Aidan O'Keeffe (alo779) and Maahe Kazmi (mzk229)
Abhigya Paudyal
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Lab 7: Diffusion and Brownian Motion

Part 1: Diffusion

Objective

The goal of this experiment was to observe the diffusion of dye in water, assess the different causes of motion of the dye within the water, and assign timescales to each of the observed behaviors.

Hypothesis

We performed a quick check wherein we put three drops of pink dye into a beaker containing roughly 250 mL of water and then marked the time at which the initial velocity, gravity, random currents, and diffusion stopped being the dominant factor influencing the visual motion. The exact method used to determine when each type of motion stopped, as well as that of how to calculate the magnitude of elapsed time, are laid out in the methods section. In any case, we obtained the following data from the quick check.

Table 1: Quick Check Data

Dominant Motion	Initial Time (s)	Final Time (s)	Elapsed Time (s)	Order of Magnitude of Elapsed Time
Initial velocity	0.00	4.77	4.77	1
Gravity	4.77	23.96	19.19	1
Random currents	23.96	60.89	36.93	2
Diffusion	60.89	104.12	43.23	2

Based on the data of Table 1, our hypothesis was the the initial velocity would be the dominant cause of motion for a duration on the order of 10 seconds, gravity would be the dominant cause of motion for a duration on the order of 10 seconds, random currents would be the dominant cause of motion for a duration on the order of 100 seconds, and diffusion would be the dominant cause of motion for a duration on the order of 100 seconds.

Materials

The materials used in this experiment were one beaker, an excess of water from a faucet, a dropper filled with an excess of pink dye, and a digital stopwatch.

<u>Methods</u>

Three trials were conducted. For each trial, first, the beaker was filled with roughly 250 mL of water from the faucet. Then, one person held the stopwatch in one hand and the dropper in the other. That person would drop three drops of dye and would start the timer when the third drop hit the water.

Four time points (besides the initial time) were recorded. The first was when dye that had gotten caught on the surface started to sink. The second was when all of the dye stopped sinking. The third was when the suspended strands of dye became invisible. The fourth was when the color of the liquid in the dye was approximately uniform. See the attached images for clarification.

To calculate the order of magnitude p of the elapsed time Δt, we used the formula

$$\tilde{p} = \log_{10}(\Delta t),$$

and then p was obtained by rounding $\tilde{p}$. This was done so that we would have a principled approach for calculating the order of magnitude, as otherwise, it might be too easy to underestimate the order of magnitude.

Figure 1: Initial Velocity Starts (0.00 s)

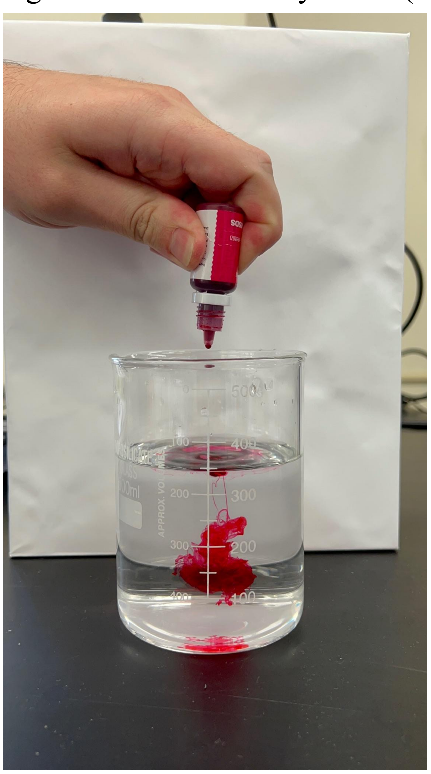


Figure 2: Gravity Starts (5.91 s)

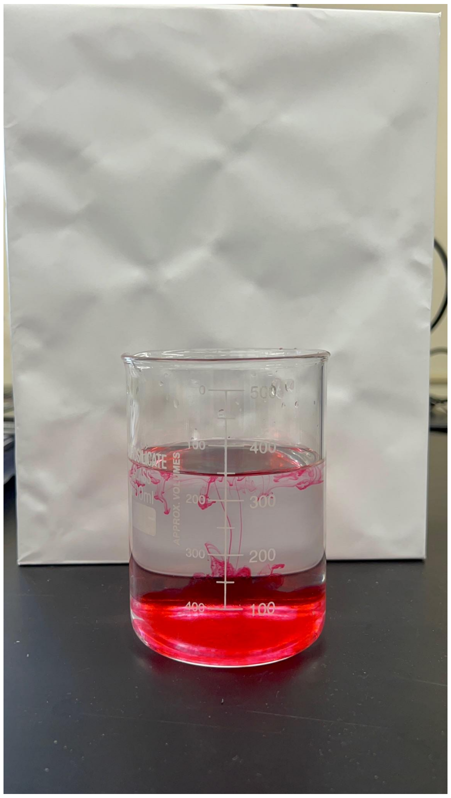


Figure 3: Random Currents Start (36.80 s)

Figure 4: Diffusion Starts (68.17 s)

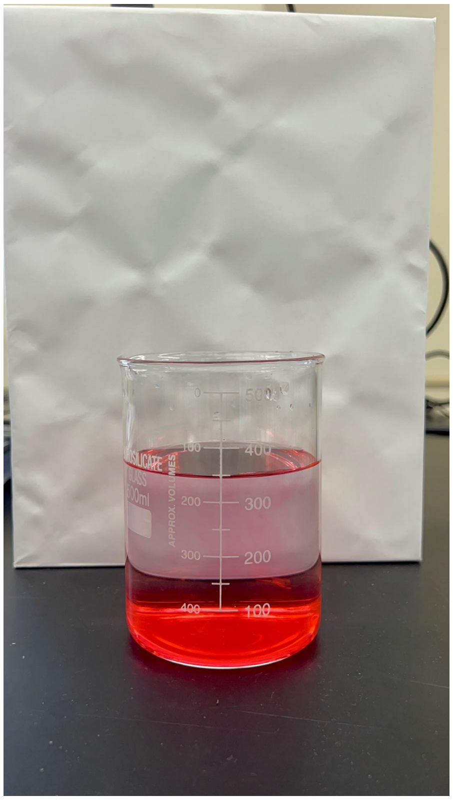
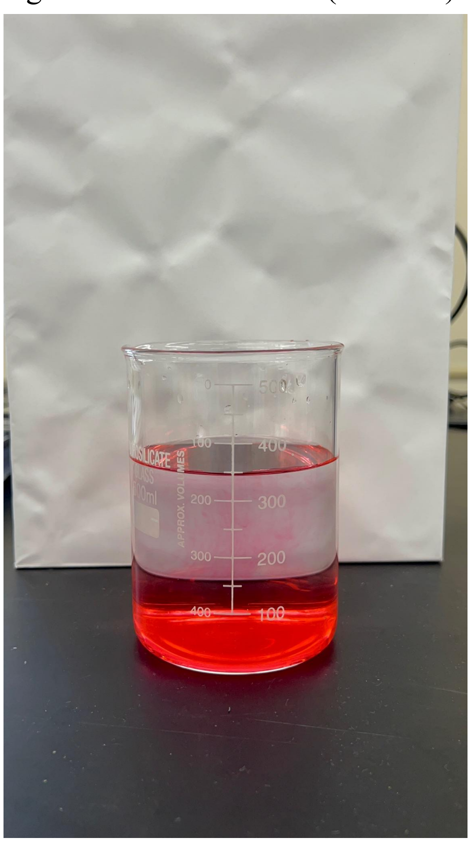


Figure 5: Diffusion Ends (171.20 s)



For Figure 5, note that the solution appeared to be mixed by eye, even if it doesn't appear mixed in the camera image.

Data

The results of this experiment are presented in Table 2.

Table 2: Results of Experiment for Part 1

Trial	Dominant Motion	Initial Time (s)	Final Time (s)	Elapsed Time (s)	Order of Magnitude of Elapsed Time
1	Initial velocity	0.00	6.47	6.47	1
	Gravity	6.47	24.39	17.92	1
	Random currents	24.39	78.95	54.56	2
	Diffusion	78.95	110.37	31.42	1
2	Initial velocity	0.00	6.11	6.11	1
	Gravity	6.11	31.86	25.75	1
	Random currents	31.86	95.80	63.94	2
	Diffusion	95.80	196.20	100.40	1
3	Initial velocity	0.00	5.91	5.91	1
	Gravity	5.91	36.80	30.89	1
	Random currents	36.80	68.17	31.37	1
	Diffusion	68.17	171.20	103.03	2

Analysis/Discussion

Unlike the other experiments in this class, this experiment was of a qualitative character, so we were not concerned with measurement uncertainties, t-scores, or anything in that vein.

The data aligned with our hypothesis for initial velocity and gravity, but deviated somewhat for random currents and diffusion. This deviation was not more than one order of magnitude, but it is deviation nonetheless.

There is one quantitative discussion to be had, and that's about our method of determining the order of magnitude. Our approach, call it method 1, comes from solving for p to the nearest integer such that

$$\Delta t = 10^{p}.$$

Another systematic approach to determining the order of magnitude, call it method 2, would be to express Δt in formal scientific notation, with some mantissa $m \in [0, 1)$ and some integer p such that

$$\Delta t = m \times 10^{p},$$

and to report the order of magnitude as p.

Either of these methods gives a comparison of Δt to powers of 10, so both are valid ways of reporting an order of magnitude. We chose to use method 1 because method 2 may assign small orders of magnitude to large numbers; for instance, method 2 would say that the order of magnitude of 9.9×10^{3} is 3, even though this number is closer to 10^{4}.

Conclusion

Our hypothesis about the order of magnitude of the time over which initial velocity and gravity were the dominant causes of motion is supported by the data; however, the data do not support our hypothesis for the order of magnitude of the time over which random currents or diffusion are the dominant cause of motion.

As always, there are improvements to be made. An actionable improvement would be to have a mechanical dropper that would dispense the desired amount of dye immediately, and the set up a photogate on the expected trajectory of the dye, that way human reaction time doesn't affect when the timer starts. It would also be good to control the temperature of the water. We took the water from the faucet, so we did not control the temperature. Theory dictates that diffusion rate increases with temperature, so this is definitely a confounding factor.

Since this is the last lab, let's have a little fun. A significant improvement over this experiment would be to use spectrophotometers to study the "stages of mixing".

The apparatus would consist of the following:

- A large cuvette.
- An array of light beams matching the size of one of the transparent walls in the cuvette. The cuvette would be placed against this array
- A corresponding array of photoreceptors opposite to the beam sources. These would measure the absorbance spectrum for some horizontal parcel of water.
- A camera to record video of the mixing process. This could be compared to the absorbance spectrum data
- If necessary, a cooling apparatus could be constructed that would prevent the light beams from heating up the water over time, so that the temperature, and by proxy the diffusion rate, could be held constant.
- The goal of this experiment would be to try to tie the diffusion rate to the time-varying absorbance spectrum of the mixture.
- For control groups, these measurements could be taken for a fully mixed dye solution—that's one extreme—and for the other extreme of a drop of dye that has just

been placed in the water, this would be difficult, but maybe it would be possible to coat a drop of dye with some kind of membrane (maybe a phospholipid bilayer), and then take the measurements through the water parcel containing the drop.

Part 2: Brownian Motion

Objective

The objective of this experiment was to observe the motion of globules of condensed milk in water, and to determine if the observed motion was Brownian motion, motion at constant velocity, or motion under constant acceleration.

Hypothesis

We predicted that we would observe Brownian motion; that is, the distance traveled by the globule by time $t \geq 0$, $x(t)$, would be a function $x(t) = c\sqrt{t}$, where c was some constant to be determined.

Materials

The materials used in this experiment were:

- One microscope
- One microscope camera
- Swift imaging
- One glass slide
- One glass coverslip
- One plastic pipette
- An excess of water
- One needle
- One can of condensed milk
- One digital stopwatch

Methods

First, the slide had to be set up. To do this, the slide was set on a dry, level surface. The pipette was used to place one drop of water on the center of the slide. Then, the needle was barely dipped into the condensed milk, so that a small droplet of condensed milk would be collected. After that, the droplet of condensed milk was placed into the drop of water on the slide. Finally, the coverslip was put on top of the liquid mixture.

The slide was transferred to the microscope, the 40x objective was put in place, and the image from the camera was put into focus. One globule of condensed milk was selected for observation.

The data was collected as follows. One person was the clicker, the other was the keeper. The clicker would click on the location of the globule and then hit the event button on the stopwatch to record the click time. The clicker typically had to look at the stopwatch while operating it, as it was surprisingly easy for both of us to accidentally hit the stop button instead of the event button. Due to this, the keeper kept their eye on the globule while the clicker was looking away. Including the initial time, ten times and ten positions were recorded for each trial. Two trials were conducted.

To be explicit, the time interval was random, but only because it was influenced by how quickly the clicker could operate.

To determine which of the equations best described the motion of the particle, we were unable to use Excel to fit a square root curve to the data directly, so instead, we calculated the square roots of the times, the times themselves, and the square of the times, and fit linear curves to the distance traveled versus each of these times. This is an equivalent procedure.

Data

Table 3: Brownian Motion Trial 1

t (s)	u (px)	v (px)	Step size (px)	x (px)
0.00	2927.00	2400.00	NA	0.00
7.24	2879.00	2188.00	217.37	217.37
15.75	2831.00	1948.00	244.75	462.12
22.33	2807.00	1788.00	161.79	623.91
27.15	2799.00	1668.00	120.27	744.18
31.63	2795.00	1576.00	92.09	836.26
37.54	2791.00	1404.00	172.05	1008.31
42.20	2807.00	1292.00	113.14	1121.45
49.59	2859.00	1152.00	149.35	1270.79
56.38	2975.00	1056.00	150.57	1421.36

Table 4: Brownian Motion Trial 2

t (s)	u (px)	v (px)	Step size (px)	x (px)
0.00	3391.00	132.00	NA	0
6.45	3207.00	216.00	202.27	202.27
15.08	3031.00	292.00	191.71	393.98
20.48	2935.00	364.00	120.00	513.98

26.03	2827.00	496.00	170.55	684.53
33.11	2639.00	556.00	197.34	881.87
39.00	2475.00	580.00	165.75	1047.62
48.73	2207.00	664.00	280.86	1328.47
54.60	1995.00	736.00	223.89	1552.37
59.72	1835.00	792.00	169.52	1721.88

Note that u and v denote the horizontal and vertical position of the globule because x was reserved for the distance traveled.

Figure 6: Globule 1 Distance Traveled versus Square Root Time

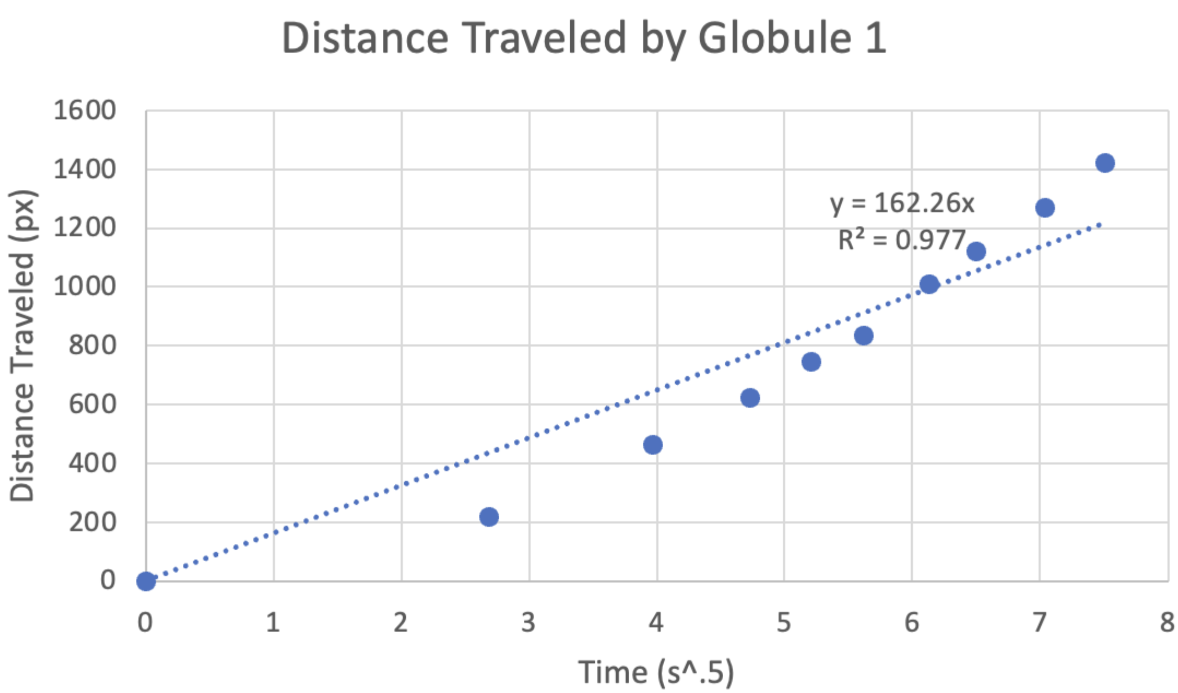


Figure 7: Globule 1 Distance Traveled versus Time

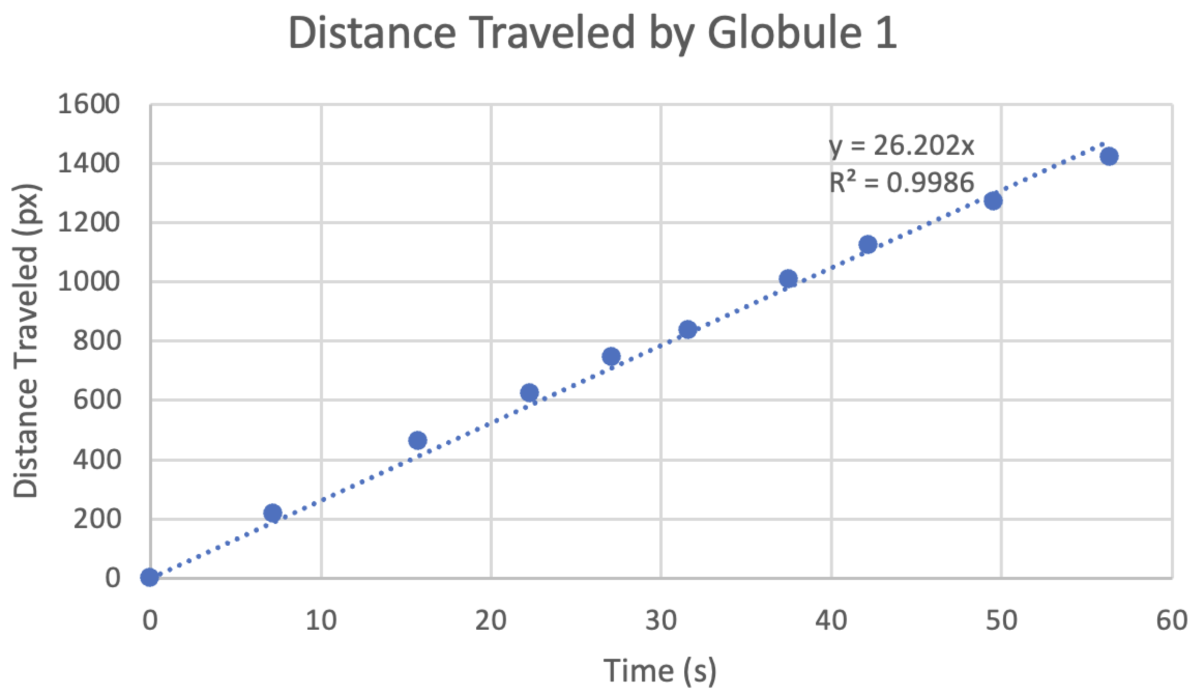


Figure 8: Globule 1 Distance Traveled versus Time Squared

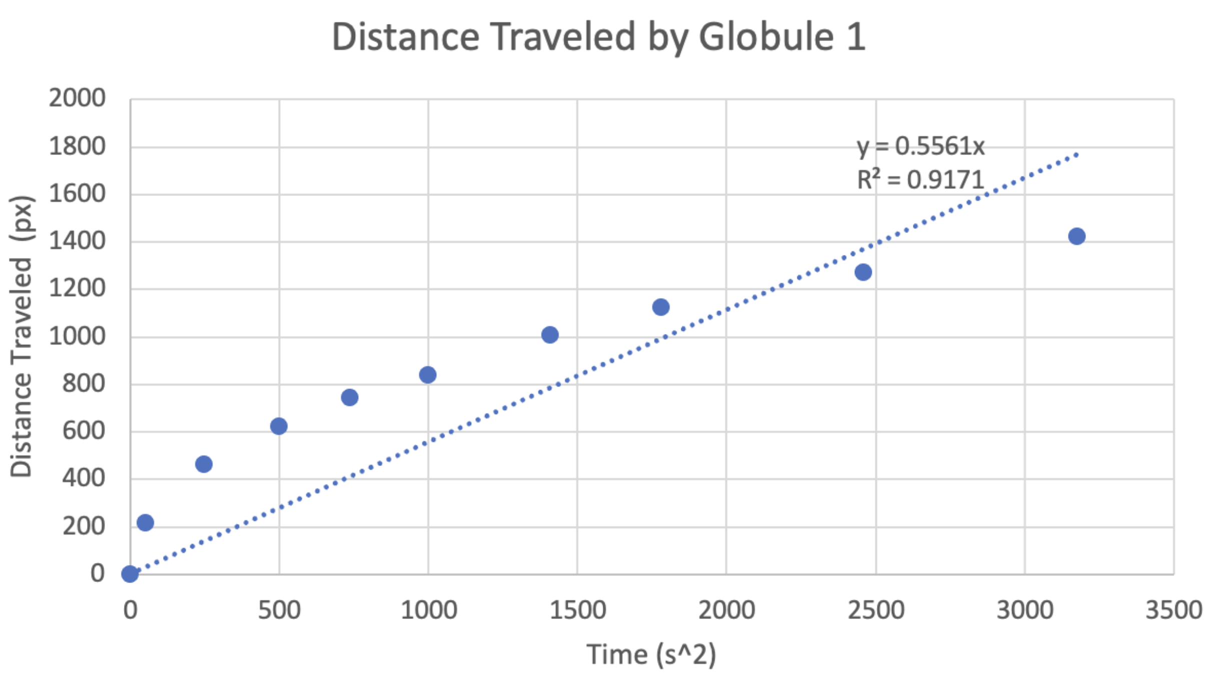


Figure 9: Globule 2 Distance Traveled versus Square Root Time

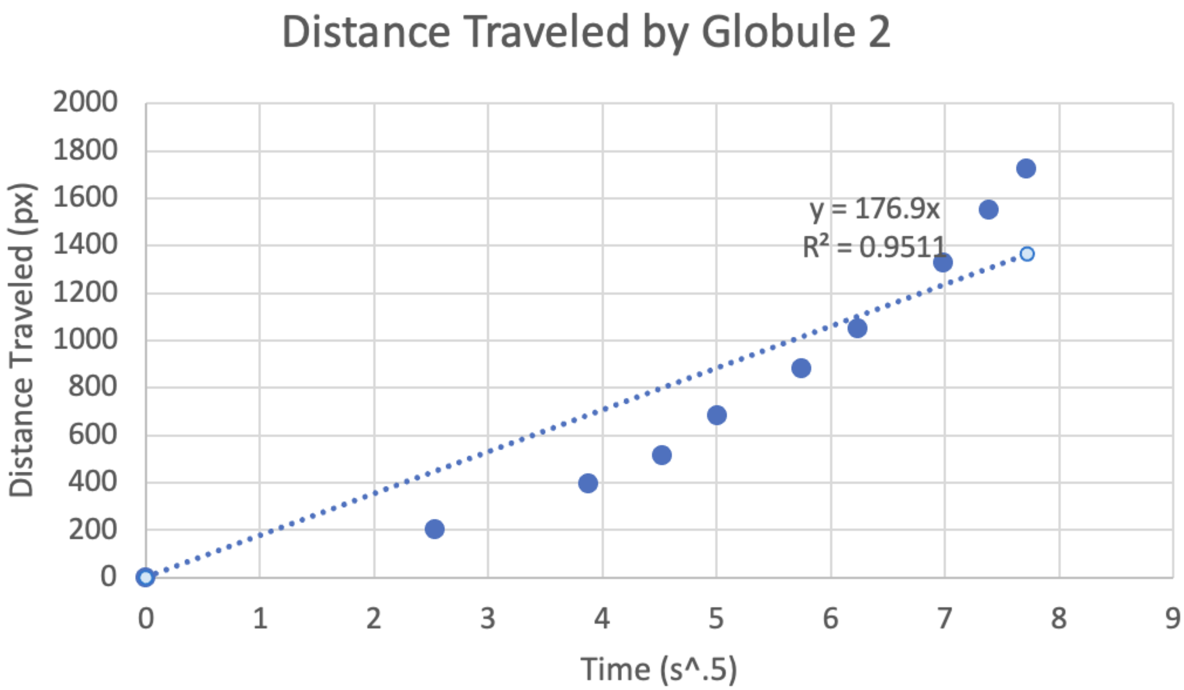


Figure 10: Globule 2 Distance Traveled versus Time

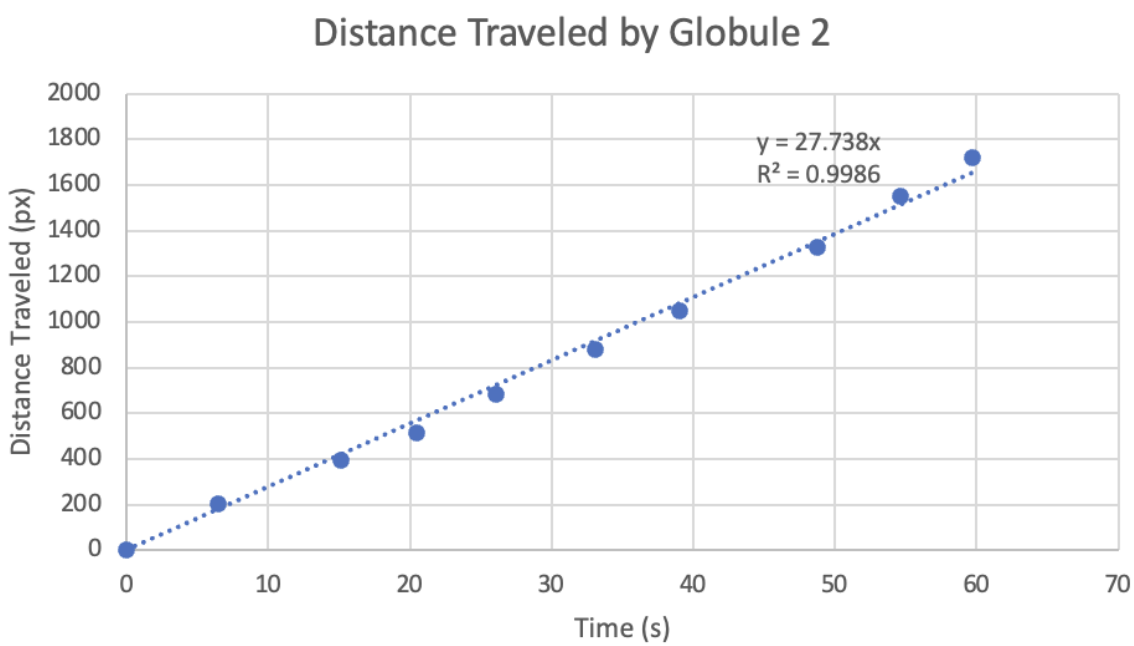
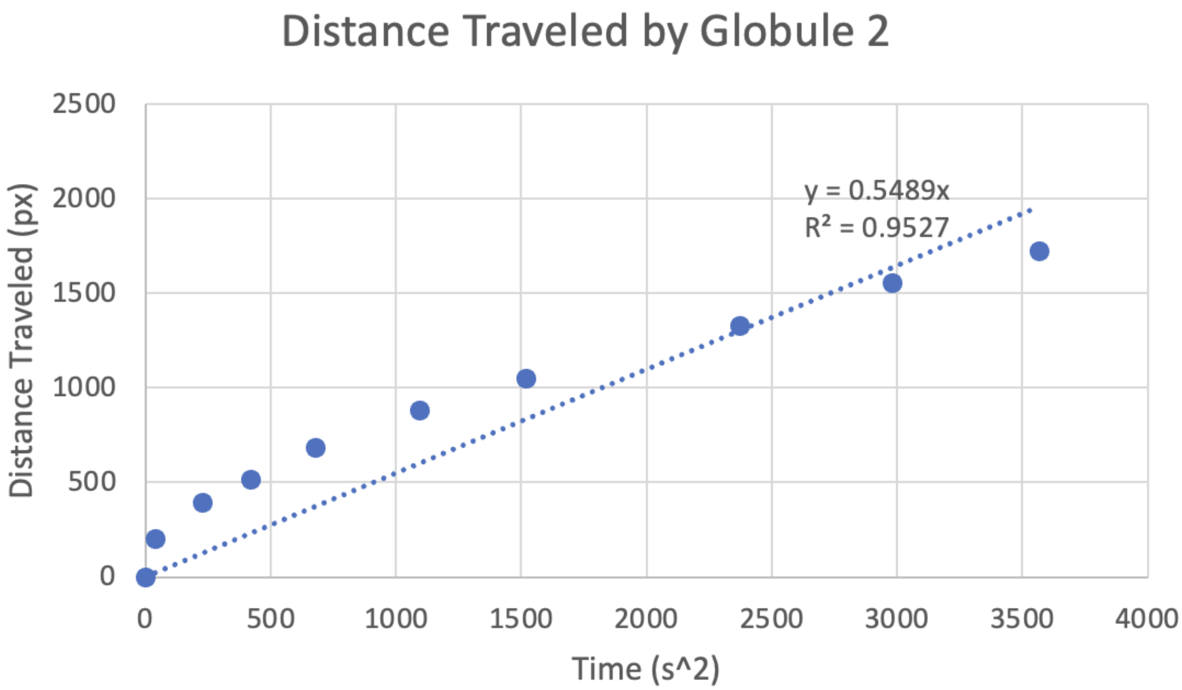


Figure 11: Globule 2 Distance Traveled versus Time Squared



<u>Analysis/Discussion</u>

Excel was used to fit three curves to the dataset (distance traveled vs. time),

$$x(t) = c\sqrt{t},\ x(t) = vt, \text{ and } x(t) = \frac{1}{2}at^2.$$

The results are presented in Table 5.

Table 5: Regression Coefficients

Trial	Quantity	$c\ (\frac{px}{s^{1/2}})$	$v\ (\frac{px}{s})$	$a\ (\frac{px}{s^2})$
Trial 1	Coefficient	162.26	25.20	1.11
	R^2	0.977	0.9986	0.9171
Trial 2	Coefficient	176.90	27.74	1.10
	R^2	0.9511	0.9986	0.9527

Note that the coefficient for *a* is twice of the coefficient appearing in the regression equations in Figures 8 and 11. This is because the model for constant acceleration has a factor of ½, so the value of *a* given here had to be doubled.

Conclusion

Based on the obsered motion of the molecules, we can conclude that we observed motion at a constant velocity. Even without taking the best fit lines, we can see just by transforming the time variables that the data was most linear over *t*, while the square root time and the time squared showed substantial visual deviation from linearity. We also see little variation in the coefficient and R^2 values between the two trials. As such, this was contrary to our hypothesis that we would observe random motion.

In terms of improvements, it would be nice to hook up a timer to the mouse itself, that way the click times would line up more accurately with the glouble positions. With the method we used in this experiment, because tehre were two different tasks that had to be done, it wasn't possible for both people to keep their eyes on the globule, something that likely caused error and flawed our results. Another improvement would be to use a level to see if the surface on which the microscope was resting was level, as if it was not, then that may explain why we observed motion at a constant velocity instead of Brownian motion.