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Lab 1 - Period of a Pendulum

Part 1: Period vs. Amplitude

Objective

A basic model for the period of a pendulum T is

$$T = 2\pi\sqrt{\frac{l}{g}},$$

where l is the length of the string in meters of the pendulum, and $g = 9.81\, ms^{-2}$ is the magnitude of acceleration due to gravity. Note that the amplitude does not appear in this equation, meaning that the prediction for the period is independent of the amplitude. The purpose of this lab was to examine if this is true: Is the period of a pendulum truly independent of its amplitude?

Materials

The materials required for this lab were: one pendulum stand, one circular protractor, one string, one mass with a hook, one stopwatch, pencil and paper to take measurements, and one computer for the analysis.

In this instance of the experiment, the mass was labeled as 100 g, and the length of the string was measured to be 27.10 +/- 0.05 cm.

Hypothesis

Table 1: Quick Check

Amplitude (degrees)	Period (s)
6	1.12 +/- 0.01
36	1.09 +/- 0.01

Based on the quick check data in Table 1, our hypothesis was that the period of the pendulum is independent of the amplitude, the reason being that, even though the period measurements were not equal within the error, human reaction time may mean that the true errors are larger than the systematic error.

Method

At four different amplitudes—6 degrees, 16 degrees, 26 degrees, and 36 degrees—we took five measurements of the period.

First, we had to start the pendulum's swinging. At 6 degrees and 16 degrees, we were able to start the swinging once, and take the five measurements. At 26 and 36 degrees, we observed that the amplitude decreased rather rapidly; therefore, we adopted a different procedure: the swinging would be started, a measurement would be taken, and then we would stop the swinging and start again.

Measurements required two people, and were taken as follows. One person, the setter, set the pendulum's initial state, and the other, the timer, managed the stopwatch. Both people would coordinate so that the pendulum was at the correct initial position and was parallel to the pendulum stand, so as to ensure the pendulum moved in a plane. The timer would simultaneously tell the setter when to release the mass and start the stopwatch. The timer would then stop the stopwatch when the mass returned to its initial amplitude. The elapsed time was recorded.

We attempted to keep the following variables constant: the mass of the pendulum, the length of the string, and the initial velocity.

Data

Table 2: Pendulum Periods at Various Amplitudes, Measured with Digital Stopwatch

Amplitude (degrees)	Trial 1 (s)	Trial 2 (s)	Trial 3 (s)	Trial 4 (s)	Trial 5 (s)
6	1.14 +/- 0.01	1.08 +/- 0.01	1.14 +/- 0.01	1.15 +/- 0.01	1.16 +/- 0.01
16	1.27 +/- 0.01	1.23 +/- 0.01	1.24 +/- 0.01	1.11 +/- 0.01	1.13 +/- 0.01
26	1.30 +/- 0.01	1.33 +/- 0.01	1.31 +/- 0.01	1.32 +/- 0.01	1.29 +/- 0.01
36	1.40 +/- 0.01	1.32 +/- 0.01	1.22 +/- 0.01	1.32 +/- 0.01	1.40 +/- 0.01

Analysis

Table 3 reports the mean period of the five trials, for each amplitude. The errors in Table 3 are the standard deviations of the means, calculated by dividing the standard deviation of each row by the $\sqrt{5}$, because 5 was the number of trials. Note that some of these random errors are larger than the systematic error.

Table 3: Mean Period for Each Amplitude (Part 1)

Angle (degrees)	Mean Period (s)
6.000 +/- 0.025	1.13 +/- 0.01
16.000 +/- 0.025	1.20 +/- 0.03

26.000 +/- 0.025	1.31 +/- 0.01
36.000 +/- 0.025	1.33 +/- 0.03

We also wanted to compare the measurements pairwise. To do this, we used the *t*-score formula provided in the analysis tools hand out,

$$t = \frac{|\overline{x_1} - \overline{x_2}|}{\sqrt{\overline{\delta x_1}^2 + \overline{\delta x_2}^2}},$$

where $\overline{x_i}$ is the mean of x_i and $\overline{\delta x_i}$ is the standard deviation of the mean of x_i, for $i = 1, 2$. We did not do these calculations by hand, but rather used Jupyter Notebooks. The code is presented in Image 1.

Image 1: *t*-score Code

```
def stdev_of_mean(data):
    N = len(data)

    # Setting ddof = 1 ensures estimate for standard deviation is unbiased
    sigma = np.std(data, ddof=1)

    # Now we get the standard deviation of the mean
    delta = sigma/sqrt(N)

    return delta
```

```
def t_score(data1, data2):
    mu1 = np.mean(data1)
    mu2 = np.mean(data2)
    delta1 = stdev_of_mean(data1)
    delta2 = stdev_of_mean(data2)

    return abs(mu1 - mu2) / sqrt( delta1**2 + delta2**2 )
```

There is an error in this code, which is that delta1 and delta2 should actually be equal to the maximum of the standard deviation of the mean and the systematic error. Fortunately, because the standard deviations of the means in Table 3 are all at least the systematic error, this programming mistake did not lead to a false calculation.

The *t*-scores and their interpretations are reported in Table 4.

Table 4: *t*-scores and Interpretations (Part 1)

Amplitude of Data Set 1	Amplitude of Data Set 2	*t*-score	Interpretation
6	16	1.7809034678264297	Inconclusive
6	26	11.221350149914782	Distinguishable

6	36	5.491531942629777	Distinguishable
16	26	3.491614183440623	Distinguishable
16	36	2.9537312507888704	Inconclusive
26	36	0.647619290350411	Indistinguishable

Note that the Interpretation column of Table 4 was population according to the following criteria: If the *t*-score was less than 1, at least 1 but less than 3, or at least 3, then the interpretation was indistinguishable, inconclusive, or distinguishable, respectively.

Conclusion

The pairwise conclusions of this experiment can be found in the Interpretation column of Table 4, with distinguishable meaning our hypothesis was false, inconclusive meaning inconclusive, and indistinguishable meaning failure to reject our hypothesis, respectively. The big picture is that, for all but one of the cases, our hypothesis was either false, or the data was inconclusive.

This means that we need to improve our experiment. As indicated in the notes on Table 3, the random error was larger than the systematic error in some cases. There are several sources of random error towards which we could direct improvement efforts. First, the reaction time of the timer introduces a large random error into the period measurement. Second and third, there is also random variation in the initial amplitude and velocity of the pendulum. Of these three sources, it seems that the reaction time of the timer is the most severe source of random error. This will be mitigated in Part 2.

Part 2: Period vs. Amplitude Redux

Objective

In Part 2, we are still interested in investigating if the period of a pendulum is truly independent of the amplitude. The difference between this and Part 1 is that we seek to make the random error much smaller.

Materials

The materials required for this lab were: one pendulum stand, one circular protractor, one string, one mass with a hook, one stopwatch, one PASCO photogate, one PASCO interface, and one computer to record the photogate output. We also used another computer for the analysis, although this could be carried out on the first computer.

Hypothesis

Based on the results of Part 1, our hypothesis was that the period of the pendulum does depend on the amplitude.

Method

This experiment required two people, one to ensure the correct amplitude was set, and another to position the mass such that the string was vertical.

Once the pendulum state was initialized, one person would hit record on the PASCO software, and the other person would release the pendulum. Six measurements of the period were taken, and then the pendulum was stopped. The first measurement was discarded, as we were instructed that it was most likely erroneous.

Unlike part 1, damping was not a problem at the amplitudes of 26 and 36 degrees. It may be that the reaction time of the timer made it appear that damping was a problem, and the photogate eliminated that apparent error. In any case, for all amplitudes, we simply let the photogate take measurements off of one initialization.

Data

Table 5: Pendulum Periods at Various Amplitudes, Measured with PASCO Photogate

Amplitude (degrees)	Trial 1 (s)	Trial 2 (s)	Trial 3 (s)	Trial 4 (s)	Trial 5 (s)
6	1.13 +/- 0.01	1.13 +/- 0.01	1.13 +/- 0.01	1.13 +/- 0.01	1.13 +/- 0.01
16	1.14 +/- 0.01	1.14 +/- 0.01	1.14 +/- 0.01	1.14 +/- 0.01	1.14 +/- 0.01
26	1.15 +/- 0.01	1.15 +/- 0.01	1.15 +/- 0.01	1.15 +/- 0.01	1.15 +/- 0.01
36	1.16 +/- 0.01	1.16 +/- 0.01	1.16 +/- 0.01	1.16 +/- 0.01	1.16 +/- 0.01

Analysis

Table 6 reports the mean period of the five trials, for each amplitude. Note that in Table 5, for each amplitude, the measurement of the period was the same across all trials. As a result, the standard deviation of the mean is 0. However, we cannot have 0 error in our measurements, so the errors in table 6 are the systematic errors of the photogate, 0.01 s.

Table 6: Mean Period for Each Amplitude (Part 2)

Angle (degrees)	Mean Period (s)
6	1.13 +/- 0.01
16	1.14 +/- 0.01
26	1.15 +/- 0.01
36	1.16 +/- 0.01

While populating this table, an interesting problem was stumbled upon: For each amplitude, every measurement of the period was identical within the systematic error, meaning that the standard deviation of the mean was 0. As a result, we used the systematic error in place of the standard deviation of the mean in this table.

As in Part 1, we were interested in comparing the periods pairwise. We used the same *t*-test and the same code, but due to the period measurement being the same across trials for each amplitude, we get extreme results.

This time, we used code that determines the correct value for the $\delta\overline{x}_i$, $i = 1, 2$.

Image 2: Corrected *t*-score Code

```
def stdev_of_mean(data):
    N = len(data)

    # Setting ddof = 1 ensures estimate for standard deviation is unbiased
    sigma = np.std(data, ddof=1)

    # Now we get the standard deviation of the mean
    delta = sigma/sqrt(N)

    return delta
```

```
def t_score(data1, data2, sys_error=0):
    mu1 = np.mean(data1)
    mu2 = np.mean(data2)
    delta1 = max( stdev_of_mean(data1), sys_error )
    delta2 = max( stdev_of_mean(data2), sys_error )

    return abs(mu1 - mu2) / sqrt( delta1**2 + delta2**2 )
```

Just to make sure that all is explained, the sys_error=0 sets sys_error to 0 by default, so that if the user does not pass a systematic error, the t_score function just uses the standard deviation of the mean. When we computed the *t*-scores in Table 7, we correctly set sys_error=0.01.

Table 7: *t*-scores and Interpretations (Part 2)

Amplitude of Data Set 1	Amplitude of Data Set 2	*t*-score	Interpretation
6	16	0.7071067811865325	Indistinguishable
6	26	1.4142135623730805	Inconclusive
6	36	2.1213203435596446	Inconclusive
16	26	0.7071067811865481	Indistingusihable
16	36	1.414213562373112	Inconclusive
26	36	0.7071067811865639	Indistinguishable

<u>Conclusion</u>

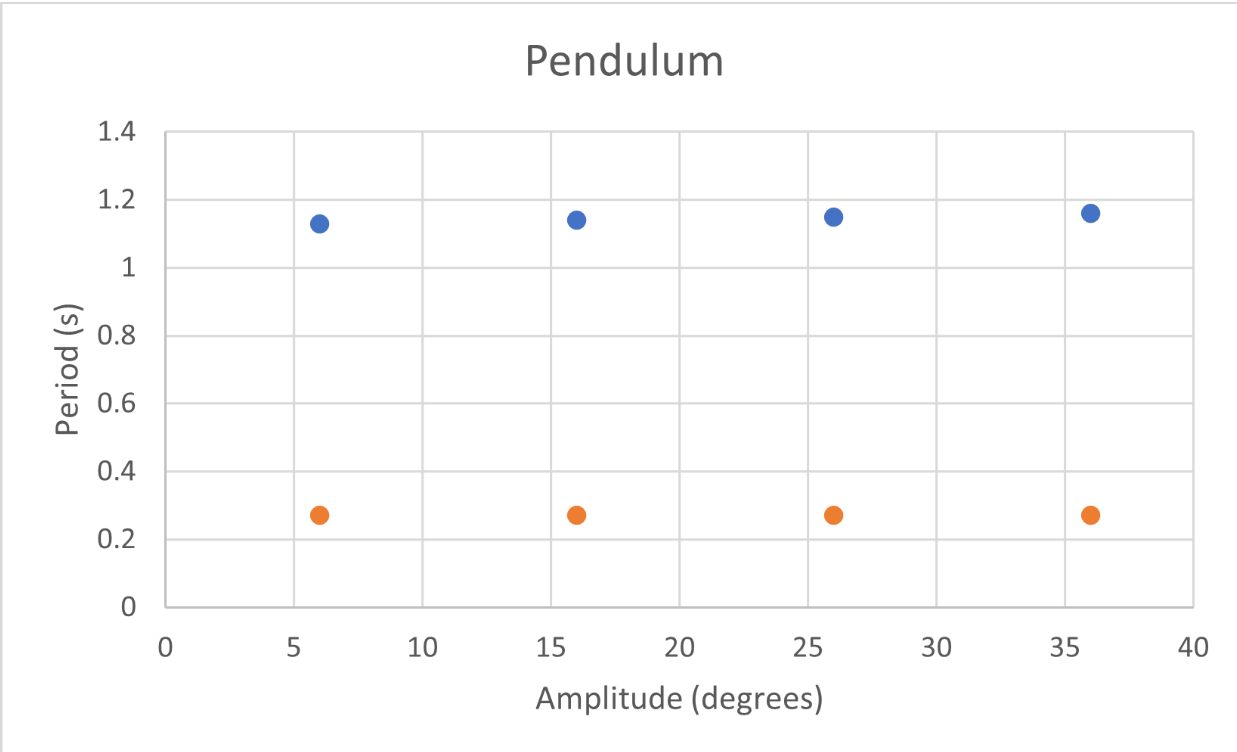
The results of this experiment are more in favor of the hypothesis of part 1, that the period does not depend on the amplitude. The inconclusive *t*-scores in Table 7 tell us that more testing is required to determine if the periods of the corresponding pairs of amplitudes are different or not, while the indistinguishable results tell us that this experiment has proven thy hypothesis of part 1 false for those pairs of amplitudes.

This result was unexpected, because (1) more accurate models of the period of a pendulum actually do depend on the amplitude, so one would expect to observe *t*-scores in the distinguishable range with a timing device that has so small a delay as a photogate, and (2) even though the *t*-scores give the results that they do, plain inspection of Table 5 implies that the period does depend on the amplitude. That being said, these are the results.

Potential improvements to this experiment are as follows. First, we could still employ methods to reduce variation in the initial position and velocity of the mass, such as building simple release mechanism that keeps the initial position and velocity constant. Second, we could test different string lengths, as even though these results disagreed with our hypothesis, a different conclusion may be reached if the string length is changed. Finally, since theory tells us that the period does depend on the amplitude, we could explore other models that introduce theta into the equation for the period, and see of they fit the measured data better.

A final note: Consider Figure 1

Figure 1: Plot of Observed Mean Periods and Precited Periods versus Amplitude



The nondescript title “Pendulum” was chosen by the prelab for Lab 2, not by us.

The important thing to note is that the predicted value was off from the measured value by quite a bit. We believe the cause for this is that we made the predictions with the length of the string, but the model for the period is derived assuming that the mass on the pendulum is a point mass; thus, when using the model for the pendulum period, it is important the the length of the weight is added to the length of the string, as this gives a more accurate prediction, see Figure 2.

Figure 2: A More Accurate Prediction