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Lab 4: Whatever Floats Your Boat

Part 1: Floating and Sinking

Objective

The objective of this experiment is to test the model for the maximum mass a boat can carry (including its own):

$$m_{max} = \rho_{water} V_{object},$$

where m_{max} is the maximum mass the boat can carry in kilograms, $\rho_{water} = 1000 \pm 5$ kilograms per cubic meter, and V_{object} is the volume of the hull of the boat in cubic meters.

Hypothesis

For the quick check of this experiment, we were told to make two different boats; one that would sink when ⅔ full of water, and one that would do the opposite and sink. We made a small "boat" made of aluminum foil with small borders around the edge and relatively large surface area; this is the boat that floated. We made a flat piece of aluminum with many layers of aluminum foil that sank immediately. It seems that qualitatively, the relation between volume and buoyant force could explain the difference between this. **Our hypothesis for this part of the experiment is that the model proposed is in fact correct.** Our calculated m_{max} was 100.00 +/- 0.50 g (see the analysis section for details about this error). Based on the model, we predicted that 100.00 +/- 0.50 g would be just enough to sink our small tupperware boat.

Materials

The materials used in this experiment were a 100 mL graduated cylinder, a large tupperware (hereafter called the container), two small tupperwares (hereafter referred to as boats), a set of steel weights with labeled masses ranging from 1 to 100 g, a digital scale, and an excess of water. A calculator was also used for analysis.

Method

First, the container was filled approximately ⅔ of the way full with water. This was not measured precisely; the important things is that the boats could be submerged in the depth of water. Next, the volume of the boat was measured. To do this, the graduated cylinder was filled to some value, and the water was poured in. After that, a set of weights slightly exceeding the calculated m_{max} was gathered. The mass of each of these weights was taken with a digital scale.

With all of this preparation complete, the trial could begin. First, a collection of weights whose total mass was less than the calculated maximum mass was added to the center of the boat, and then the boat was gently placed in the center of the container. It was important to avoid imparting any vertical velocity on the boat, as this might have sunk the boat with less than the correct amount of mass. We verified that the boat did not sink, and then we added small weights—namely those labeled 5 g and 1 g—until the boat sank. The value of the mass that caused the boat to sink was recorded as the maximum mass, the idea being that this value was just above m_{max}.

A note about errors. At first, we thought that we should use the propagation of error formula to get more accurate estimates of the systematic error for the experimental maximum mass and the boat volume. However, we were instructed by the substitute LI that these errors were negligible.

Data

Table 1: Parameters and Results for Part 1

Trial	Boat Volume (mL)	Theoretical Maximum Mass (g)	Experimental Maximum Mass (g)
1	100.00 +/- 0.05	100.00 +/- 0.50	100.13 +/- 0.01

Analysis

To see the model uncertainty, let

$$m = f(\rho, V) = \rho V.$$

This is the model in the hypothesis, but we've removed the subscripts for brevity. Now, the systematic error of the model is given by

$$\delta m = \delta f = \sqrt{(\frac{\partial f}{\partial \rho}\delta\rho)^2 + (\frac{\partial f}{\partial V}\delta V)^2}.$$

It was given that $\delta\rho = 5\ kg/m^3$, and the error from the graduated cylinder, converted to consistent units, was $\delta V = 5 \times 10^{-8}\ m^3$. Since the partial derivatives were provided, we have that

$$\delta m = \sqrt{(V \times 5)^2 + (\rho \times 5 \times 10^{-8})^2}.$$

Plugging in the values from Table 1 (converted into the correct units) gives

$$\delta m = 5.024938 \times 10^{-4}\ kg = 0.50\ g,$$

where here we have not rounded until the end of the calculation.

Using the standard procedure for *t*-scores, we calculated a *t*-score of 0.26, meaning that the model's prediction and the data were indistinguishable.

Conclusion

The data above suggests serves as evidence towards our hypothesis because the *t*-value was very small.

As always, there are a number of improvements that could be made to make the experiment better. For one, it would be good to use sand or steel filings to add mass to the boat. These are denser than water, so the would work similarly to the weights, but it would be possible to add mass in much smaller increments, giving a closer estimate of the maximum mass the boat could hold. For another, we could get more accurate measurements of the boat volumes if we were to fill the boat with concrete, submerge it in a large graduated cylinder, and then measure the change in the volume reading. This would give a more accurate measure of the boat volume, as surface tension wouldn't be there to give us a wide range of volume measurements. Finally, it would be accurate, though slower, to measure the mass of the boat and the added mass every time we added more mass, as that way we could take the scale's systematic error, and we would know that it was correct, rather than just neglecting the extra propagation of error.

Part 2: Floating and Sinking Redux

Objective

Because we obtained a positive result in part 1, the objective of this experiment was to test the hypothesis on the other boat.

Hypothesis

As in part 1, we hypothesized that the model

$$m_{max} = \rho_{water} V_{object}$$

would provide an accurate prediction of the maximum mass the boat could carry and just barely float. The prediction can be found in Table 2 in the Theoretical Maximum Mass column.

Materials

The materials in this part were exactly the same as in part 1, except for that we used a different boat.

Method

The method of this part was exactly the same as in part 1.

Data

Table 2: Parameters and Results for Part 2

Trial	Boat Volume (mL)	Theoretical Maximum Mass (g)	Experimental Maximum Mass (g)

1	173.00 +/- 0.05	173.00 +/- 0.87	173.46 +/- 0.01

<u>Analysis</u>

The propagation of error formula is exactly the same as it was in the analysis section of part 1. Using the volume in Table 2 and the provided density, we end up with

$$\delta m = 8.664438816 \times 10^{-4}\, kg = 0.87\, g.$$

Then, using this value in the *t*-score formula, and with systematic error of the experimental maximum mass equal to 1×10^{-5} kg, we obtain a *t*-score between the theoretical and experimental masses of 0.53, which again means that the model prediction was indistinguishable from the data.

<u>Conclusion</u>

As before, this data lends evidence to our hypothesis. Because constraints on time or materials prevented us from implementing the improvements mentioned in the conclusion of part 1, those improvements would also apply to part 2, especially the volume measurement improvement, as this boat had an extra little volume under the lip where the lid goes on, the measurement of the volume of which was heavily influenced by the surface tension of the water.