1.3

a. to find the distribution of Y|X=x we put the second function into the first one resulting in:

$$f_{xy}(Y|X=x)=\begin{cases}0.7 & \text{if } X=0.4\\ 0.3 & \text{if } x=-0.7\end{cases}\cdot\left(\frac{1}{2\sqrt{2\pi}}\right)e^{\left[-\frac{1}{2}\left(\frac{Y-x}{2}\right)^2\right]}$$

so $f_{xy}(Y|X=x)$ is a normal distribution with mean x and st.dev. 2

b. Yes, X and Y are independent since the product of the marginal distribution is equal to the joint distribution of the 2, that can be easily verified using $f_Y(y)$ (which we calculate in the next point) and calculate $f(x)\cdot f_Y(y)$ which is going to be equal to $f_{xy}(x,y)$.

c. As said in the lecture it is always possible to recover $f_Y(y)$ from the joint distribution:

$$f_Y(y)=\int f(x)\cdot\left(\frac{1}{2\sqrt{2\pi}}\right)e^{\left[-\frac{1}{2}\left(\frac{Y-x}{2}\right)^2\right]}=$$

$$0.7\cdot\left(\frac{1}{2\sqrt{2\pi}}\right)\cdot\int e^{\left[-\frac{1}{2}\left(\frac{Y-0.4}{2}\right)^2\right]}dx+0.3\cdot\left(\frac{1}{2\sqrt{2\pi}}\right)\cdot\int e^{\left[-\frac{1}{2}\left(\frac{Y-(-0.7)}{2}\right)^2\right]}dx$$

the 2 integrals represents 2 normal distributions of means 0.4 and -0.7 and st.dev. 2 so $f_Y(Y)$ is a mixture of normals.