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Lab 3: A Balancing Act

Part 1: A Balancing Act

Objective

A condition under which it is proposed that two masses on a balance will be in rotational static equilibrium is

$$m_1 x_1 = m_2 x_2$$

where m_1 is the mass of the object on, say, the left side of the pivot, x_1 is the distance of the left object from the pivot, m_2 is the mass of the object on the right side of the pivot, and x_2 is the distance of the right object from the pivot. Both masses are measured in the same units, and both lengths are too.

The purpose of this experiment is to determine if this model "works", meaning that if the condition is satisfied, then the system will be in rotational static equilibrium.

Materials

The materials used in this experiment were one balance stand, one pivot, one meter stick with a hole drilled through the center of the broad side, one level, two hangers, each of mass 10.0 +/- 0.5g, and a set of weights provided by the lab.

A note about the error here: we were instructed to use 0.5 g as the error. Below, you will see errors of 1.0 g on the mass measurements; this is because the masses placed on the balances were composed of the hanger *and* the weight, so we were instructed to add the uncertainties.

Hypothesis

For our quick check, we hung the 10.0 +/- 0.5 g hangers at each end of the meter stick to ensure we were able to achieve a static configuration. We were successful, and the configuration roughly seemed to conform to the model, so our hypothesis is that the model was correct. Based on previous understandings of balance and weights and the quick check using the hanger masses, we predict that the given model is scientifically accurate. Although we are unable to see exact angles of the meter stick, we believe that the unseen tilt of the meter stick is insignificant to the model.

To formulate a testable hypothesis, we rearranged to model to the form

$$m_1 = \frac{m_2 x_2}{x_1},$$

so that the hypothesis could now be stated as: The model will accurately predict the value of m_1 required to attain a static configuration for given values of m_2, x_2, and x_1.

Method

To start this experiment, we positioned the pivot to the exact halfway point of the meter stick. We decided it was at the 50 cm mark within the uncertainty. There is uncertainty in the positioning of the pivot, but we determined that it would not have a significant impact on the results, the reason being that friction between the meter stick and the pivot would give us some room for error in the positioning of the masses. For this part of the experiment, we were instructed to take 3 trials with 3 unique positionings of masses; however, as will be shown, the *t*-scores for trials 2 and 3 were too large, so we conducted two more trials. An explanation of the issue for trials 2 and 3 is proposed.

To test the accuracy of the model, we placed the masses at different points on the meter stick, and moved them on the hangers until the system was in a static configuration. The values of m_1, m_2, x_1, and x_2 were then recorded.

The independent variables were m_2, x_1, and x_2. The dependent variable was m_1. The control variables included the configuration of the balance when no masses were hung (call it the baseline configuration), and all of the equipment used.

<u>Data</u>

Table 1: Data for Part 1

Trial	Mass 2 (g)	Distance 1 (cm)	Distance 2 (cm)	Experimental Mass 1 (g)	Theoretical Mass 1 (g)
1	110.0 +/- 1.0	21.90 +/- 0.05	12.00 +/- 0.05	60.0 +/- 1.0	60.3 +/- 0.6
2	210.0 +/- 1.0	22.50 +/- 0.05	6.60 +/- 0.05	60.0 +/- 1.0	61.6 +/- 0.6
3	210.0 +/- 1.0	29.70 +/- 0.05	30.00 +/- 0.05	210.0 +/- 1.0	212.1 +/- 1.1
4	30.0 +/- 1.0	10.00 +/- 0.05	6.60 +/- 0.05	20.0 +/- 1.0	19.8 +/- 0.7
5	210.0 +/- 1.0	10.00 +/- 0.05	10.00 +/- 0.05	210.0 +/- 1.0	210.0 +/- 1.8

This table presents the data for part 1 of this lab. Note that the errors on the theoretical mass 1 measurements were obtained via the propagation of error formula. An example of such a calculation is carried out below.

<u>Analysis</u>

To analyze our results, we calculated *t*-scores between the trial and the model, given by

$$t = \frac{|m_{1,exp} - m_{1,th}|}{\sqrt{(\delta m_{1,exp})^2 + (\delta m_{1,th})^2}},$$

where $m_{1,exp}$ is the measured value for m_1, $m_{1,th}$ is the theoretical or calculated value for m_1, $\overline{\delta m_{1,exp}}$, because we only took one trial, is the systematic error of the experimental mass (1.0 g), and $\overline{\delta m_{1,th}}$ is the error of the model, obtained below.

Let

$$m_{1,th} = f(m_2, x_1, x_2) = \frac{m_2 x_2}{x_1}.$$

Then the error in $m_{1,th}$ is given by the propagation of error formula; that is,

$$\delta f = \sqrt{(\frac{\partial f}{\partial m_2}\delta m_2)^2 + (\frac{\partial f}{\partial x_1}\delta x_1)^2 + (\frac{\partial f}{\partial x_2}\delta x_2)^2}$$

$$= \sqrt{(\frac{x_2}{x_1} \times 1.0)^2 + (\frac{m_2 x_2}{x_1^2} \times 0.05)^2 + (\frac{m_2}{x_1} \times 0.05)^2},$$

because δm_2 = 1.0 g and $\delta x_1 = \delta x_2$ = 0.05 cm.

For trial 1, m_2 = 110.0 g, x_1 = 21.90 cm, and x_2 = 12.00 cm. Substituting these values and evaluating the expression, we obtain

$$\delta f = \sqrt{(0.5479452055)^2 + (0.1376118096)^2 + (0.6182661545)^2} = 0.6 \text{ g},$$

where we have not rounded until the end of the calculation in order to reduce rounding error.

With δf in hand, we can calculate the *t*-score

$$t = \frac{|60.0\ g - 60.3\ g|}{\sqrt{(0.1\ g)^2 + (0.6\ g)^2}} = 0.49.$$

t being less than 1 means that, in the case of trial 1, the experimental mass 1 is indistinguishable from the theoretical mass 1, lending evidence towards the hypothesis. All of the *t*-scores are reported in Table 2.

Table 2: *t*-scores for Part 1

Trial	1	2	3	4	5
t-score	0.49	2.63	1.92	0.28	0.00
Interpretation	Indistinguish able	Inconclusive	Inconclusive	Indistinguish able	Indistinguish able

Conclusion

After the beginning of this experiment, we concluded that the evidence and data supports the hypothesis. To explain our *t*-scores, we draw from the concept of friction, specifically at the pivot. We demonstrated to the instructor by showing that, given the placement of one mass, the other mass used to balance it could be placed in a range of about .5 cm in width. The most likely explanation is that static friction at the pivot was preventing the balance from moving, and allowing us to place the masses in incorrect locations. This was rectified in trials 4 and 5 by finding the furthest and closest positions one of the masses could be placed such that the balance would move, then sliding the mass to the center of that range, bumping the balance upward to overcome static friction, and then adjusting until bumping the balance resulted in not only a stable configuration, but a level one. To improve upon this experiment, we recommend several options. First, by using the corrective bumping method described above at the outset, you may be able to obtain more accurate results. Second, by implementing the use of a digital scale, you are more likely to get accurate mass measurements, as opposed to using the printed mass weight on the masses themselves. Last, to ensure the accuracy of the balance stand, use a level measurement tool to prove the levelness of the table. The last improvement was not implemented in part two of this experiment due to time constraints.

Part 2: Beyond the Balancing Act

<u>Objective</u>

The purpose of this part of the experiment was to measure the weight of a mystery object. For our mystery object, we chose a pair of headphones in a case with a small clip on one side of it. We used the condition of rotational static equilibrium described in part 1 to calculate the mystery object's mass.

<u>Materials</u>

Our materials for this part of the experiment were exactly the same as in the part above, but this part includes the mystery object (the headphones in the case with a clip). Instead of using the provided lab weights on both sides of the hanger, we used the weight of the headphones on one side, and the provided lab weights on the other.

<u>Hypothesis</u>

For this part of the experiment, we were unsure what to have as a hypothesis, as this is more of an application than an experiment. However, we again hypothesized our new model will accurately predict the value of m_1 required to attain a static configuration for given values of m_2, x_2, and x_1. This model would calculate the correct weight, and mirror the weight given by the digital scale.

<u>Method</u>

Our method for this part of the experiment was almost identical to the method in the part above. We used the same corrective bump method described in the previous sections to ensure proper results were obtained. To start this process, we placed the headphones on the hanger and put the hanger on the meter stick balance. We then adjusted the 510 +/- 1g weight on the other side of the meter stick with the hanger until the meter stick was balanced.

Data

Table 3: Data for Part 2

Mass 2 (g)	Distance 1 (cm)	Distance 2 (cm)	Calculated Mystery Mass (g)
510.0 +/- 1.0	40.00 +/- 0.05	6.5 +/- 0.05	82.88 +/- 0.67

The error on the mystery mass is calculated below.

The mass of the mystery object was measured with the digital scale to be 82.72 +/- 0.01 g.

Analysis

First, the calculated mystery mass was calculated by evaluating *f* at the values in the first three columns of Table 3.

Next, we calculate the error on the mystery mass. Again, we use the formula

$$\delta f = \sqrt{\left(\frac{x_2}{x_1} \times 1.0\right)^2 + \left(\frac{m_2 x_2}{x_1^2} \times 0.05\right)^2 + \left(\frac{m_2}{x_1} \times 0.05\right)^2},$$

where the values for the independent variables can be found in Table 3. This error comes out to

$$\sqrt{(0.1625)^2 + (0.10359375)^2 + (0.6375)^2} = 0.67 \text{ g}.$$

With this error in hand, we can now calculate the *t*-score:

$$t = \frac{|82.72 - 82.88|}{\sqrt{(0.01)^2 + (0.67)^2}} = 0.24,$$

which means that the value calculated with the balance and the value measured with the digital scale are indistinguishable.

Conclusion

In conclusion, the hypothesis was correct; the model was successful in measuring the weight of the mystery object.

As always, improvements can be made to this set up. The first would be to have used the digital scale to measure the mass of the weight we were using to calculate the mass of the mystery object, instead of using the measurement printed on the weight, and the larger systematic error that came with it. This would have made the denominator of the *t*-score larger, but it would have also been a more exact result. Another improvement would be to reduce the friction between the meter stick and the pivot; while this would make the balance harder to use, it would obviate the need for the “bumping” step described above, so that we could be more confident that the first static equilibrium obtained would be the correct one. The unimplemented improvement of part 1—to make sure the table was level—would also improve the set up used here.