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Lab 6: Reproducible Experiments Part 2

Note that the feedback for the instructions provided by the other group have been appended to the end of the report.

Objective

The objective of this experiment was to repeat another group's—hereby dubbed the instructors—experiment. The instructors gave instructions for the "Inflating Balloons" setup, meaning that we were testing the model

$$P = k|\Delta s|,$$

where P is the pressure exerted on the interior of the balloon, Δs is the change in some measure of the balloon's size, and k is the spring constant. The instructors stated that s is the diameter of the balloon.

Hypothesis

Using the provided equation that $P = \rho g h$—where $\rho = 1,000\ \frac{kg}{m^3}$ is the density of water, $g = 9.81 \frac{m}{s^2}$ is acceleration due to gravity, h is the height of the water in the tube (to be clarified below), and P is the pressure at the bottom of the tube—the instructors provided an equation for k:

$$k = \frac{\rho g h}{|\Delta s|}.$$

No quantitative hypothesis was provided, but, in the same vein as Lab 5, **we expected to see that as the volume of water in the balloon increased, the values of *k* would initially be very high, then they would be essentially constant for a while, and then they would drop off beyond a certain volume of water.**

The independent variable was the volume of water added to the tube, and the dependent variable was k. Control variables were the increment in the volume of water added, the environment in which the experiment was conducted, and all of the equipment used.

This is the first time that the independent variable does not appear in the expression of the dependent variable. Instead, the volume of water in the balloon influences the height h, and that is what appears in the formula for k. As a result, the error in k is influenced by the error for h rather than by that in the volume.

Materials

The materials used in this experiment were a large plastic bin filled approximately 75% of the way with water, a plastic tube, a stand to hold the tube, balloons, rubber bands, a graduated cylinder, a meterstick, and a soft tape measure. Note taking materials were used, and a calculator was used for analysis.

<u>Methods</u>

To set up the experiment, the tube was released from the stand, the balloon was stretched over the lower end of the tube such that the part of the balloon gripping the tube was roughly as long as the width of the rubber band. The rubber band was then used to secure the balloon to the tube, and the tube was inserted back into the water so that the balloon was just barely submerged. Finally, the tube was resecured with the stand. [put trial 0 steps here]

Next, the following was repeated until the balloon popped. [put trial 1-20 steps here].

<u>Data</u>

First, we had to convert all of the quantities into consistent units. *s* and *h* are measured in centimeters, so the density of water is

$$\rho = 1,000 \frac{kg}{m^3} \times 10^{-6} \frac{m^3}{cm^3} = 0.001 \frac{kg}{cm^3}$$

and acceleration due to gravity is

$$g = 9.81 \frac{m}{s^2} \times 10^2 \frac{cm}{m} = 981 \frac{cm}{s^2}.$$

To find the dimensions of *k*, we simplified the dimensions in the right hand side of the formula for *k*,

$$[k] = \frac{\frac{kg}{cm^3} \cdot \frac{cm}{s^2} \cdot cm}{cm} = \frac{kg}{cm^2 \cdot s^2}.$$

Also note that $\delta s = 0.05$ cm and $\delta h = 0.05$ cm. We have been permitted to make a simplifying assumption that $\delta|\Delta s| = 0.05$ cm. We have taken *g* and ρ to be errorless, and because the total volume does not appear in the expression for *k*, we have omitted the error in those measurements too.

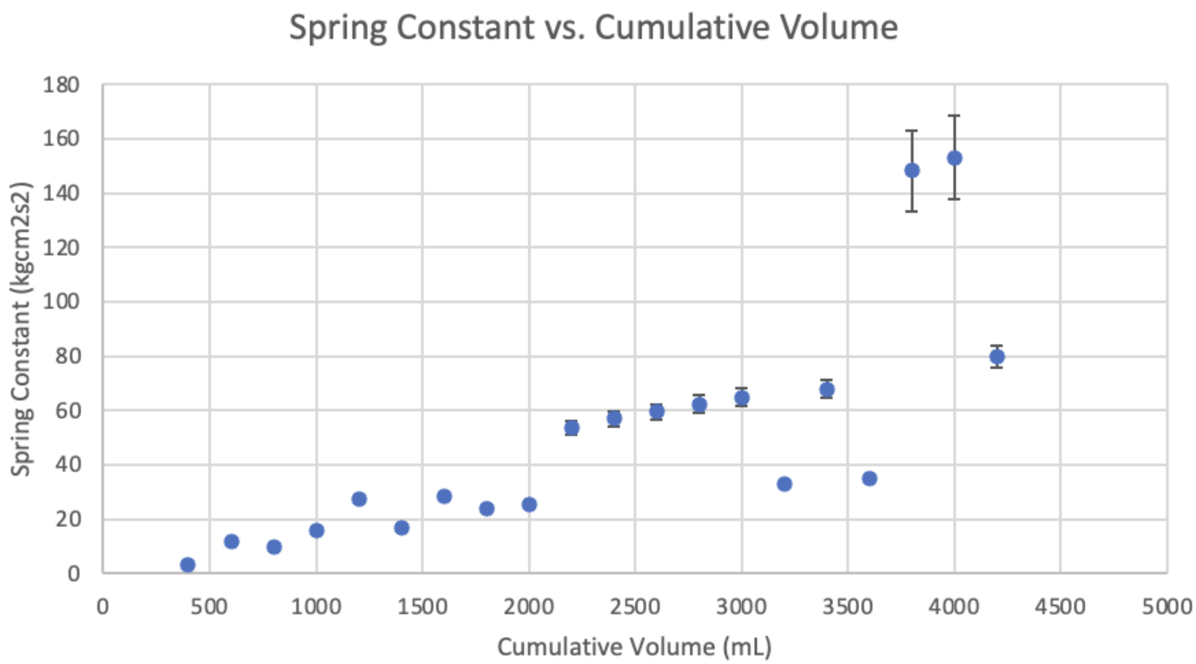
Table 1: Experimental Data

Trial	Total Volume* (mL)	s (cm)	h (cm)	$\lvert\Delta s\rvert$ (cm)	k ($\frac{kg}{cm^2 \cdot s^2}$)
0	200	NA	NA	NA	NA
1	400	24.50 +/- 0.05	48.00 +/- 0.05	15.40 +/- 0.05	3.06 +/- 0.01
2	600	28.00 +/- 0.05	41.50 +/- 0.05	3.50 +/- 0.05	11.63 +/- 0.17
3	800	32.00 +/- 0.05	39.00 +/- 0.05	4.00 +/- 0.05	9.56 +/- 0.12
4	1000	34.50 +/- 0.05	39.50 +/- 0.05	2.50 +/- 0.05	15.50 +/- 0.31
5	1200	36.00 +/- 0.05	41.50 +/- 0.05	1.50 +/- 0.05	27.14 +/- 0.91

6	1400	38.50 +/- 0.05	43.00 +/- 0.05	2.50 +/- 0.05	16.87 +/- 0.34
7	1600	40.00 +/- 0.05	46.00 +/- 0.05	1.50 +/- 0.05	28.15 +/- 1.00
8	1800	42.00 +/- 0.05	48.50 +/- 0.05	2.00 +/- 0.05	23.79 +/- 0.60
9	2000	44.00 +/- 0.05	52.00 +/- 0.05	2.00 +/- 0.05	25.51 +/- 0.64
10	2200	45.00 +/- 0.05	54.50 +/- 0.05	1.00 +/- 0.05	53.46 +/- 2.67
11	2400	46.00 +/- 0.05	58.00 +/- 0.05	1.00 +/- 0.05	56.90 +/- 2.84
12	2600	47.00 +/- 0.05	60.50 +/- 0.05	1.00 +/- 0.05	59.35 +/- 2.97
13	2800	48.00 +/- 0.05	63.50 +/- 0.05	1.00 +/- 0.05	62.28 +/- 3.12
14	3000	49.00 +/- 0.05	66.00 +/- 0.05	1.00 +/- 0.05	64.75 +/- 3.24
15	3200	51.00 +/- 0.05	67.00 +/- 0.05	2.00 +/- 0.05	32.86 +/- 0.82
16	3400	52.00 +/- 0.05	68.50 +/- 0.05	1.00 +/- 0.05	67.75 +/- 3.36
17	3600	54.00 +/- 0.05	71.00 +/- 0.05	2.00 +/- 0.05	34.83 +/- 0.87
18	3800	54.50 +/- 0.05	75.50 +/- 0.05	0.50 +/- 0.05	148.13 +/- 14.81
19	4000	55.00 +/- 0.05	78.00 +/- 0.05	0.50 +/- 0.05	153.04 +/- 15.30
20	4200	56.00 +/- 0.05	81.50 +/- 0.05	1.00 +/- 0.05	79.95 +/- 4.00
21	4400	NA	NA	NA	NA

Note that the balloon popped on trial 21.

Figure 1



Based on the graph here we can see how k was essentially constant until a certain volume of water was in the balloon. For us, that threshold volume was somewhere between 3,600 mL and 3,800 mL. For the most part, the uncertainties seemed to grow with volume; this appears to be related to h increasing with the volume after an initial dip. Also, the observations are not consistent with our hypothesis. Instead of starting high, settling to some constant for an intermediate range, and then dropping off, we instead see the spring constant starting constant, then increasing until the balloon popped. Unlike the wires, which stretched when the force applied was beyond some critical value, it seems like the balloon was stretching less and less as the volume increased.

Analysis/Discussion

First, we obtain an expression for δk. Recall that

$$k = \frac{\rho g h}{|\Delta s|} = \frac{0.981h}{|\Delta s|}.$$

With the assumption that g and ρ are errorless, the only errors to consider are δh and $\delta|\Delta s|$, the values of which are provided above. Now,

$$\delta k = \sqrt{\left(\frac{\partial k}{\partial h} \times \delta h\right)^2 + \left(\frac{\partial k}{\partial |\Delta s|} \times \delta|\Delta s|\right)^2}$$

$$= \sqrt{\left(\frac{\rho g}{|\Delta s|} \times 0.05\right)^2 + \left(\frac{\rho g h}{|\Delta s|^2} \times 0.05\right)^2} = \sqrt{\left(\frac{0.04905}{|\Delta s|}\right)^2 + \left(\frac{0.04905h}{|\Delta s|^2}\right)^2}$$

$$= 0.04905\sqrt{\left(\frac{1}{|\Delta s|}\right)^2 + \left(\frac{h}{|\Delta s|^2}\right)^2},$$

where $[\delta k] = \frac{kg}{cm^2 \cdot s^2}$.

With this, we were able to populate Table 2. Recall that the weight of the i-th trial is $w_i = \frac{1}{(\delta k_i)^2}$.

Table 2: Errors in k and Weights

Trial	δk ($\frac{kg}{cm^2 \cdot s^2}$)	w (($\frac{kg}{cm^2 \cdot s^2}$)$^{-2}$)
1	0.01	9199.68
2	0.17	35.96
3	0.12	69.23
4	0.31	10.36
5	0.91	1.22
6	0.34	8.75
7	1.00	0.99
8	0.60	2.82
9	0.64	2.46
10	2.67	0.14
11	2.84	0.12
12	2.97	0.11
13	3.12	0.10
14	3.24	0.10
15	0.82	1.48
16	3.36	9.54e-2
17	0.87	1.48
18	14.81	4.56e-3
19	15.30	4.30e-3
20	4.00	6.26e-2

From table 2, we were able to calculate the weighted average and the error of the weighted average, given by

$$\overline{k}_{weighted} = \frac{\sum_{i=1}^{N} k_i w_i}{\sum_{i=1}^{N} w_i} \text{ and } \delta\overline{k}_{weighted} = \frac{1}{\sqrt{N-1}}\sqrt{\frac{\sum_{i=1}^{N} (k_i - \overline{k}_{weighted})^2 w_i}{\sum_{i=1}^{N} w_i}},$$

respectively. With the aid of Jupyter notebooks, the results were that $\overline{k}_{weighted} = 3.20 \frac{kg}{cm^2 \cdot s^2}$ and $\delta\overline{k}_{weighted} = 0.32 \frac{kg}{cm^2 \cdot s^2}$.

Conclusion

The data collected in this experiment are not aligned with our hypothesis. Rather than seeing the *k* versus volume plot follow a roughly inverse sigmoidal shape as predicted, we see the points remaining small over a range of volumes, before increasing sharply just before the popping volume.

In retrospect, this makes sense. The wires tested were ductile, so that past a certain tension, they would stretch, whereas rubber is elastic, so that it exerts a greater force to restore its resting shape.

As always, there are improvements to be made to this experiment. For one, it would have been better to measure the diameter of the balloon instead of the circumference, using a Vernier caliper instead of a tape measure. Theoretically, they should be equivalent, since one is proportional to the other, but in practice, it is easier to measure the diameter than the circumference. The diameter is the length of the largest line segment between two points on the balloon's surface, while the circumference is the supreme arclength of intersections between planes and the balloon; since the balloon was not spherical, identifying the supremum by eye is challenging. However, we wanted our results to be comparable with the other group's. Another improvement would be to report the uncertainties in the volume, it's just good practice.

Instruction Feedback

The instructions were clear, legible, and readable. It would have been better to break the document into clear sections, although this not being the case did not impair our experiment.

The instructions were fully specified. The procedures were either stated clearly or could be easily inferred.

Yes, a reasonably reproducible quantitative result was given in the form of the equation to test,

$$k = \frac{\rho g h}{|\Delta s|},$$

and they also followed the rule of not providing any data.