ECON 471 Problem Set 1

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1 Probability Theory

1.1 Probability spaces and random variables

- 1. Angrist and Evans
  - Ω: Households in the U.S.
  - $\omega \in \Omega$: a single household
  - X: Number of children
  - Y: womens hours per week
- 2. Baird et al.
  - Ω: Children in Kenya
  - $\omega \in \Omega$: A single child
  - X: number of children receiving deworming treatment
  - Y: effect of deworming treatment (years of primary school enrolled; hours worked each week; miscarriage rates)
- 3. Black
  - Ω: residents with children who go to school in Massechusetts
  - $\omega \in \Omega$: A single resident
  - X: childrens' elementary school test scores
  - Y: willingness to pay for housing
- 4.
  - Ω: People of sweden
  - $\omega \in \Omega$: One person in sweden
  - X: vector of job training, cancer research, university scholarship program (the effect of these programs existing)
  - Y: vector of wealth, health of person
- 5. Papers 2 and 3 can be used to evaluate the goals of the Swedish government. Paper 2 talks about how the act of increasing "deworming" treatment results in an increase of labor and some other statistics. This can be compared to the Swedish government's initiative to fund its cancer research program as it is proven in Kenya's case in paper 2 that investing in health programs can lead to improvement of wealth and health of the people. Paper 3 draws comparison to Sweden's education as it talks about how investing in the elementary school system led to higher test scores and some other parameters that demonstrate an increase in overall wealth and health of the people. This is comparable to Sweden's initiative to increase university program funding. If increasing elementary funding increased overall health and wealth, then that speaks to the effect of Sweden increasing their university program funding.

1.2 Distribution of a function of random variables

- 1. The distribution of X_1 is

$$f_x(X_1) = \begin{cases} 0.5 & \mathrm{X}_1 = 0 \\ 0.5 & \mathrm{X}_1 = 1 \end{cases}$$

- 2. The distribution of X_2 is

$$f_x(X_2) = \begin{cases} 0.5 & \mathrm{X}_2 = 0 \\ 0.5 & \mathrm{X}_2 = 1 \end{cases}$$

- 3. a function of a random variable is also a random variable itself and here is why:

$$f(x) = 3x$$

or

$$f(\omega) = 3x(\omega)$$

- 4. The distribution of $avg(X_1, X_2)$ is

$$f_{X_1 X_2}(1/2X_1 + X_2) = \begin{cases} 0.25 & \mathrm{avg}(\mathrm{X}_1, X_2) = 0 \\ 0.50 & \mathrm{avg}(\mathrm{X}_1, X_2) = 0.5 \\ 0.25 & \mathrm{avg}(\mathrm{X}_1, X_2) = 1 \end{cases}$$

- 5. The distribution of $avg(X_1, X_2, X_3)$ is

$$f_{X_1 X_2 X_3}(1/3X_1 + X_2 + X_3) = \begin{cases} 0.125 & \mathrm{avg}(\mathrm{X}_1, X_2, X_3) = 0 \\ 0.375 & \mathrm{avg}(\mathrm{X}_1, X_2, X_3) = .333 \\ 0.375 & \mathrm{avg}(\mathrm{X}_1, X_2, X_3) = .667 \\ 0.125 & \mathrm{avg}(\mathrm{X}_1, X_2, X_3) = 1 \end{cases}$$