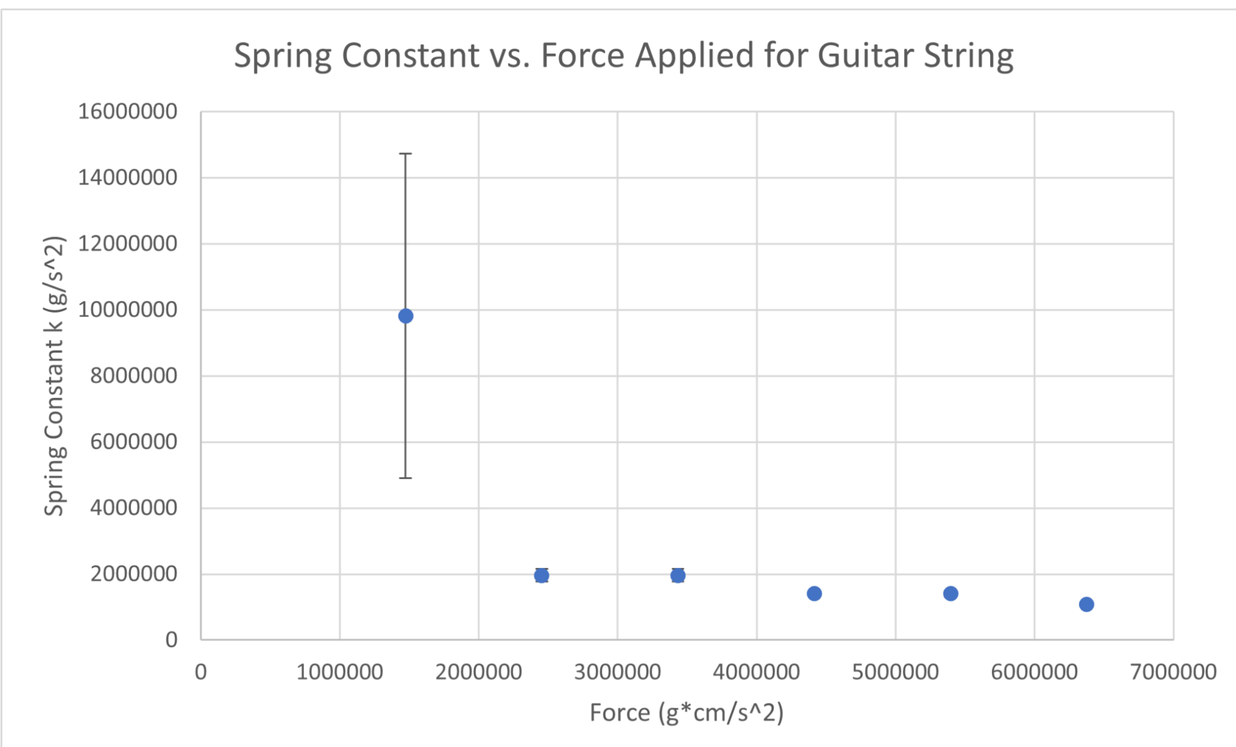
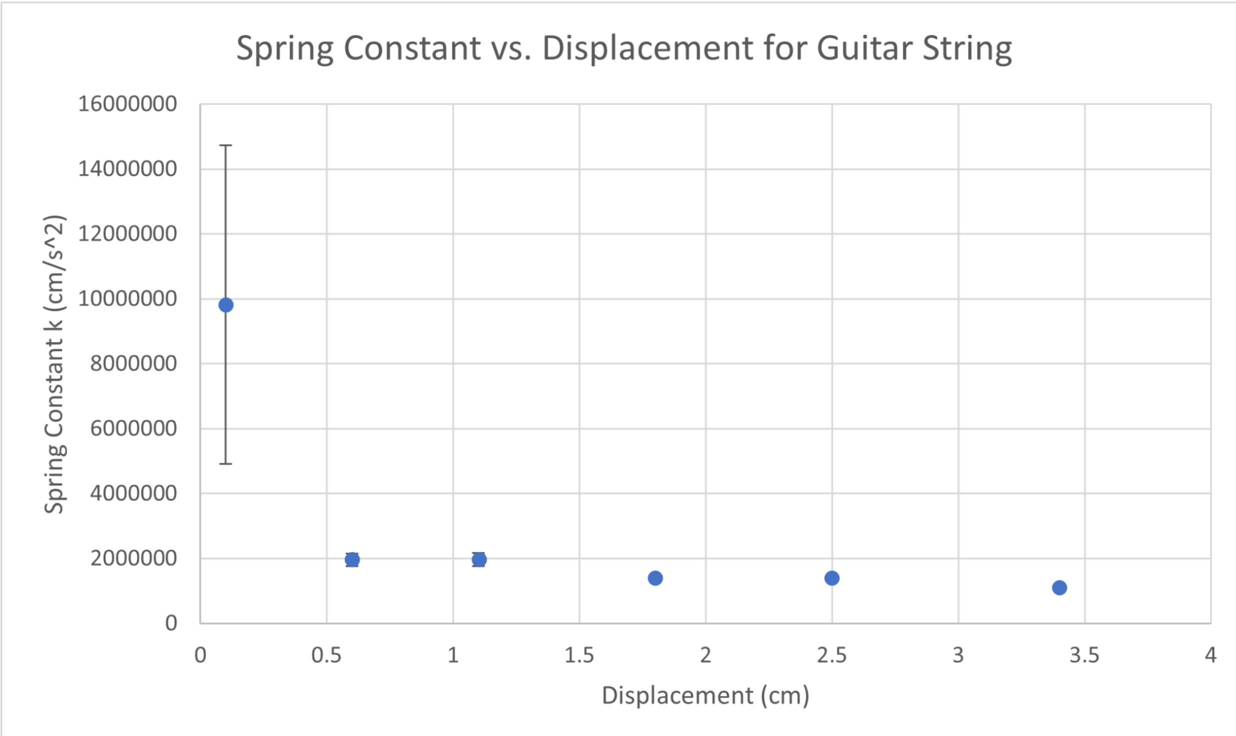


Figure 4



The plots for the nylon string are consistent with our hypothesis, that the spring constant would be essentially constant within a range, and then it would drop off once the force applied was large enough. For us, that value is somewhere between 2,159,014.23 and 3,139,376.58 $\frac{g \cdot cm}{s^2}$.

The plots for the guitar string are not consistent with our hypothesis. They show that the spring constant initially drops off, and is then essentially constant. It seems the string started out below the lower bound on the regime of validity, and then we did not add enough weight to get past the upper bound of that region. From this data, it seems that the string becomes taut somewhere between 491,265.18 and 1,472,785.11 $\frac{g \cdot cm}{s^2}$.

Note that we did not study linear fitting, which we believe was the necessary tool to determine regimes of validity in a quantitative way, so we have resorted to a qualitative discussion.

There is no need to analyze the spring constant vs. displacement plots, because they are equivalent to the spring constant vs. force plots up to a linear transformation of the independent variable.

Conclusion

The evidence from the nylon string was consistent with our hypothesis, while the evidence from the guitar string suggests that we observed the region below the regime of validity.

There are many improvements that could be made to this experiment. The first, and the most obvious, would be to use the magnifying glass to measure displacements, rather than the meterstick. In retrospect, this is what we were supposed to do, but we didn't realize this until after the fact. Another improvement would be to keep the mass increment constant between trials. We actually did implement this improvement for the guitar string, but only after realizing that we should from the nylon string. It would also be helpful to use a jack instead of hanging masses. This would have a number of benefits, namely that we could measure the force directly instead of calculating it from hanging mass, that this would do a better job of keeping the string orthogonal to the floor, and that this would prevent the string from spinning. The string would spin if enough mass was place on it, and while we took measures to control this, accidentally ignoring it for a period of time could lead to the string shortening from twisting or possibly snapping.

We look forward to seeing how the other group's data compared to ours.

Hanging Wires

Quantitative Result

The main idea of this whole thing is to see if Hooke's law,

$$|F| = k|x|,$$

is valid for the hanging wires setup, where $|x|$ is the length of the wire, $|F|$ is the magnitude of the force exerted on the wire, and k is the spring constant. To test this, we we see if

$$k = \frac{|\Delta F|}{|\Delta x|}$$

is a constant, and if it is not, then where is it not (as far as we know, we have not yet been taught the analytical tool for this, unfortunately).

Part 1: The Nylon Wire

Objective

The objective of this experiment is to measure the spring constant of a nylon string.

Materials

You will need:

- The hanging string set up
- Tape
- One 200 g weight
- Four 500 g weights
- Two 1000 g weights
- The digital scale at the front of the room
- A notebook, a writing utensil, and a calculator for convenience

Physical Setup

1. Stand up and face the hanging wires so that they are between you and the wall.
2. Locate the nylon string and the guitar string immediately to the left of it. These are the two wires whose spring constants you will measure.
3. Ensure that there is a meter stick fixed to the right of all the strings.
4. If present, remove any tape from the nylon string that may be left over from previous groups.
5. Visually estimate the distance between the nylon string and the meter stick. Hold this distance in your memory.
6. Get the tape, and tear off a piece of tape at least twice as long as the distance you estimated above.
7. Locate the halfway point on the length of tape you have torn off. Find a position on the nylon string that is
    a. Above the knot

    b. Below the hole through which the string passes
    c. High enough that if the tape moves down, it will still give you a reading on the meter stick.
8. Stick the halfway point of the tape to the nylon string at the point you identified in step 6. Stick the two pieces of tape on either side of the half way point so that you have a long piece of tape sticking out that can reach the meter stick. This is how you will take measurements.
    a. Make sure that none of the sticky side is exposed. The tape should form a neat, clean ribbon, essentially. Make sure that the tape is not so tight that it curves, and make sure not to capture any part of the knot under the string, as this will also curve the tape.
    b. Also make sure that the tape can reach the meter stick.
    c. If these conditions are not met, remove the tape and try again.

Data Setup: Trial 0

1. Make a table in your notebook. It will have 9 columns labeled
    a. Trial
    b. m (g)
    c. x (cm)
    d. F (g*cm/s^2)
    e. $|\Delta x|$ (cm)
    f. $|\Delta F|$ (g*cm/s^2)
    g. k (g*cm/cm*s^2)
    h. δk (g*cm/cm*s^2)
    i. w ((g*cm/cm*s^2)^-2)
2. To make your life easier, make the F and $|\Delta F|$ columns wide. You can make the x and $|\Delta x|$ columns narrower to accommodate this.
3. Add 7 rows to the table. Fill the trial column from 0 to 6.
4. Go ahead and, somewhere well away from the table, write down the value of acceleration due to gravity **in cm/s^2.** You will be using this value a lot to populate the F column.
5. For trial 0, enter "NA" into every column **except m, x, and F**
6. Now, take the 200 g mass to the scale. Record its value in the m column of trial 0.
    a. This is a baseline weight, mainly it's there to straighten the string out so that you can measure x more accurately.
7. Place the 200 g mass on the loop at the end of the nylon string. Extend the tape to the meterstick, and record the meterstick reading for the top of the strip of tape. Enter this value in the x column of trial 0.
    a. Note: When you straighten out the tape, the string has a tendency to curve with the tape. This will lead to incorrect measurements. It helps to place a couple fingers on the side of the weight to prevent the string from turning, but **it is**

important that you apply no vertical force to the mass, as this will also lead to incorrect measurements.

8. Using the value in the m column, calculate the value of F=mg and write it in the F column of trial 0.
    a. Note, because the units are g*cm/s^2, you should expect numbers at least on the order of 100,000 g*cm/s^2.

Trials

1. For each trial:
    a. If you are conducting trial 1, 2, 3, or 4 pick up a 500 g mass.
    b. If you are conducting trial 5 or 6, pick up a 1000 g mass.
2. Take the mass you have picked up to the scale. Record its mass, **add** this recorded mass to the mass value of the trial directly before the one you're on, and record **this sum** into the m column of the trial you're on.
3. Go ahead and calculate F using the m value you have just calculated, and record it in the F column of the trial you're on.
4. Attach the weight the mass of which you just measured to the weight already hanging in the string. (There is a small bar on the bottom of each weight that the hook can hold on to).
5. Using the tape, measure the new meter stick reading for the top of the ribbon. Record this value in the x column of the trial you're currently on.
    a. As before, you can hold the mass to prevent the string from curving, but do not apply any vertical force to the mass.
    b. These masses have been chosen so that there should be a noticeable difference in the meterstick reading from trial to trial. If there is not, then make sure that the tape ribbon is level with the ground. One of the experimenters can take a step back and eyeball it to see if its level.
    c. If your tape ribbon happens to be long enough, then you can gently wrap it around the meterstick. This can also help you to make sure the tape is level (because the edge will be parallel to the meter stick markings), and it can help you get accurate measurements.
6. Leave the weight hanging (so over the course of the trials you'll have a chain of weights hanging from the string).
7. Calculate $|\Delta x|$ and $|\Delta F|$ by subtracting the new value of x from the previous one and taking the absolute value, and then the new value of F from the previous one and taking the absolute value.
8. Calculate $k = |\Delta F|/|\Delta x|$, and record this value in the k column of the trial you're currently on.
    a. The division by $|\Delta x|$ is why such heavy masses are required, as without them, it's likely that $|\Delta x|$ would come out to 0, which would make the trial useless.

9. Skip the last two columns for now, they can be filled in after the data is collected.
10. Move onto the next trial. Continue until trial 6 is completed.

Part 2: The Guitar String

Objective

The objective of this experiment is to measure the spring constant of a guitar string.

Materials

You will need:

- The hanging string set up
- Tape
- One 500 g weight
- Six 1000 g weights
- The digital scale at the front of the room
- A notebook, a writing utensil, and a calculator for convenience

Physical Setup

The setup is the same as in Part 1, except this time, you'll use the string directly to the left of the nylon string. You'll need a longer piece of tape.

Data Setup: **Trial 0**

The table will still be 6 rows by 9 columns, and all the labels will be the same. You'll still have NA in the table entries specified in part 1.

The only difference is that instead of using a 200 g mass as the baseline, you'll use the 500 g mass as the baseline.

Trials

The trials will proceed as they did in part 1. The only difference is that now, you're adding 1000 g weights each trial. This is actually much simpler (we didn't really know what we were doing in part 1).

Analysis for Both Part

Let $u = |\Delta x|,\ v = |\Delta m|$. (You will have to calculate $|\Delta m|$ for each trial in each part, this was only noticed after the fact).

Now, we can express k as

$$k(u, v) = \frac{vg}{u},$$

where g is acceleration due to gravity in centimeters per second squared. Note that δv = 0.01 g and δu = 0.05 cm. With this in hand, the propagation of error formula is

$$\delta k = \sqrt{\left(\frac{vg}{u^2} \times 0.05\right)^2 + \left(\frac{g}{u} \times 0.01\right)^2},$$

which is all that's needed to calculate the δk and $w = \frac{1}{(\delta k)^2}$ columns.