# Homework #2

Wednesday, September 7, 2022 2:26 PM

1. $f(x_1, x_2) = 2x_1^2 - 4x_1x_2 + 1.5x_2^2 + x_2$

$$\nabla = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 4x_1 - 4x_2 \\ -4x_1 + 3x_2 + 1 \end{bmatrix}$$

$$\frac{\partial}{\partial x_1} f = 4x_1 - 4x_2$$

$$\frac{\partial}{\partial x_2} f = -4x_1 + 3x_2 + 1$$

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$$

$$f(1,1) = 0.5$$

$x_1 + x_2$ where $g(x_1,x_2)=0$: $4x_1 - 4x_2 = 0$ $\quad x_1 = 1$

$-4x_1 + 3x_2 = -1$ $\quad x_2 = 1$

$\partial x_i = x_i - 1$

$i=1$: $\partial x_1 = 0$

$i=2$: $\partial x_2 = 1$

$f(x_1, x_2) = f(1,1) + (a\partial x_1 - b\partial x_2)(c\partial x_1 - d\partial x_2)$

$f(x_1, x_2) = 0.5 + (a(0) - b(1))(c(0) - d(1))$

$f(x_1, x_2) = 0.5 + bd$

b times d must be less than 0 for there to be downward slopes

$bd < 0$

2. $x_1 + 2x_2 + 3x_3 = 1$

point: $(-1, 0, 1)$

$A=1 \quad B=2 \quad C=3 \quad D=1$

$$d = \frac{|Ax_1 + Bx_2 + Cx_3 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

3. $a^T x = c$

there will need to be two points $x_1$ and $x_2$

$\forall x \in H$

$\forall y \in H$

$\lambda x + (1-\lambda) y \in H \quad 0 \le \lambda \le 1$

$\lambda a_1 x_1 + \dots + \lambda a_n x_n = \lambda c$

$(1-\lambda) a_1 y_1 + \dots + (1-\lambda) a_n x_n = (1-\lambda) c$

$\lambda a_1 x_1 + (1-\lambda) a_1 y_1 + \dots + \lambda a_n x_n + (1-\lambda) a_n y_n = \lambda c - (1-\lambda) c$

$a_1(\lambda x_1 + (1-\lambda) y_1) + \dots + a_n(\lambda x_n + (1-\lambda) y_n) = c$