



# Discrete Codebook World Models for Continuous Control

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University of Edinburgh  
Finnish Center for Artificial Intelligence (FCAI)  
Aalto University

# World Models

$$p(s_{t+1}, r_t \mid s_t, a_t)$$

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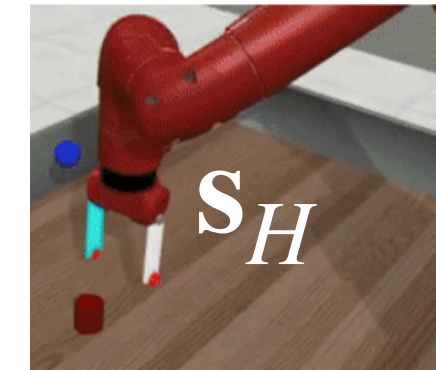
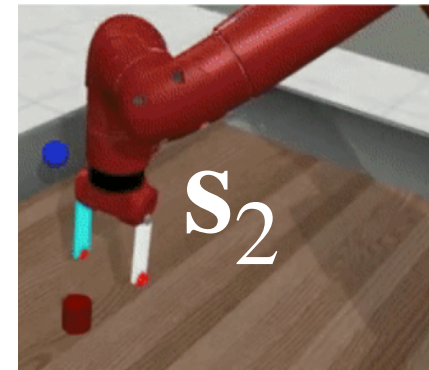
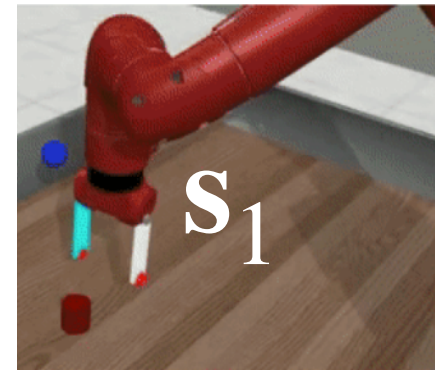
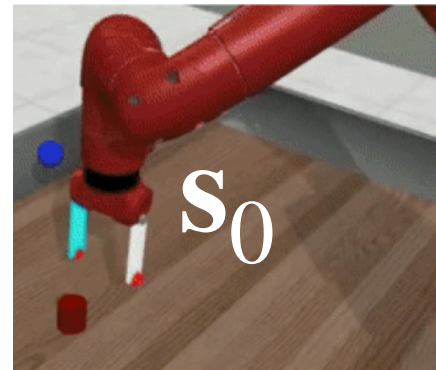
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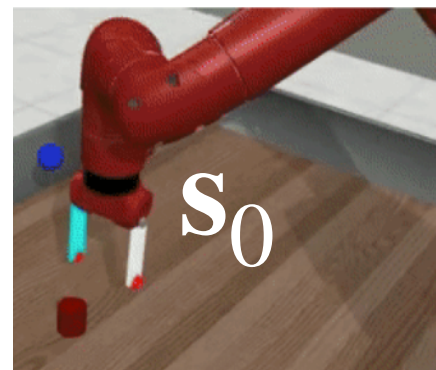
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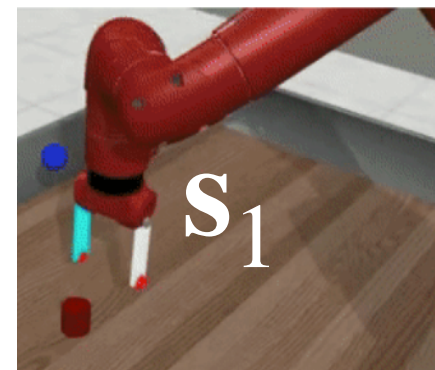
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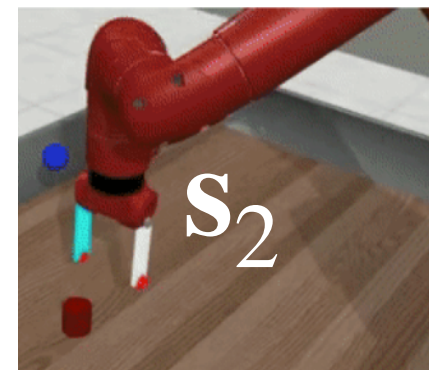
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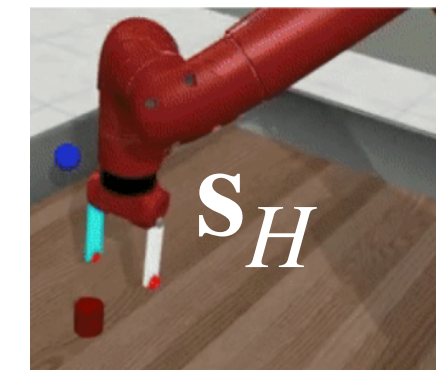
Encoder



Encoder

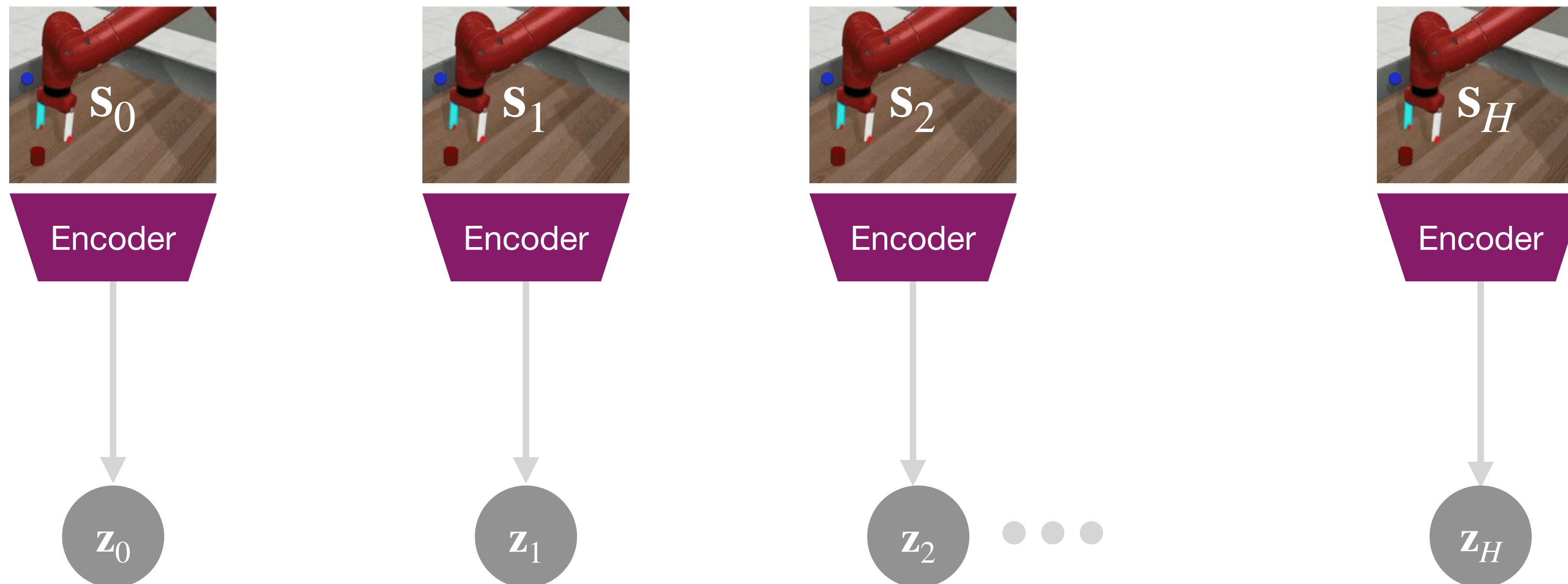


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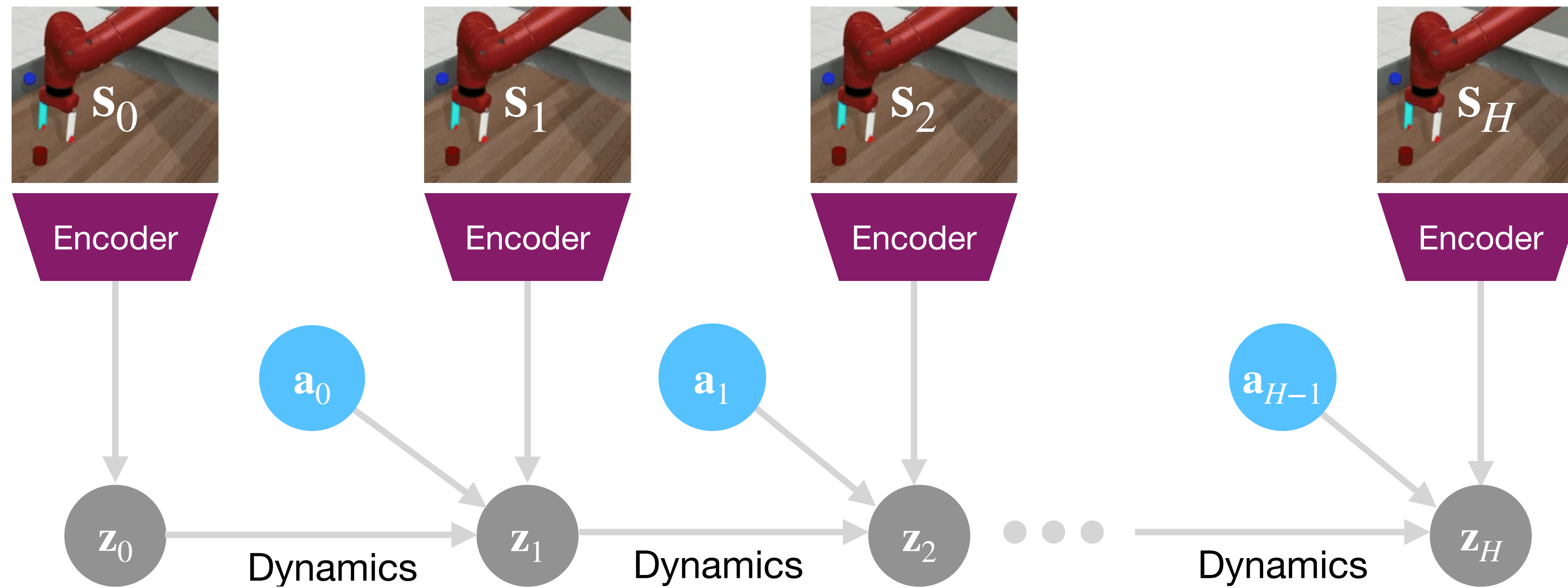


Encoder

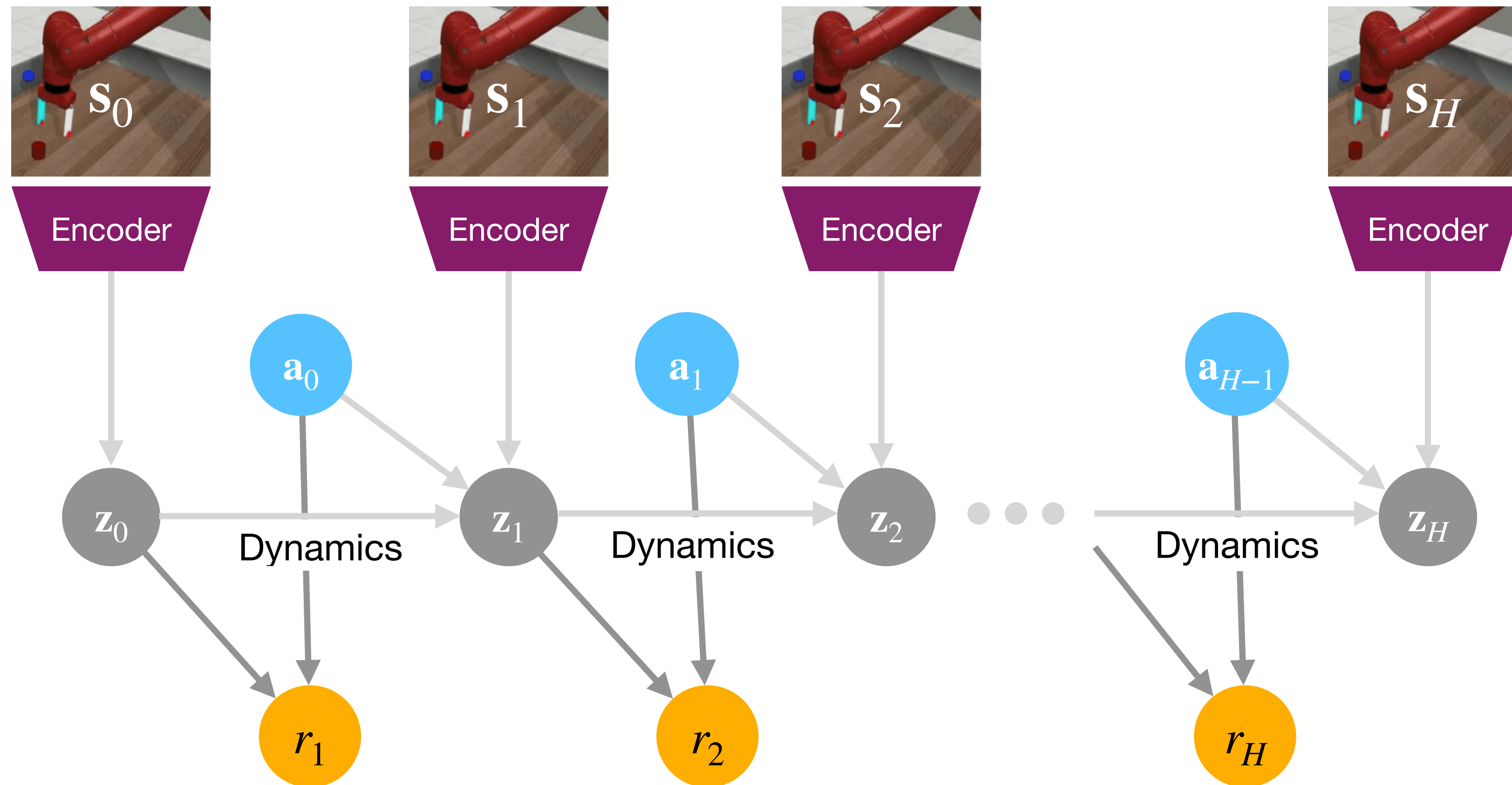
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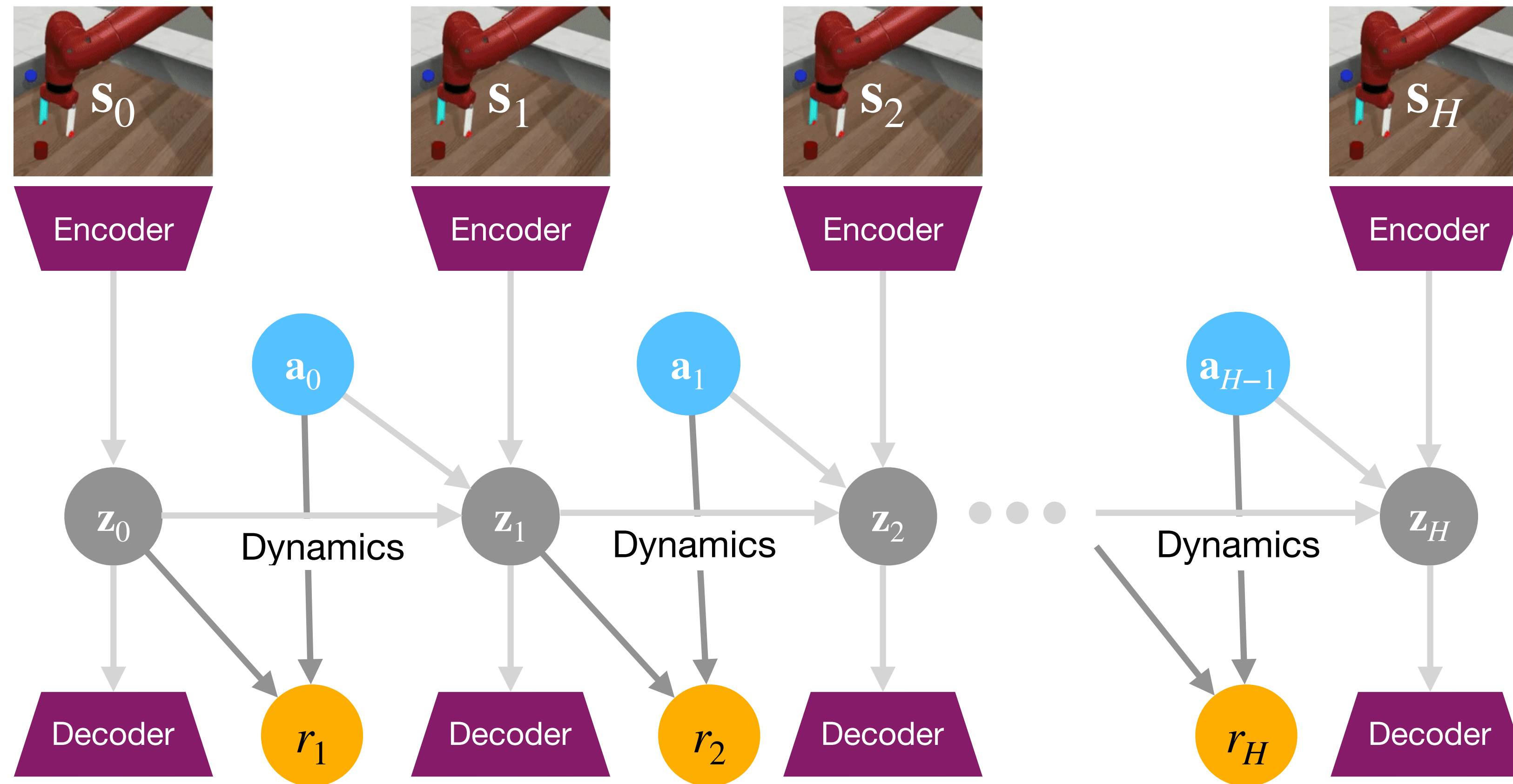
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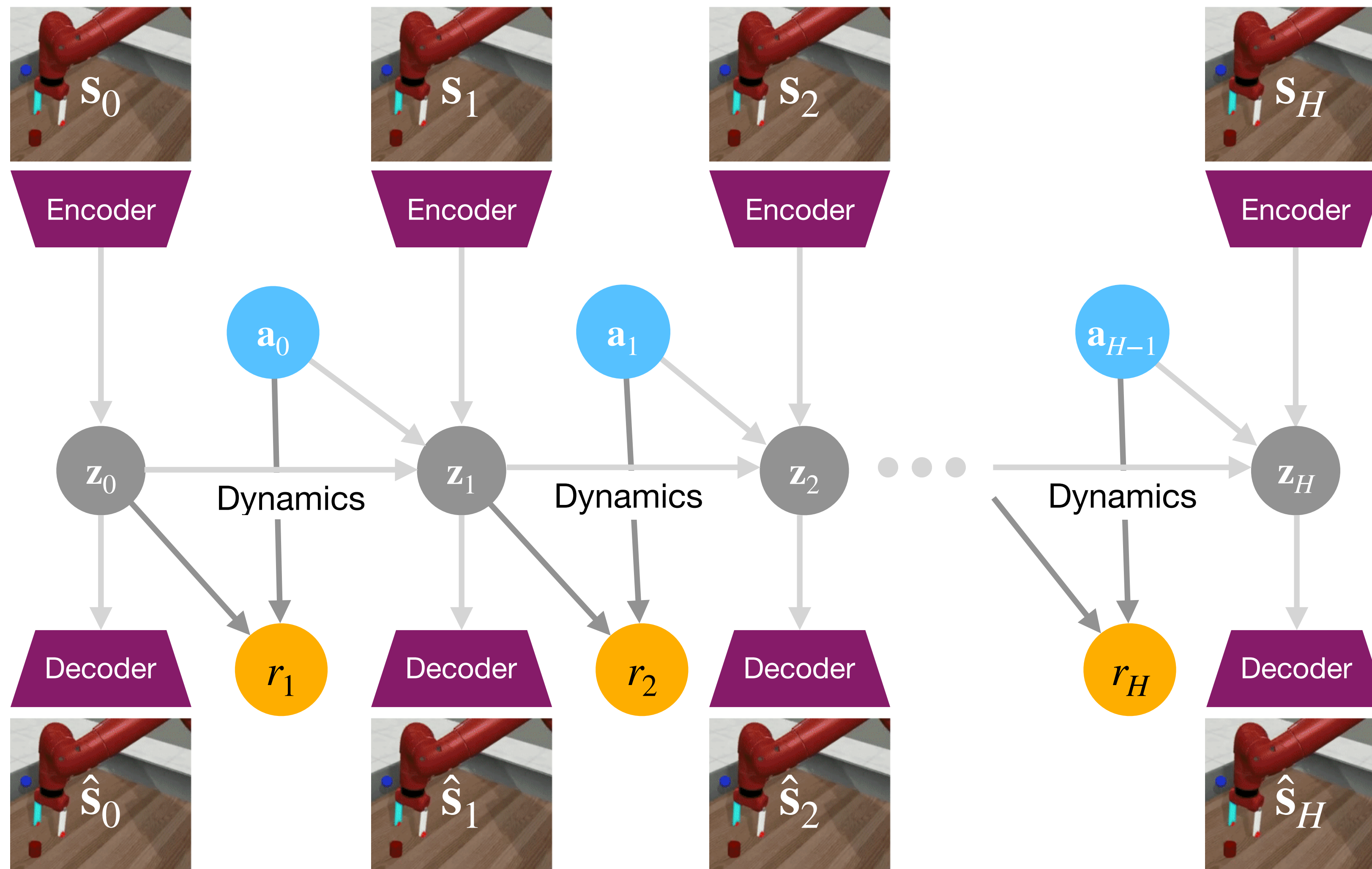
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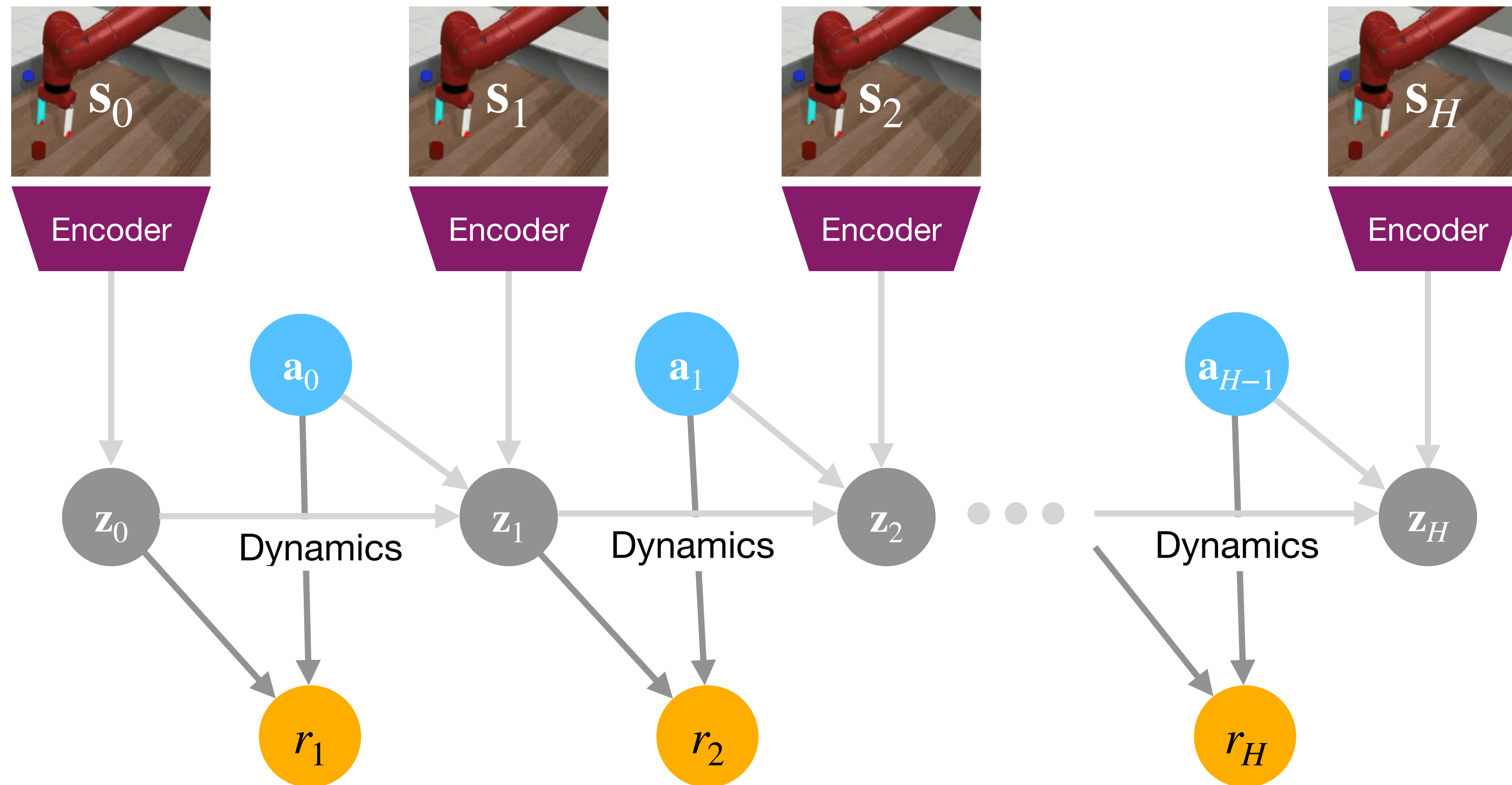
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# Latent Space Design Choices

# Latent Space Design Choices

	Discrete Latent States?	Discrete Encoding Type	Stochastic Dynamics?	Reconstruction?
DreamerV3	✓	One-hot	✓	✓
TD-MPC2	✗	N/A	✗	✗

Danijar Hafner, et al. Mastering diverse domains through world models. arXiv preprint arXiv:2301.04104, 2023.

Nicklas Hansen, et al. TD-MPC2: Scalable, Robust World Models for Continuous Control. In The Twelfth International Conference on Learning Representations, October 2023.

# Latent Space Design Choices

**1. Do discrete latent spaces offer benefits over continuous ones?**

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1. Do discrete latent spaces offer benefits over continuous ones?
2. How does the choice of discrete encoding (e.g., one-hot, label, or codebook encodings) affect performance?
3. Are there advantages to modelling the latent dynamics stochastically rather than deterministically?

# Latent Space Design Choices

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DC-MPC (ours)	✓	Codebook	✓	✗

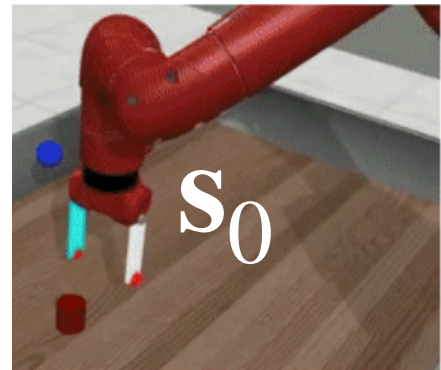
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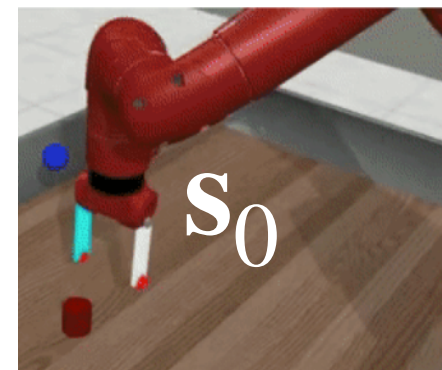
# DC-MPC: World Model Training



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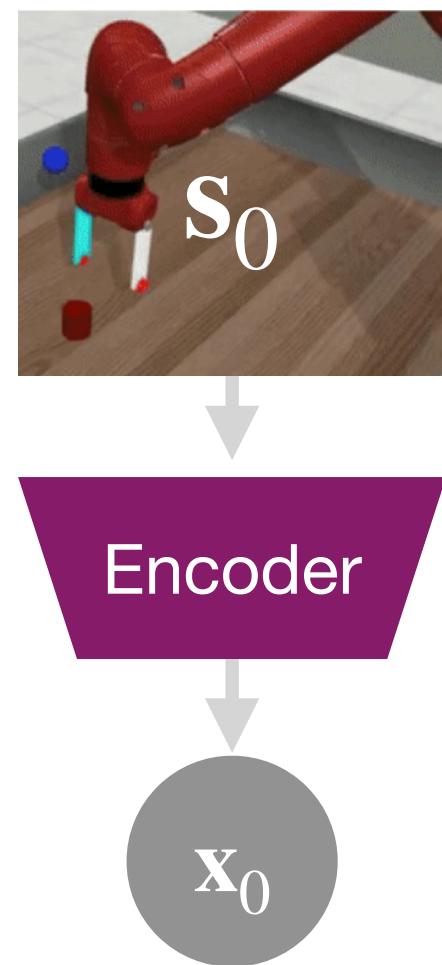


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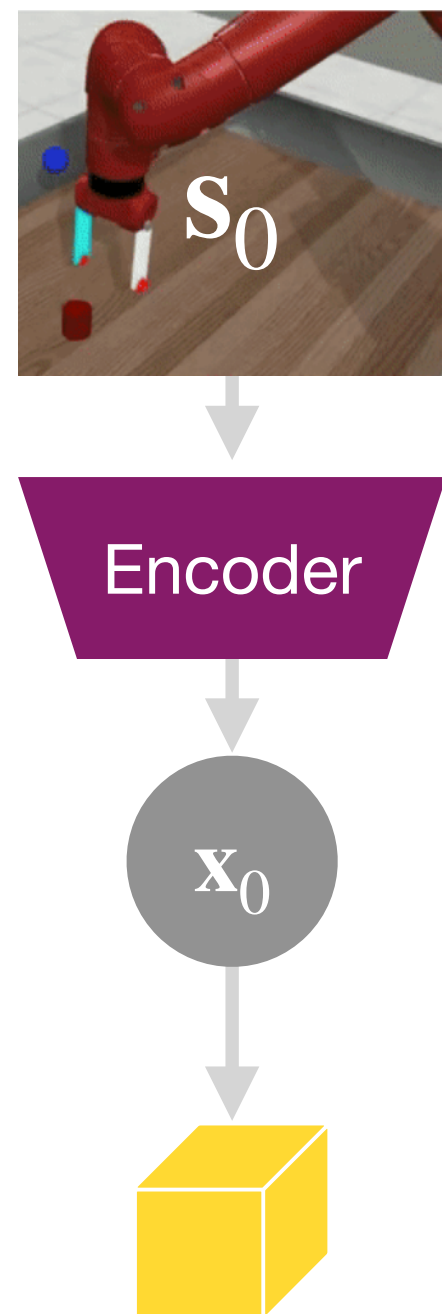


Encoder

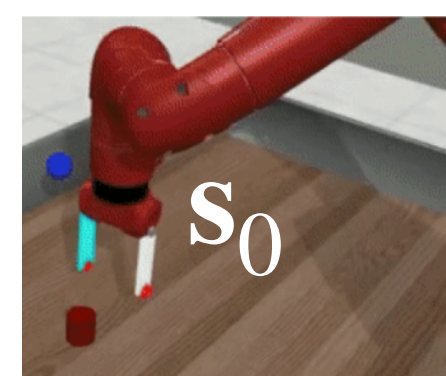
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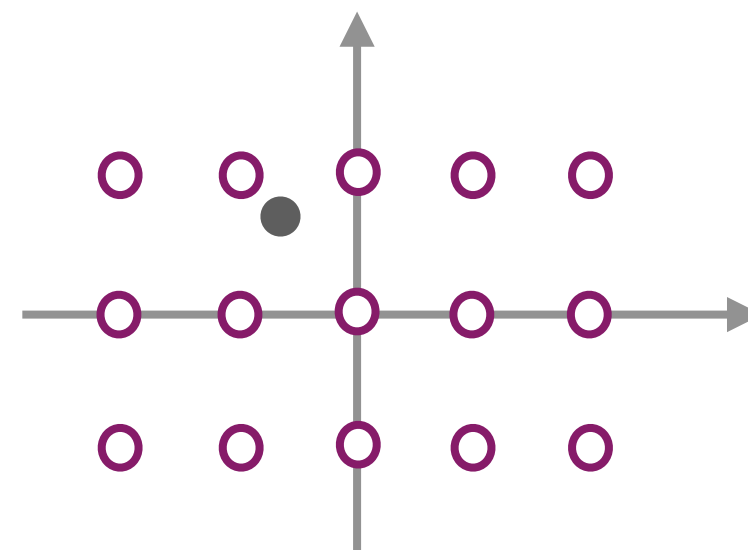
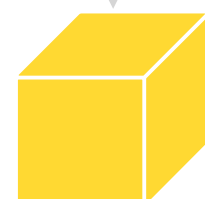


# DC-MPC: World Model Training



Encoder

$\mathbf{x}_0$



Codebook  $\mathcal{C}$

$\mathbf{c}^{(1)}$



$\mathbf{c}^{(2)}$



$\mathbf{c}^{(3)}$



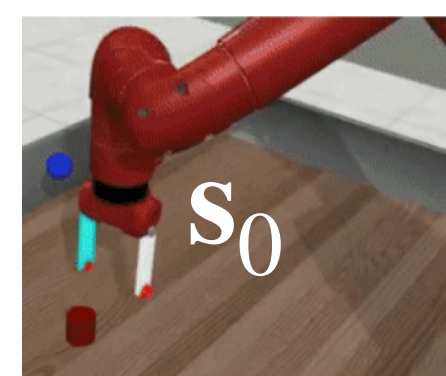
$\mathbf{c}^{(|\mathcal{C}|)}$



Number of  
codes in  
codebook  
 $|\mathcal{C}| = 15$

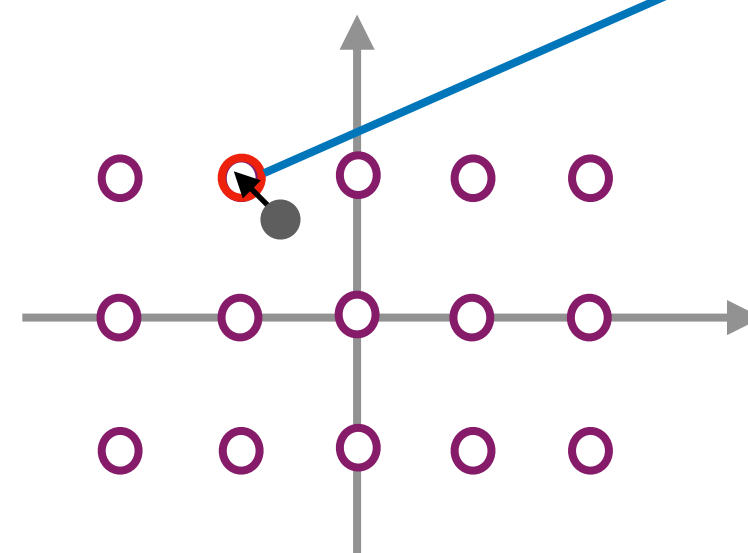
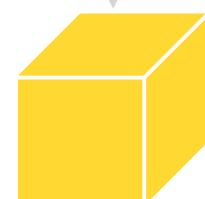
Number of  
channels  
 $b = 2$

# DC-MPC: World Model Training



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$\mathbf{x}_0$



Codebook  $\mathcal{C}$

$\mathbf{c}^{(1)}$



$\mathbf{c}^{(2)}$

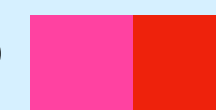


$\mathbf{c}^{(3)}$



$\vdots$

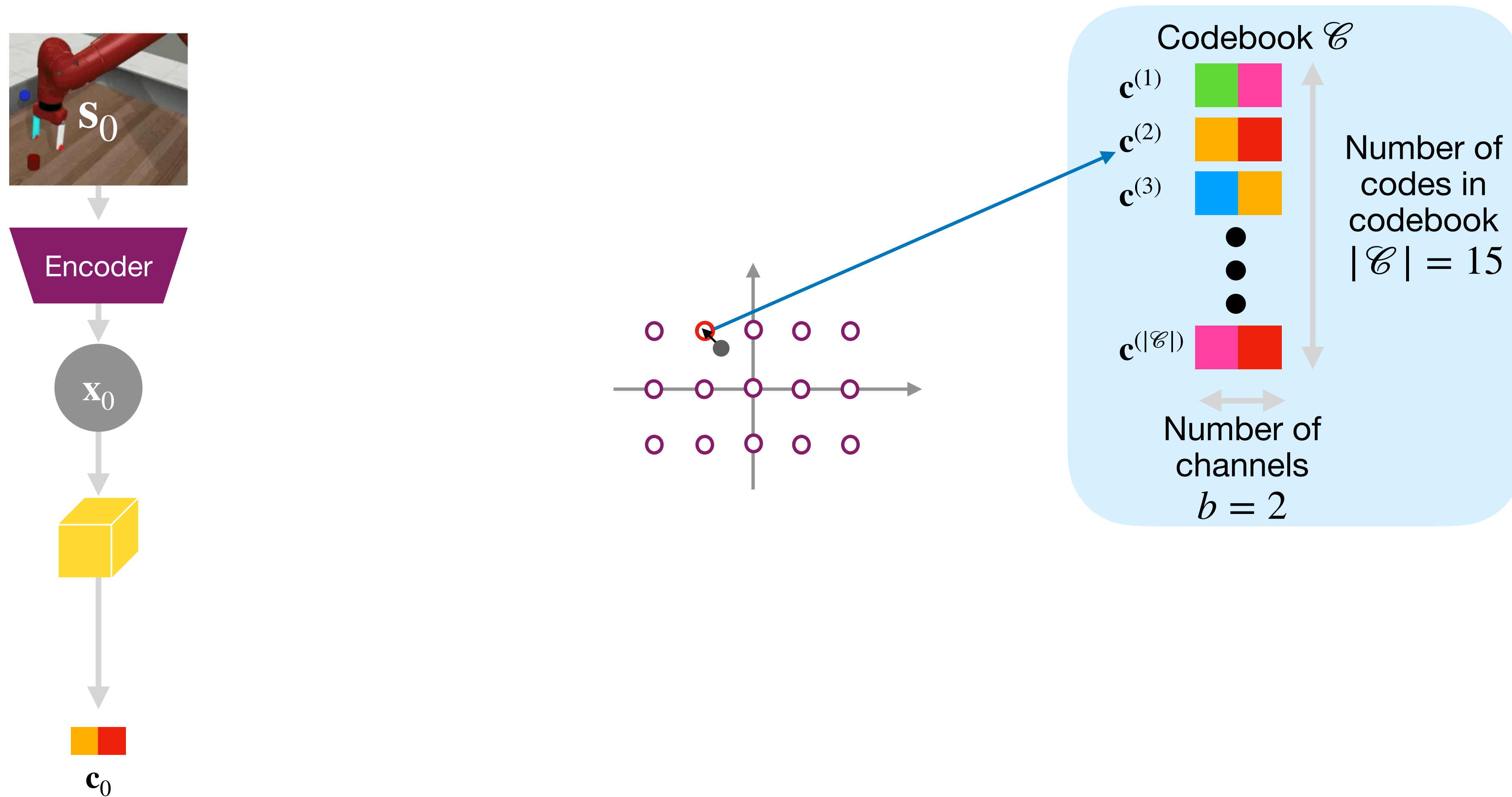
$\mathbf{c}^{(|\mathcal{C}|)}$



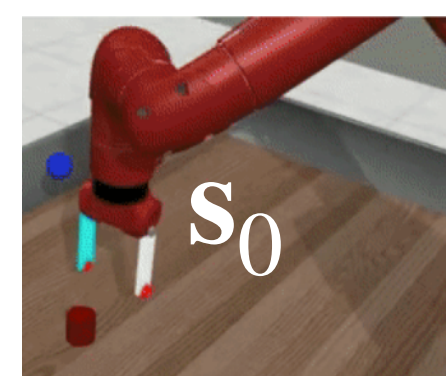
Number of  
codes in  
codebook  
 $|\mathcal{C}| = 15$

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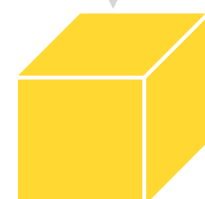


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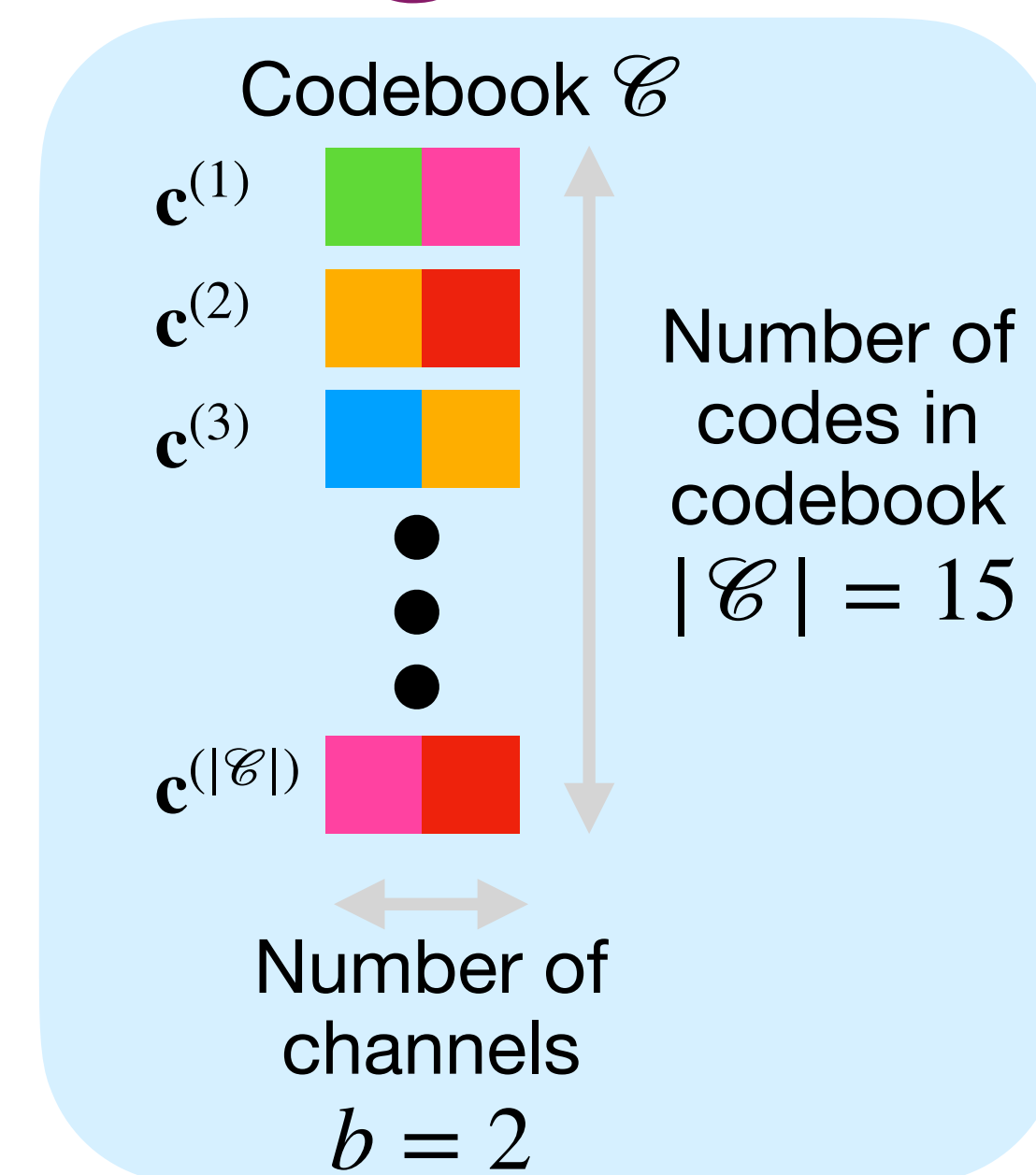


Encoder

$\mathbf{x}_0$

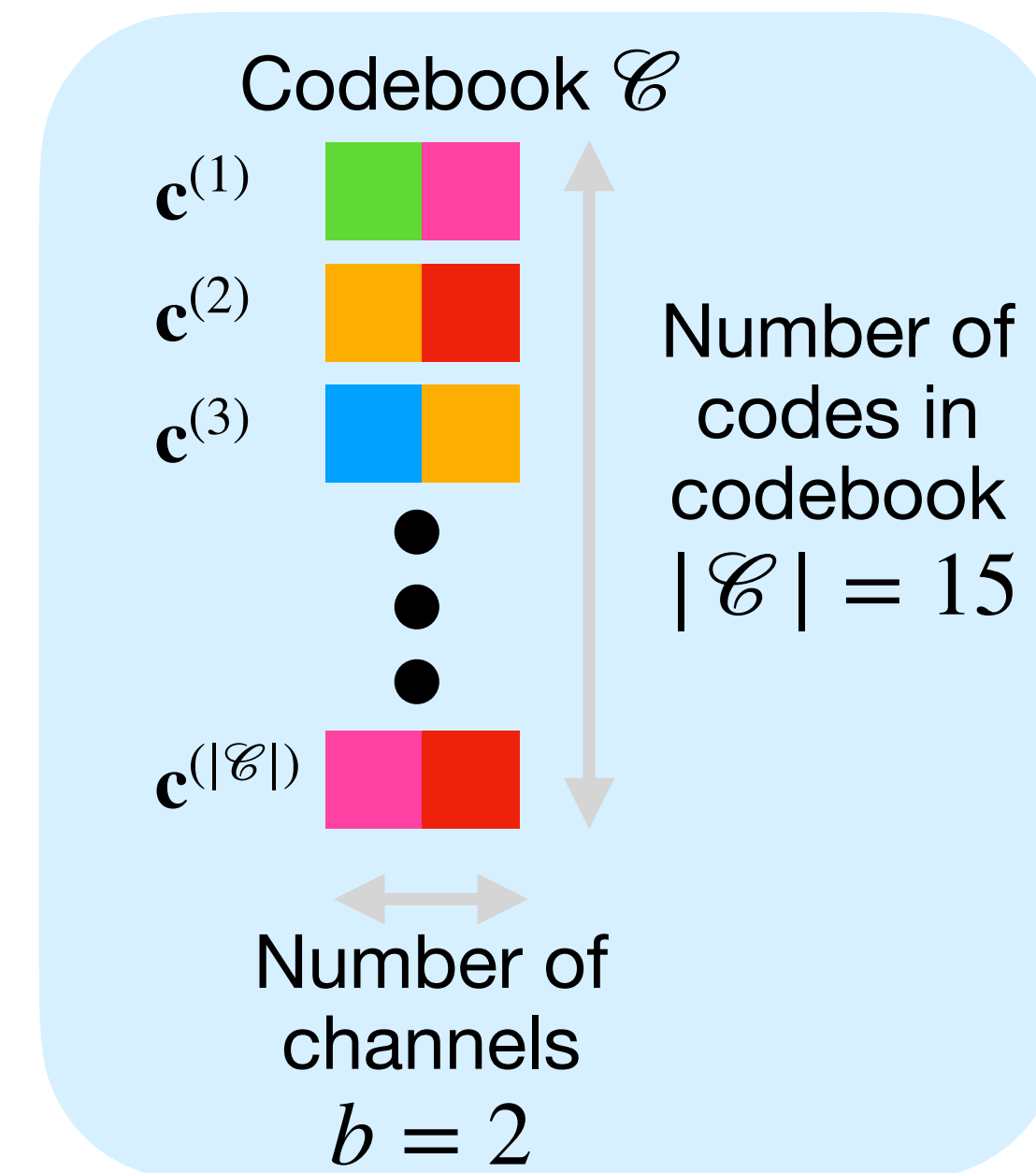
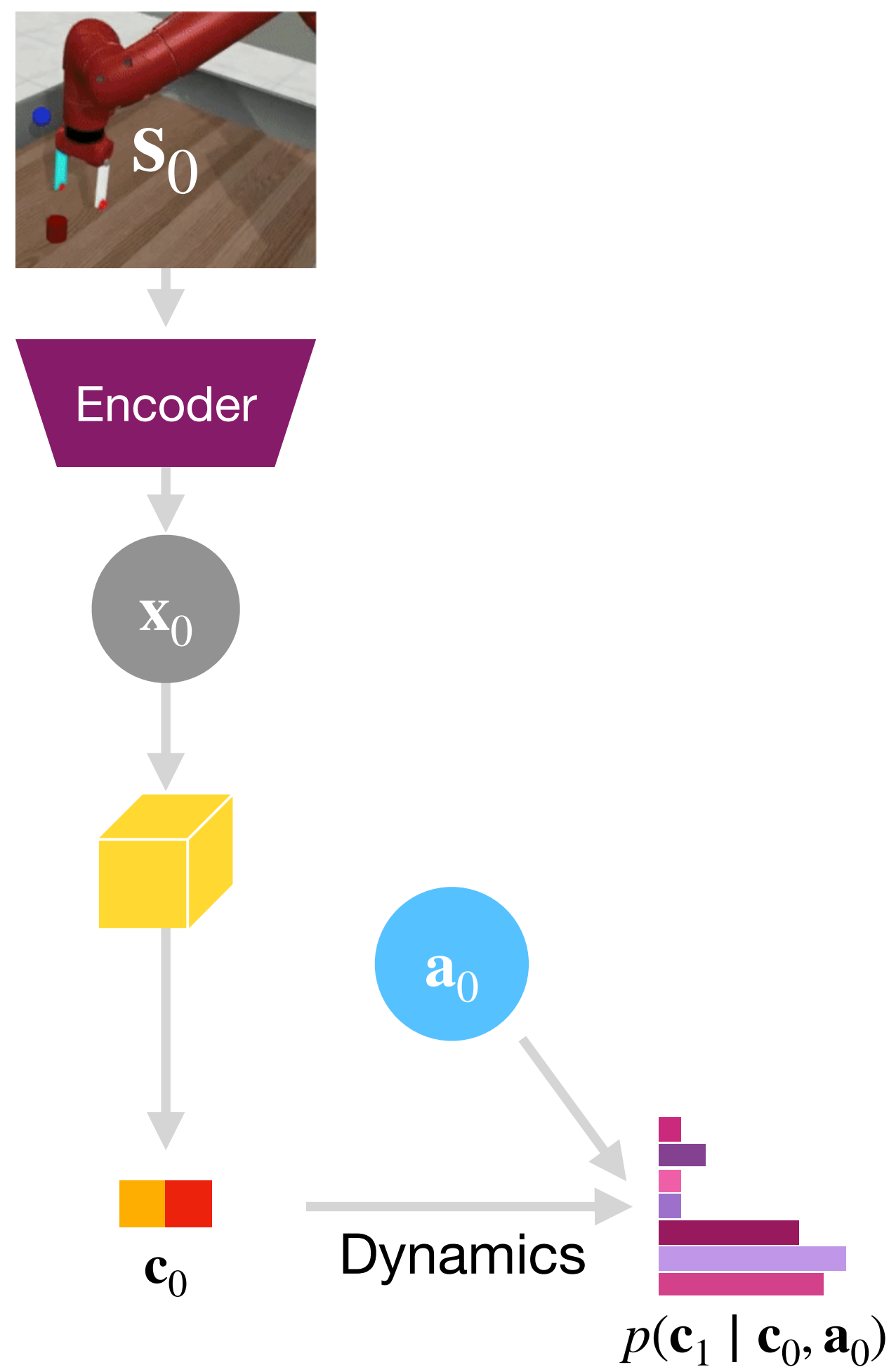


$\mathbf{c}_0$

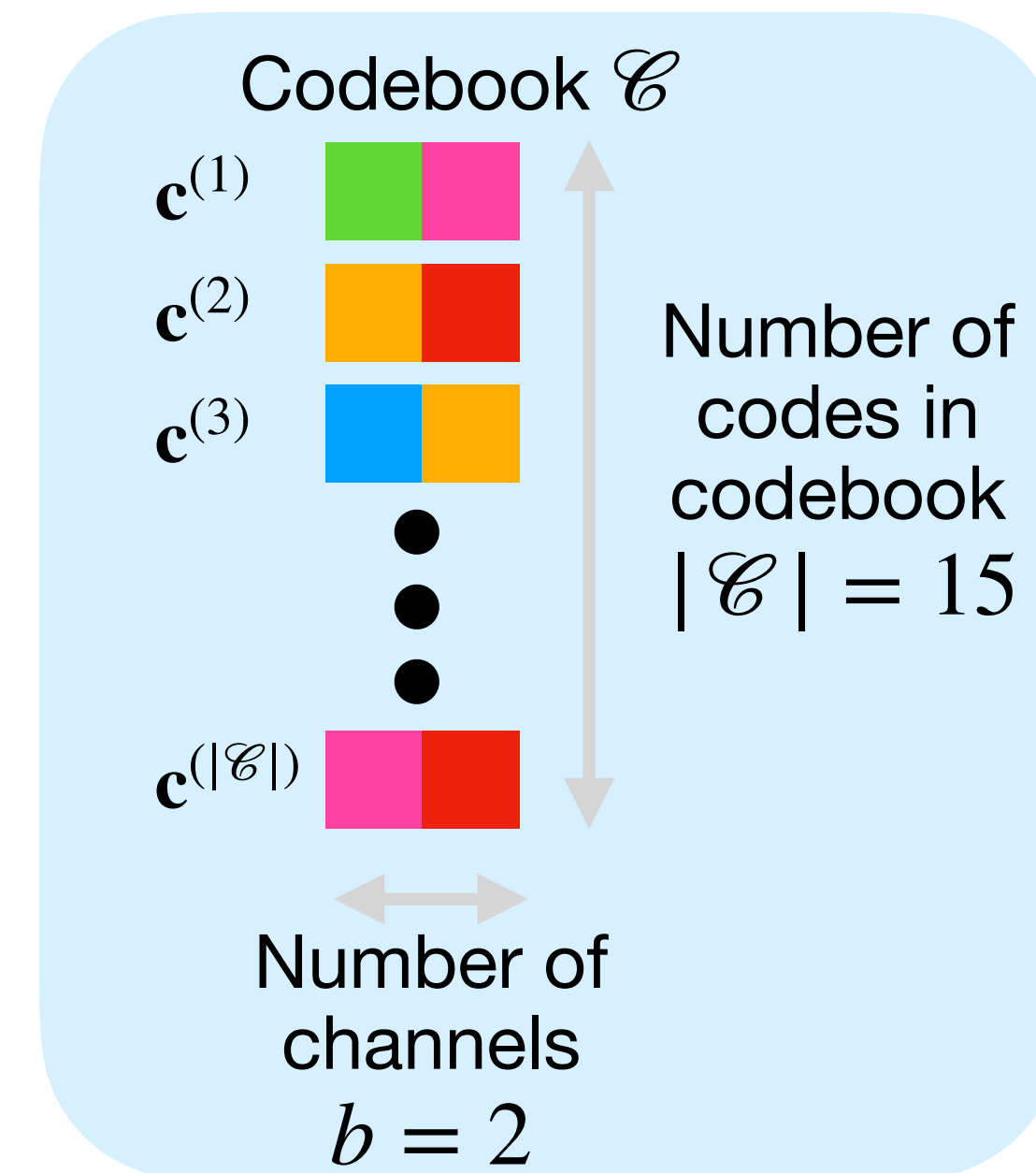
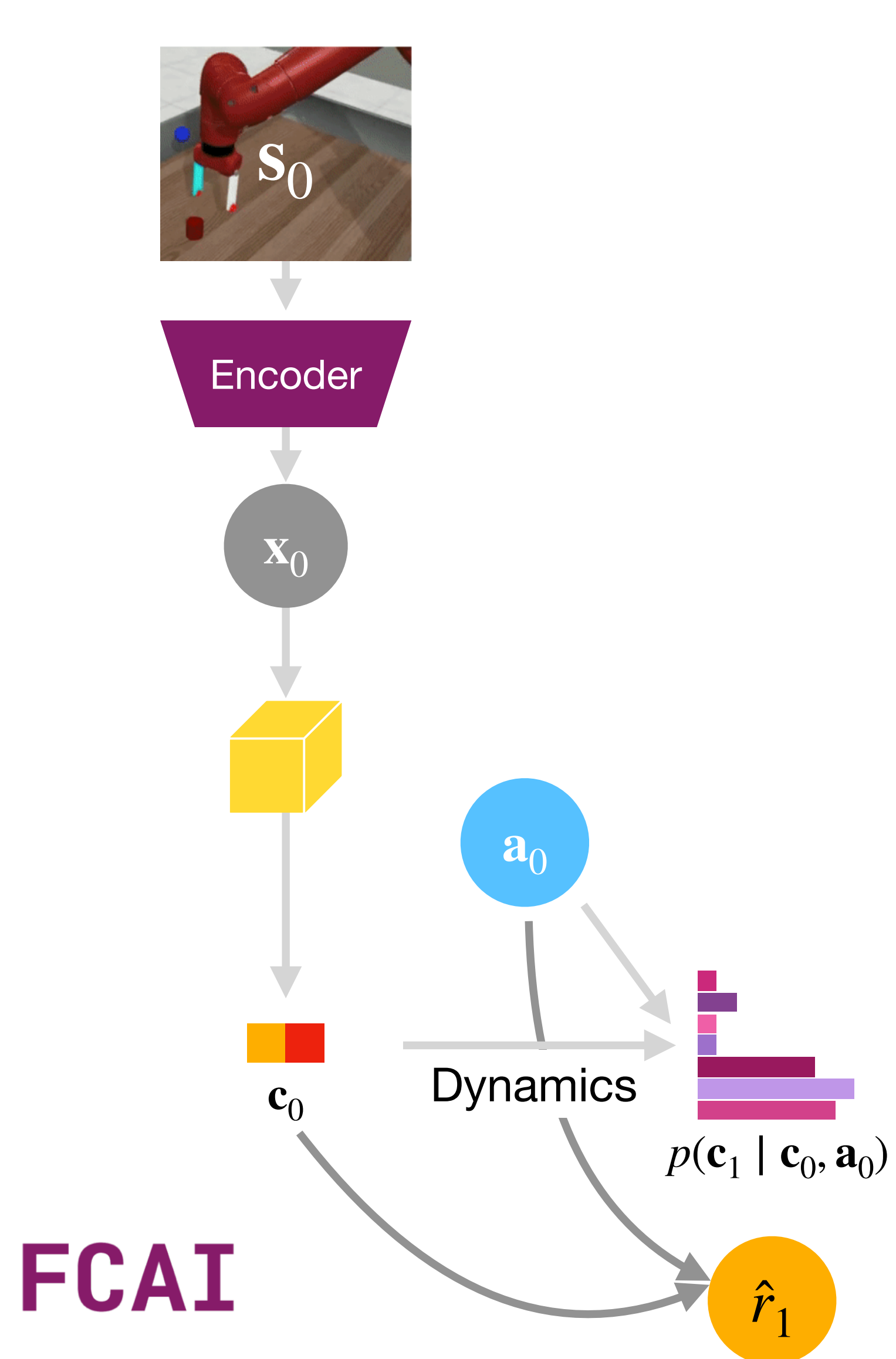




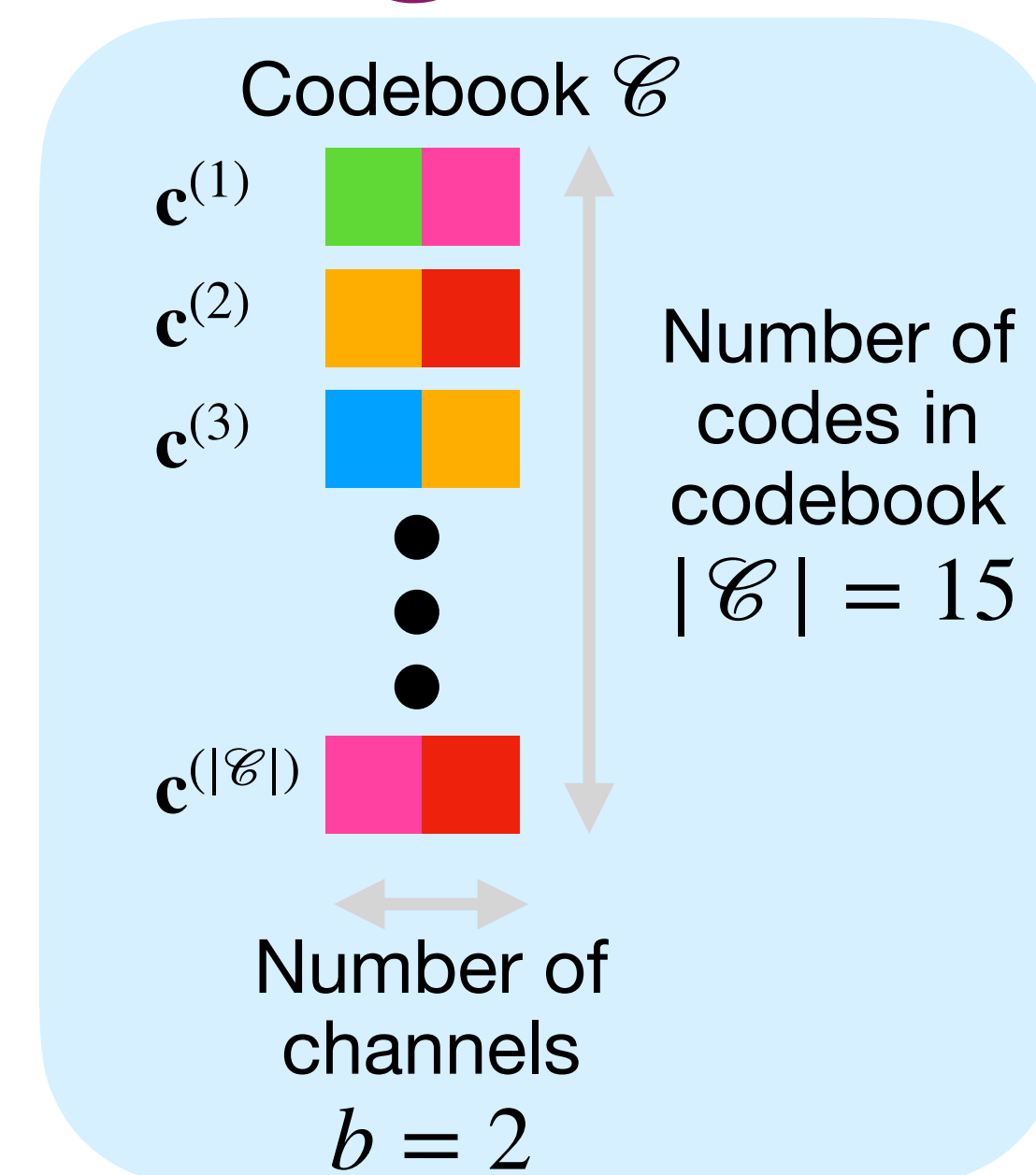
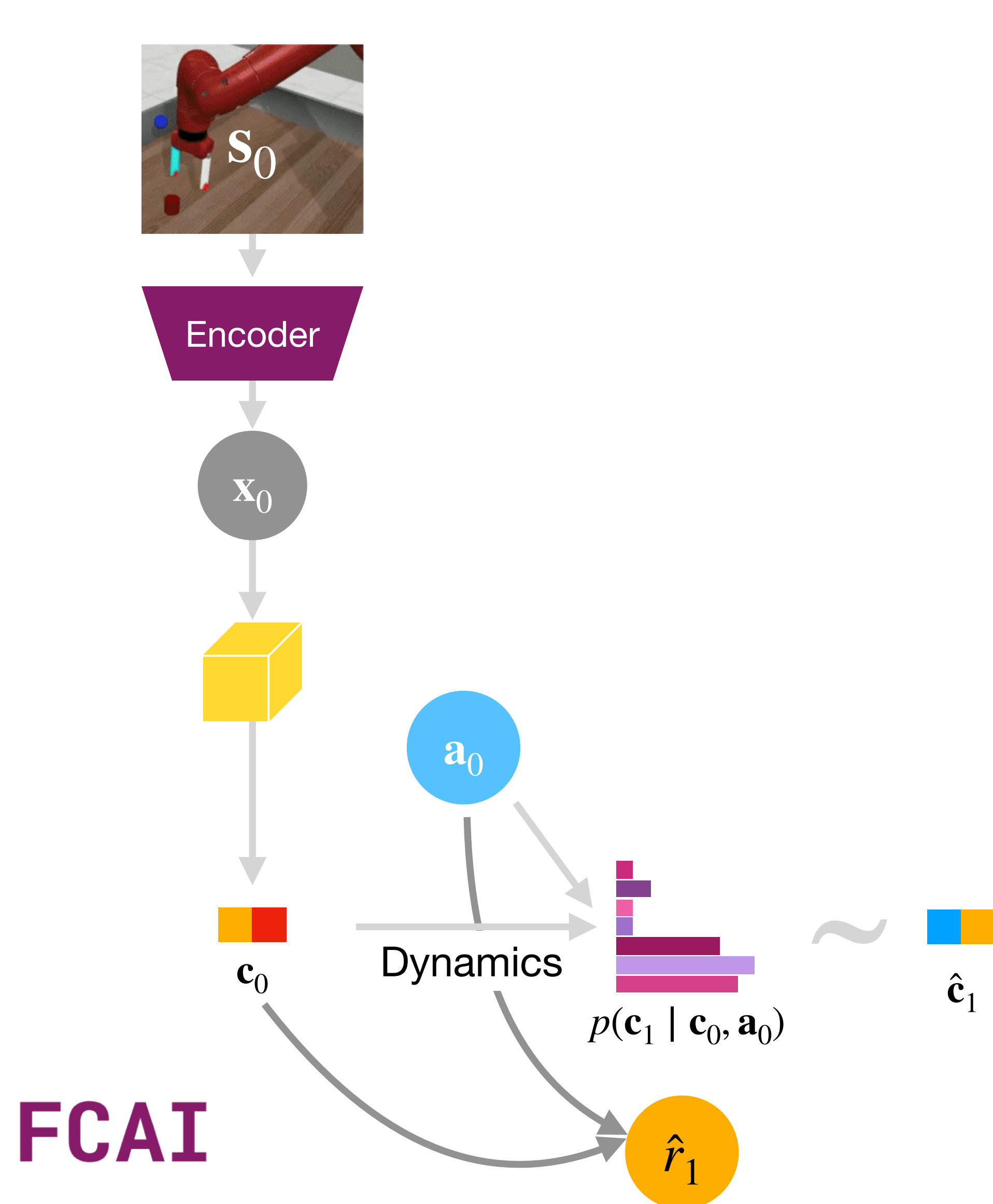
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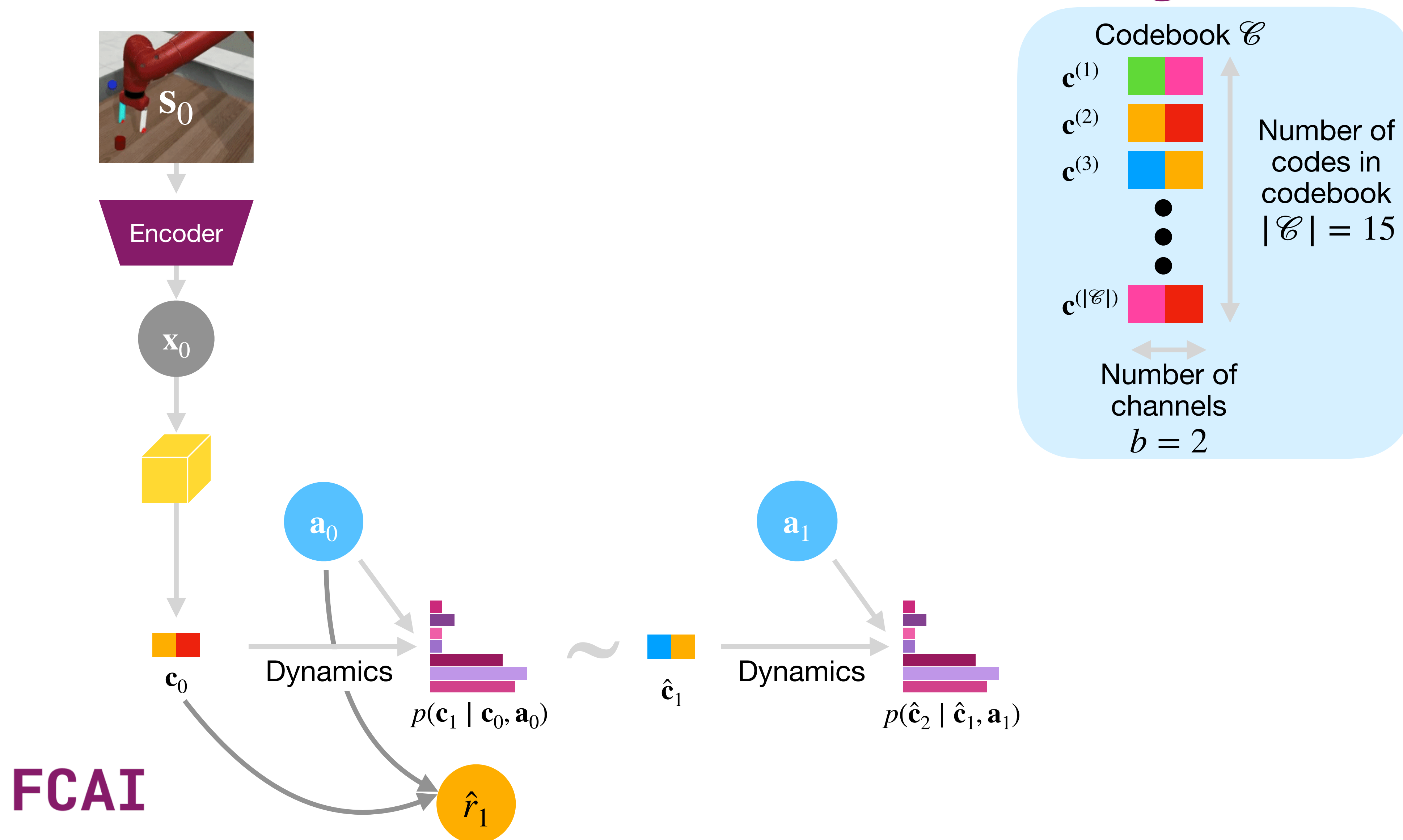
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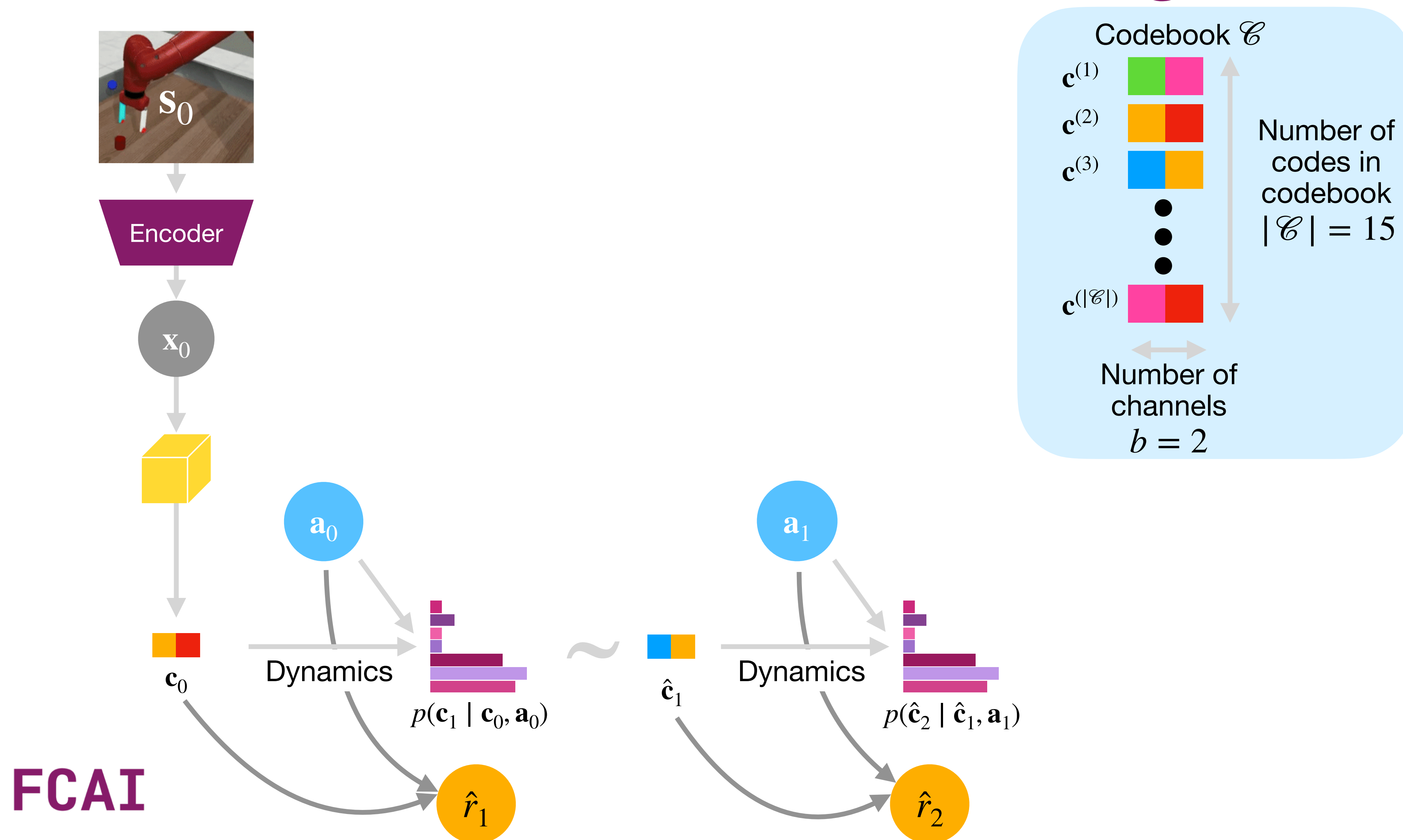
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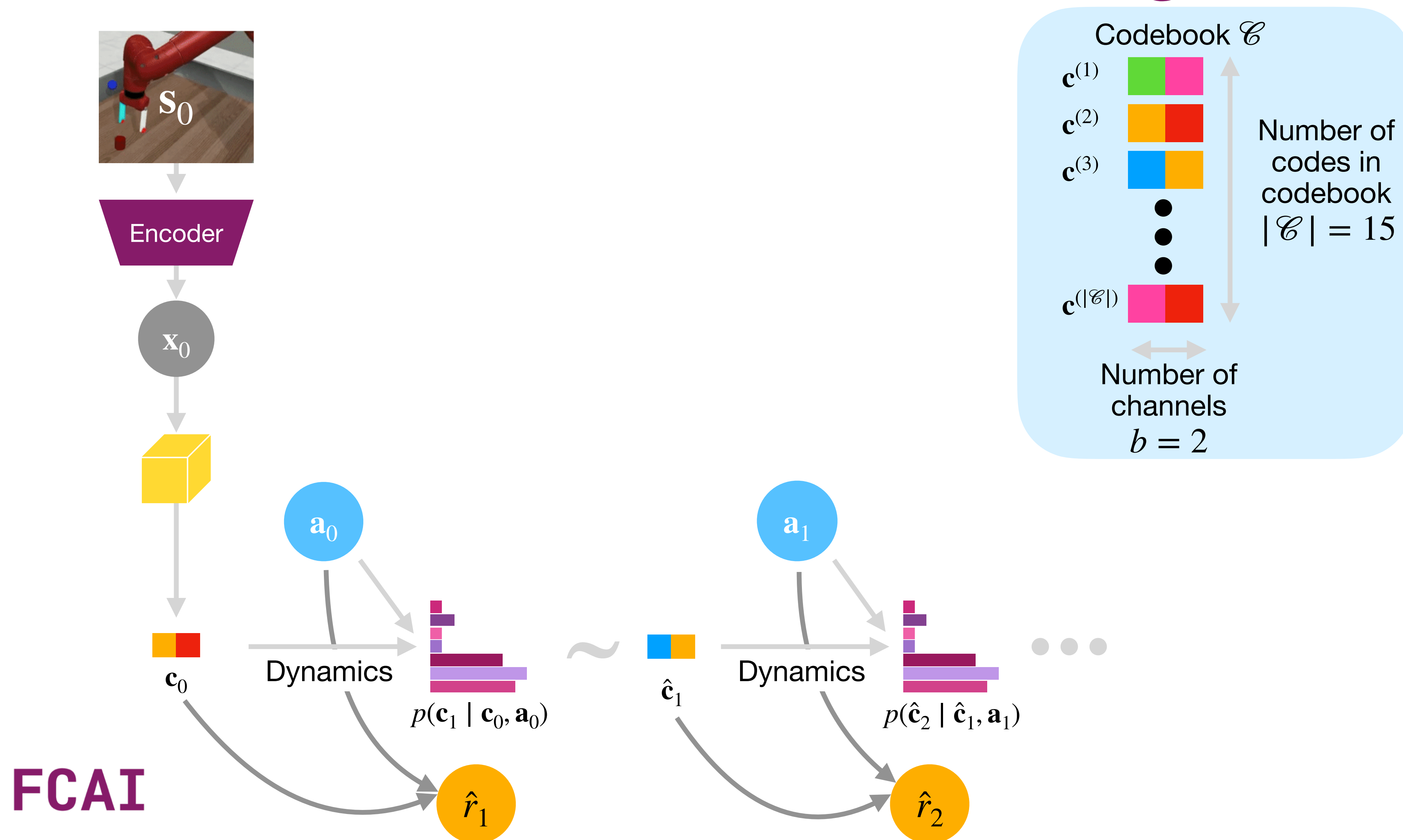
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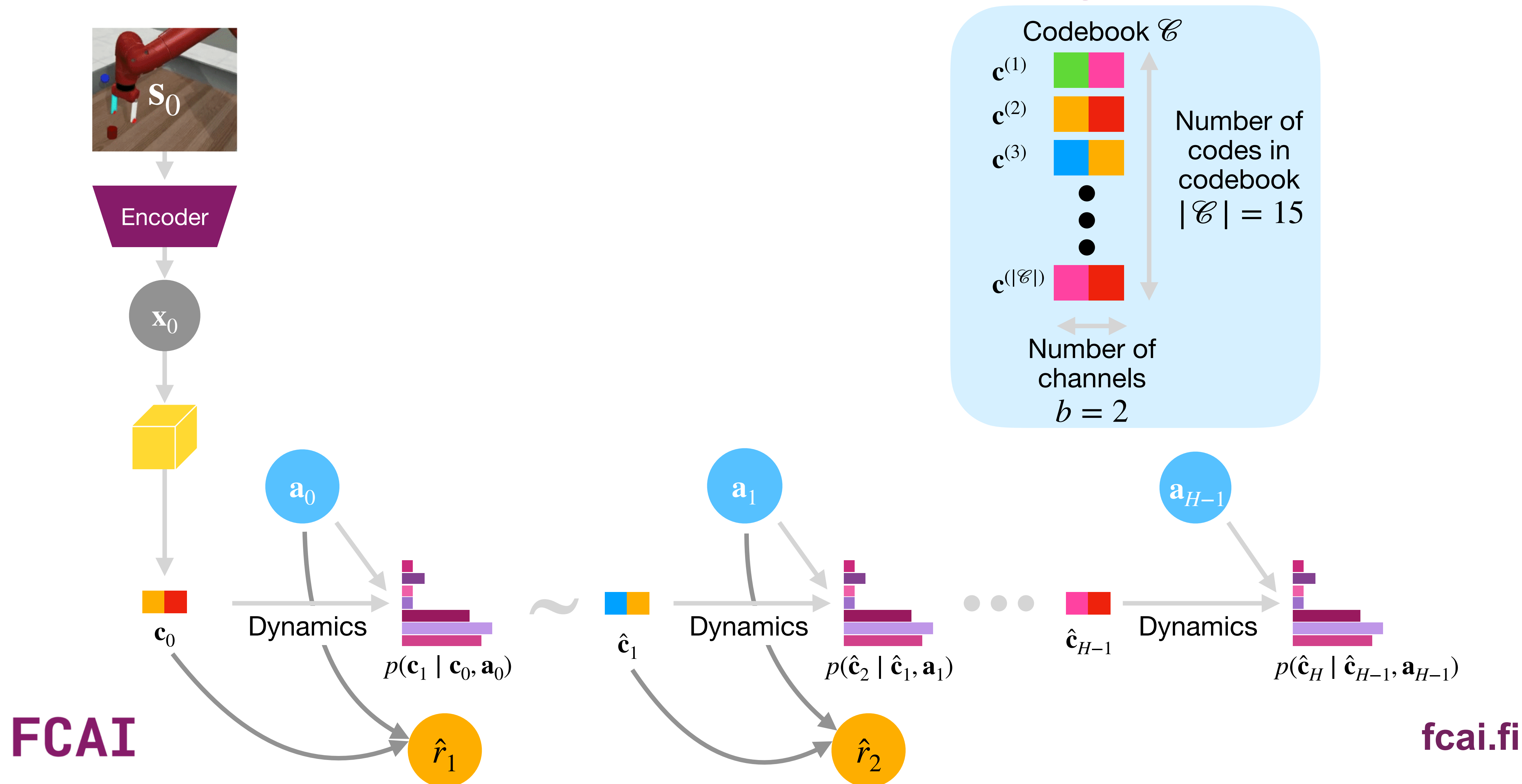
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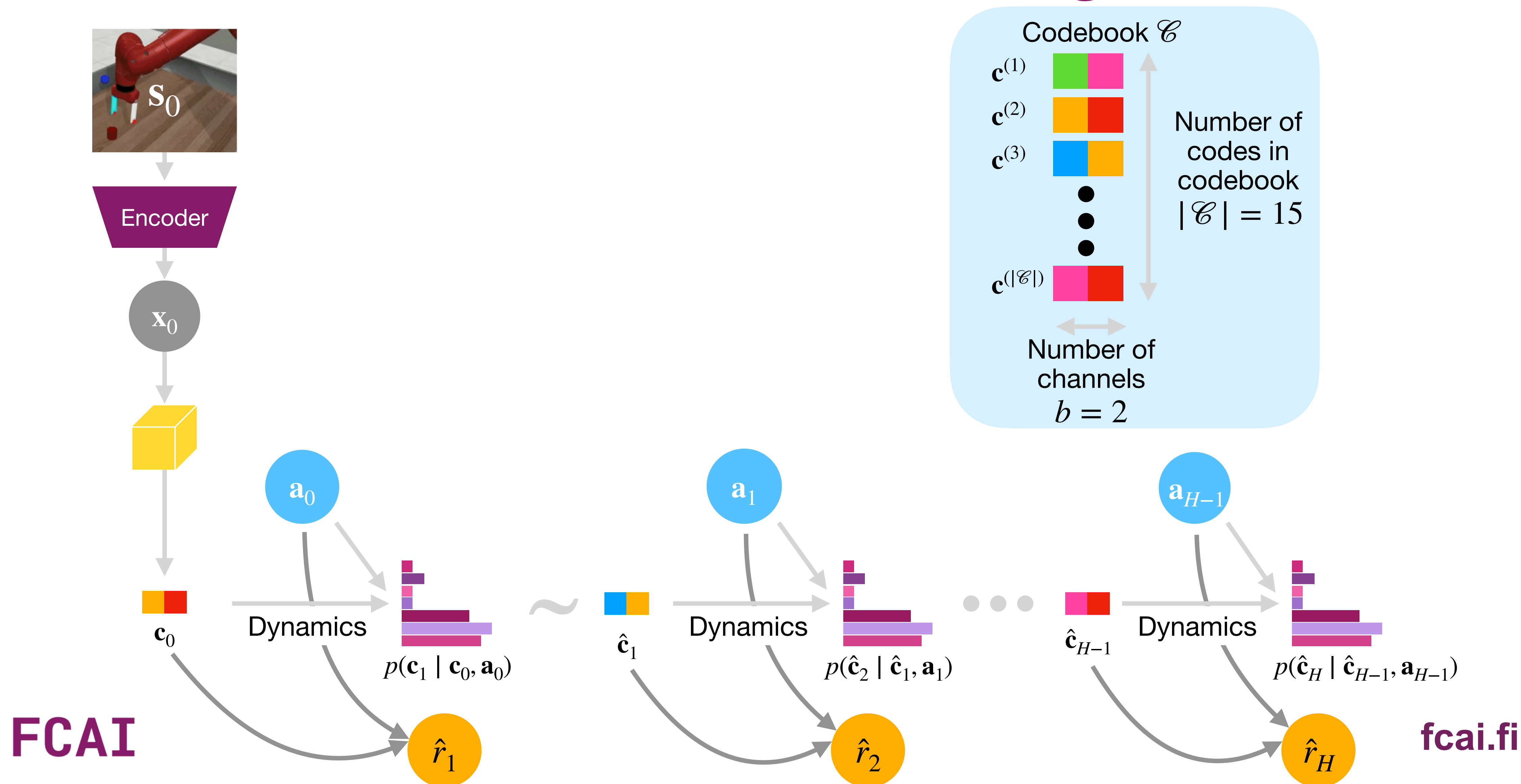


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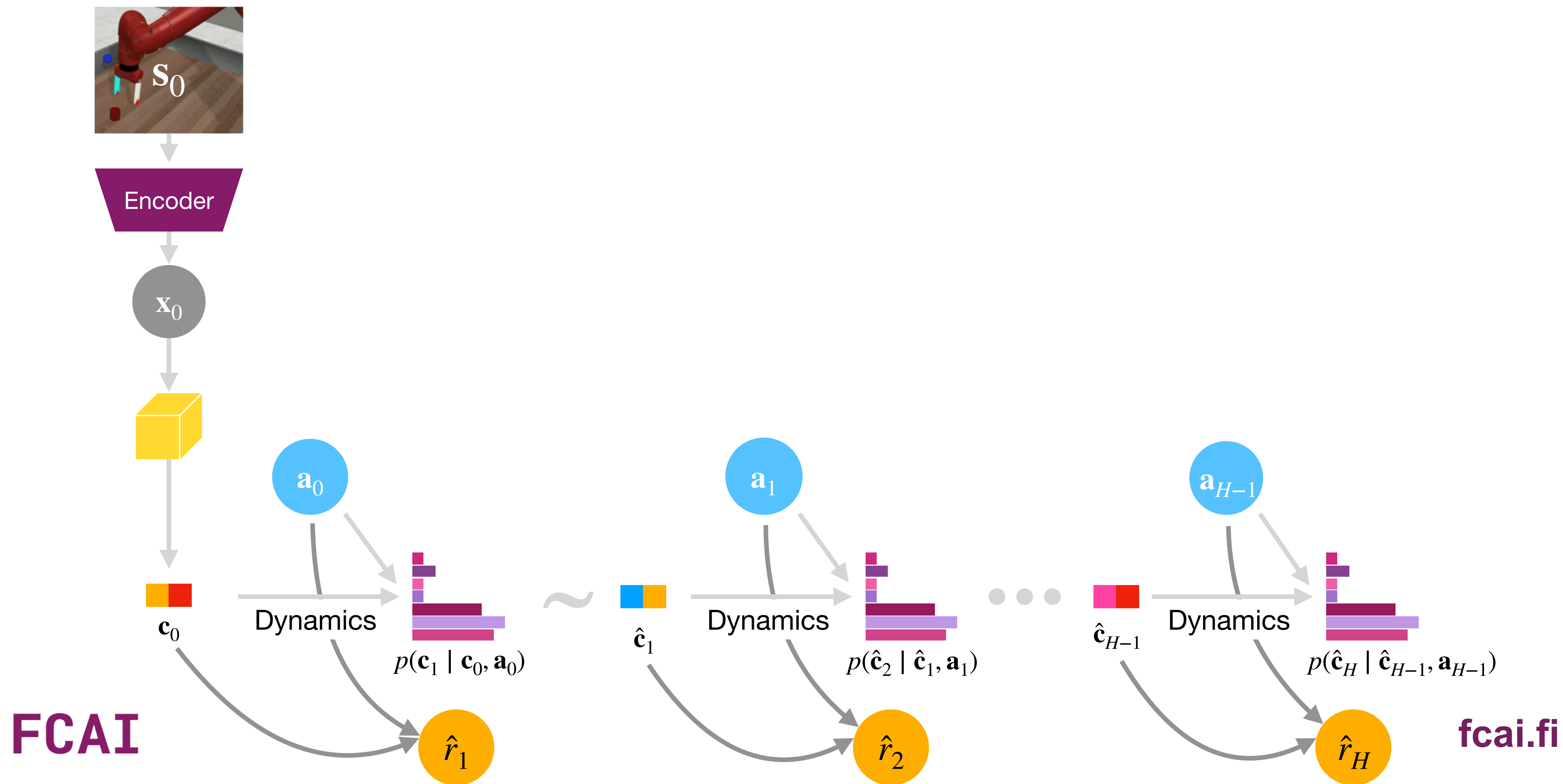


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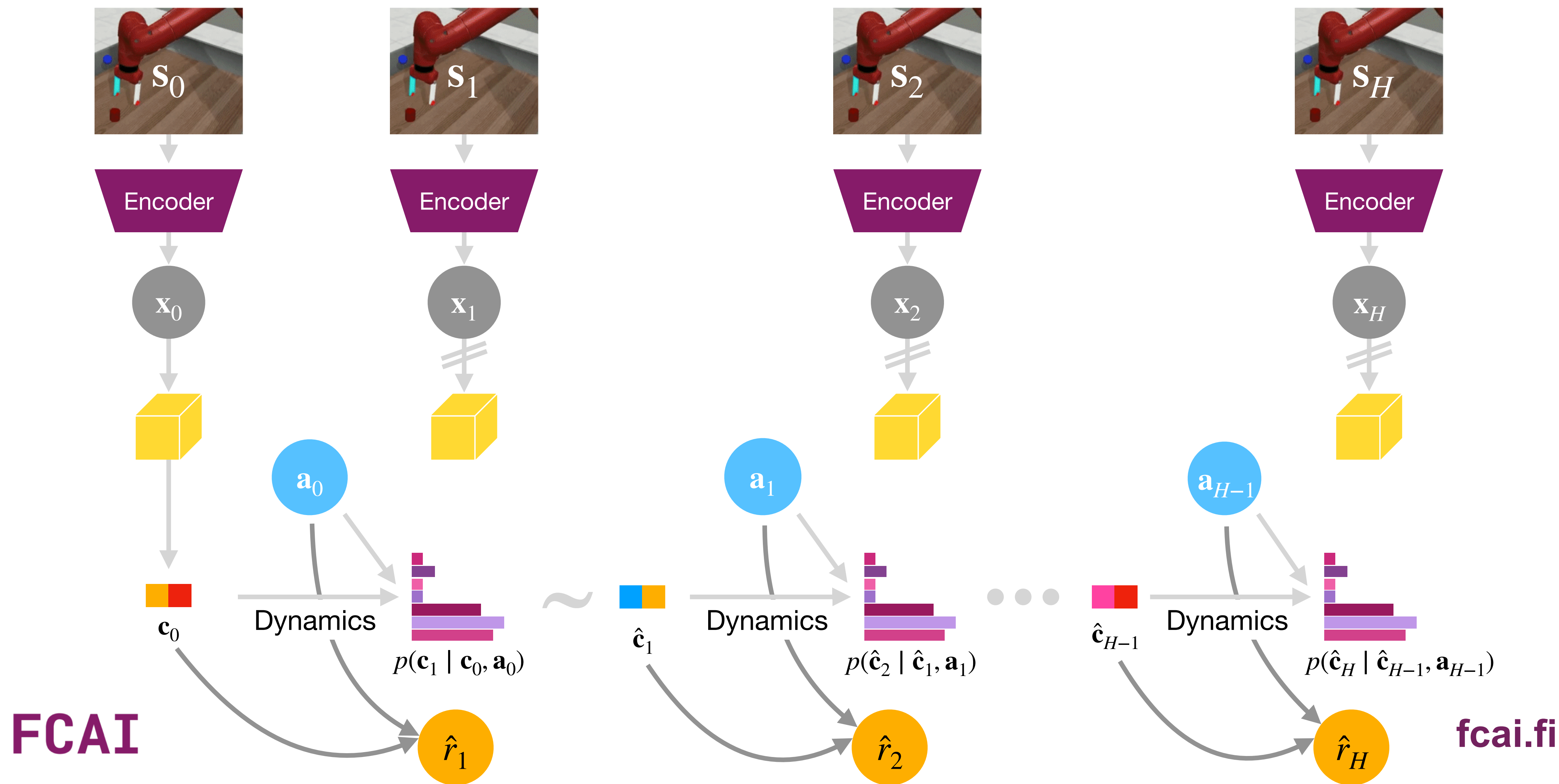




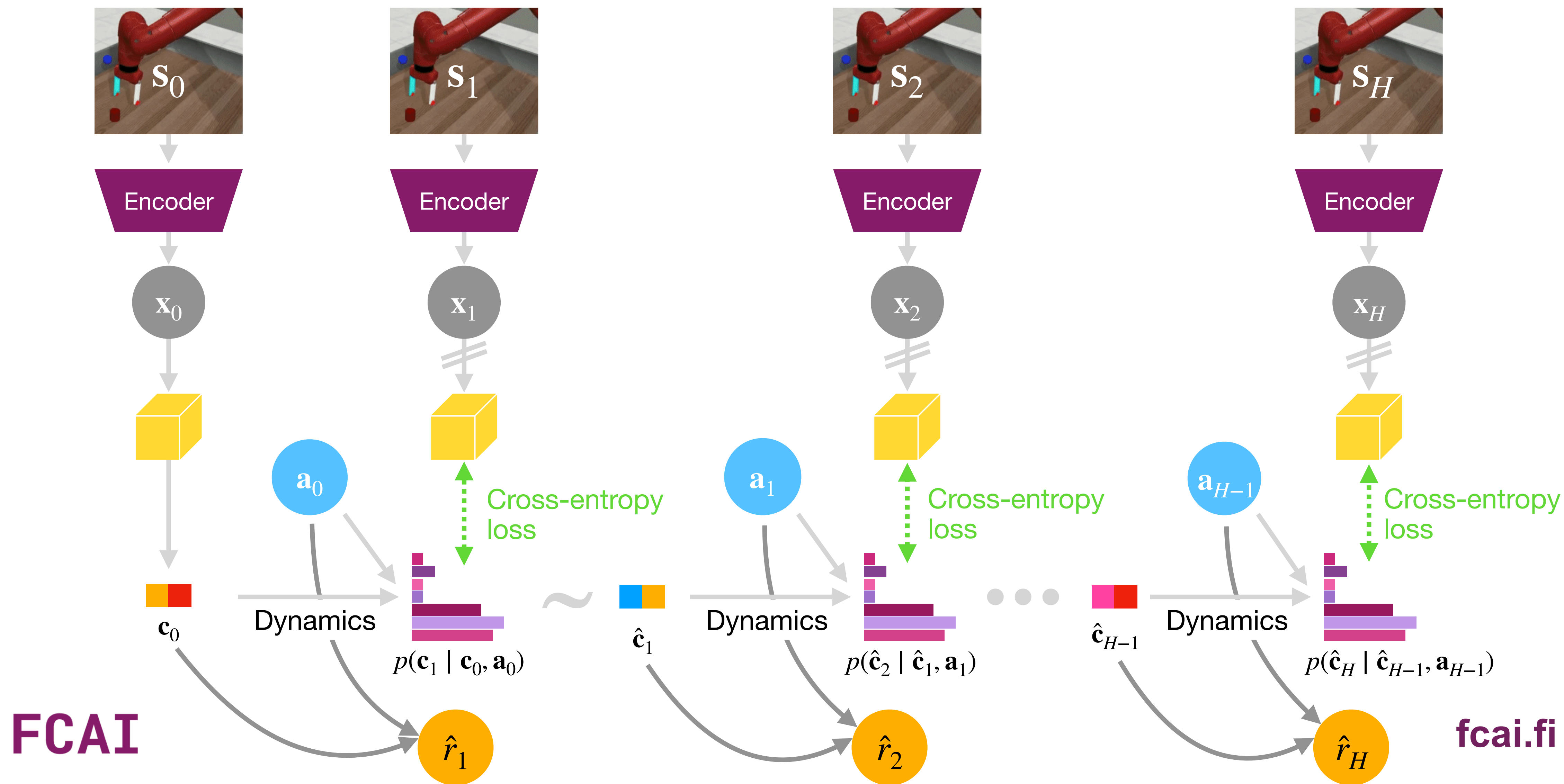
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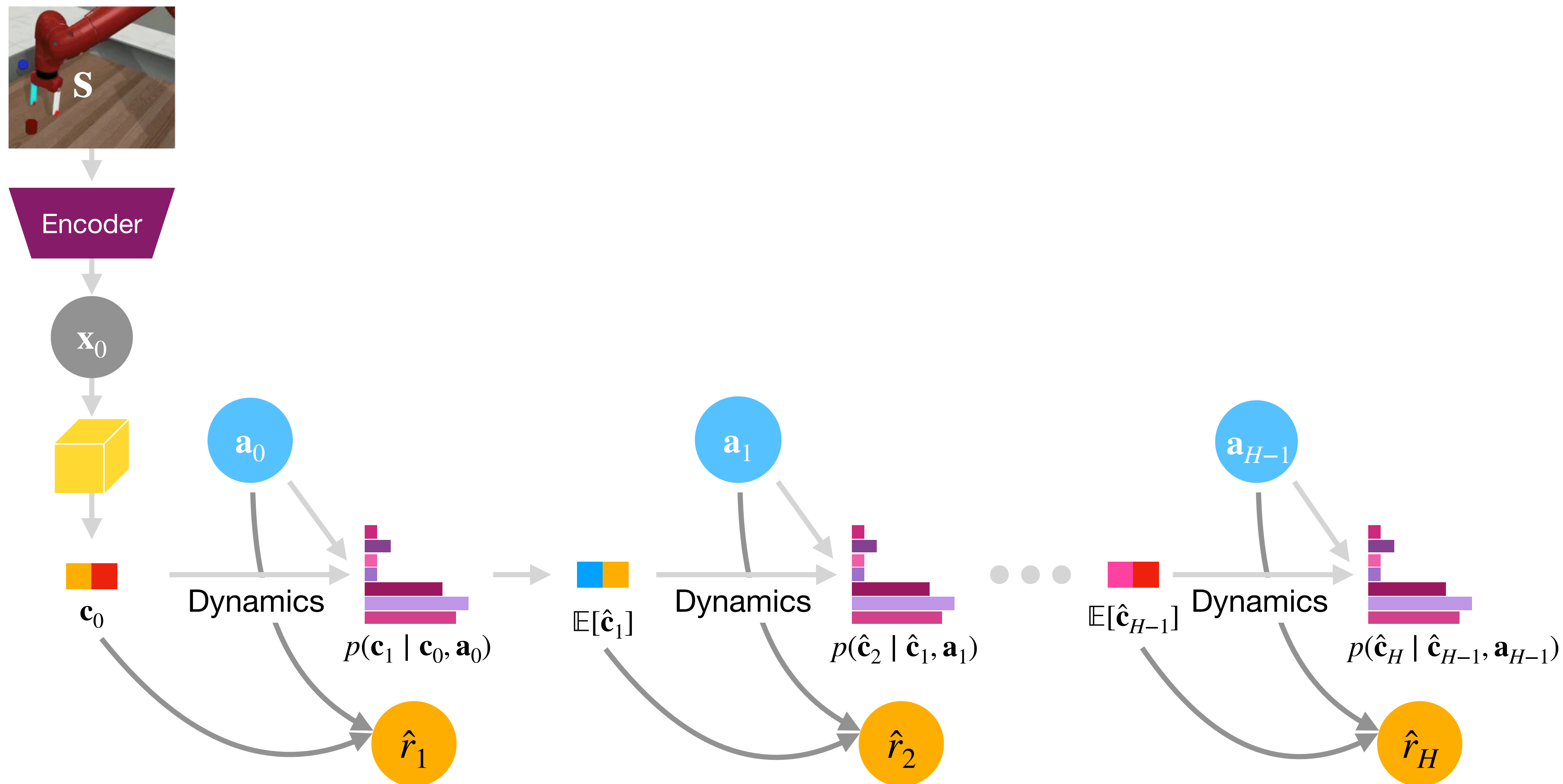
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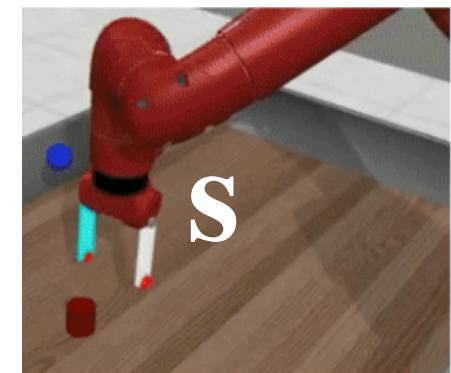
# DC-MPC: World Model Training



# DC-MPC: Decision-time Planning

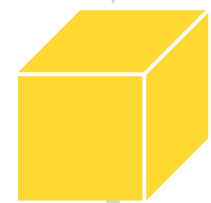


# DC-MPC: Decision-time Planning



Encoder

$\mathbf{x}_0$



$\mathbf{c}_0$

Dynamics

$\mathbf{a}_0$

$p(\mathbf{c}_1 | \mathbf{c}_0, \mathbf{a}_0)$

$\mathbb{E}[\hat{\mathbf{c}}_1]$

Dynamics

$\mathbf{a}_1$

$p(\hat{\mathbf{c}}_2 | \hat{\mathbf{c}}_1, \mathbf{a}_1)$

$\mathbb{E}[\hat{\mathbf{c}}_{H-1}]$

Dynamics

$\mathbf{a}_{H-1}$

$p(\hat{\mathbf{c}}_H | \hat{\mathbf{c}}_{H-1}, \mathbf{a}_{H-1})$

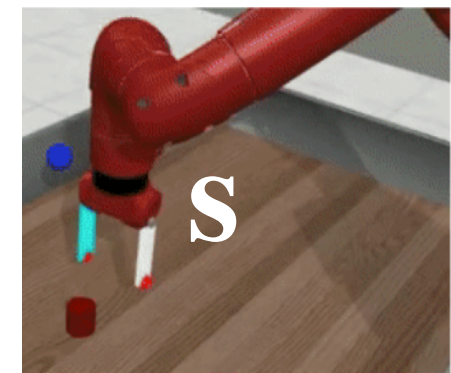
$$J(\mathbf{a}_{0:H}, \mathbf{s}) = \gamma^H Q_\psi(\hat{\mathbf{c}}_H, \mathbf{a}_H) + \sum_{h=0}^{H-1} \gamma^h R_\xi(\hat{\mathbf{c}}_h, \mathbf{a}_h)$$

$\hat{r}_1$

$\hat{r}_2$

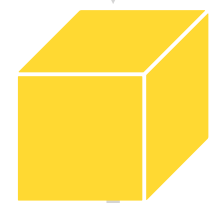
$\hat{r}_H$

# DC-MPC: Decision-time Planning



Encoder

$\mathbf{x}_0$



$\mathbf{c}_0$

Dynamics

$\mathbf{a}_0$

$p(\mathbf{c}_1 | \mathbf{c}_0, \mathbf{a}_0)$

$\mathbb{E}[\hat{\mathbf{c}}_1]$

Dynamics

$\mathbf{a}_1$

$p(\hat{\mathbf{c}}_2 | \hat{\mathbf{c}}_1, \mathbf{a}_1)$

$\mathbb{E}[\hat{\mathbf{c}}_{H-1}]$

Dynamics

$\mathbf{a}_{H-1}$

$p(\hat{\mathbf{c}}_H | \hat{\mathbf{c}}_{H-1}, \mathbf{a}_{H-1})$

Reward func.

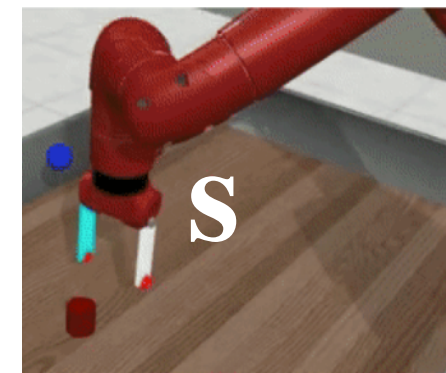
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$\hat{r}_1$

$\hat{r}_2$

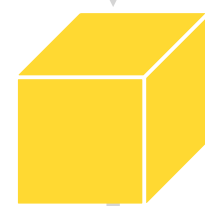
$\hat{r}_H$

# DC-MPC: Decision-time Planning



Encoder

$\mathbf{x}_0$



$\mathbf{c}_0$

Dynamics

$p(\mathbf{c}_1 | \mathbf{c}_0, \mathbf{a}_0)$

$\mathbf{a}_0$

$\hat{r}_1$

$\mathbf{a}_1$

Dynamics

$p(\hat{\mathbf{c}}_2 | \hat{\mathbf{c}}_1, \mathbf{a}_1)$

$\mathbb{E}[\hat{\mathbf{c}}_1]$

$\hat{r}_2$

$\mathbf{a}_{H-1}$

Dynamics

$p(\hat{\mathbf{c}}_H | \hat{\mathbf{c}}_{H-1}, \mathbf{a}_{H-1})$

$\mathbb{E}[\hat{\mathbf{c}}_{H-1}]$

$\hat{r}_H$

Bootstrap with  
action-value

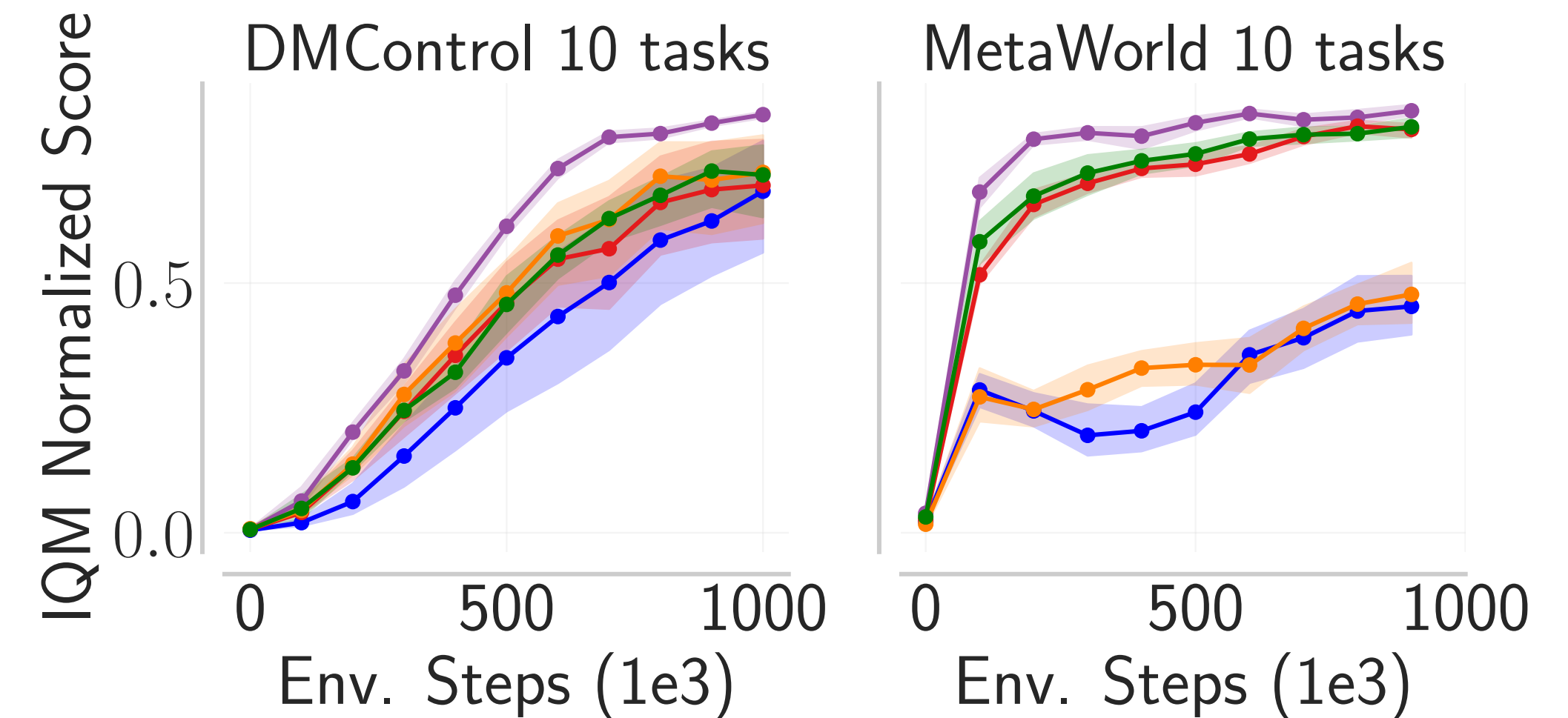
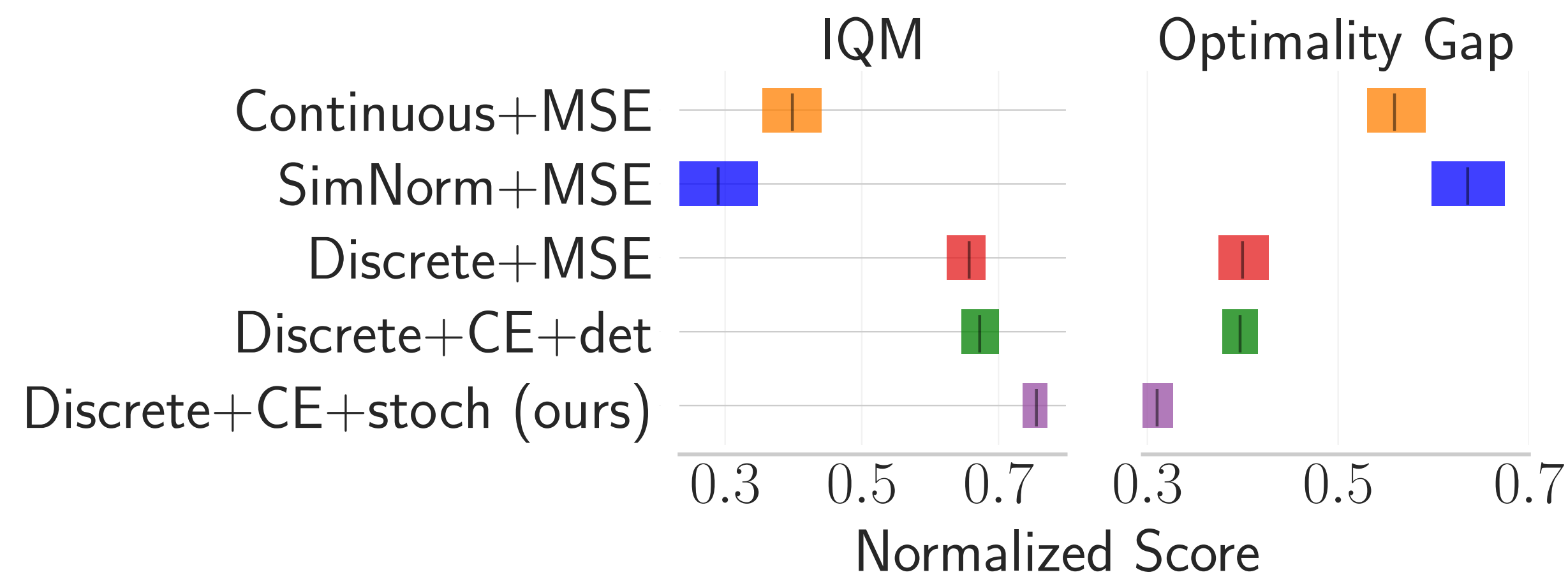
Reward func.

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# Why Does DC-MPC Work So Well?

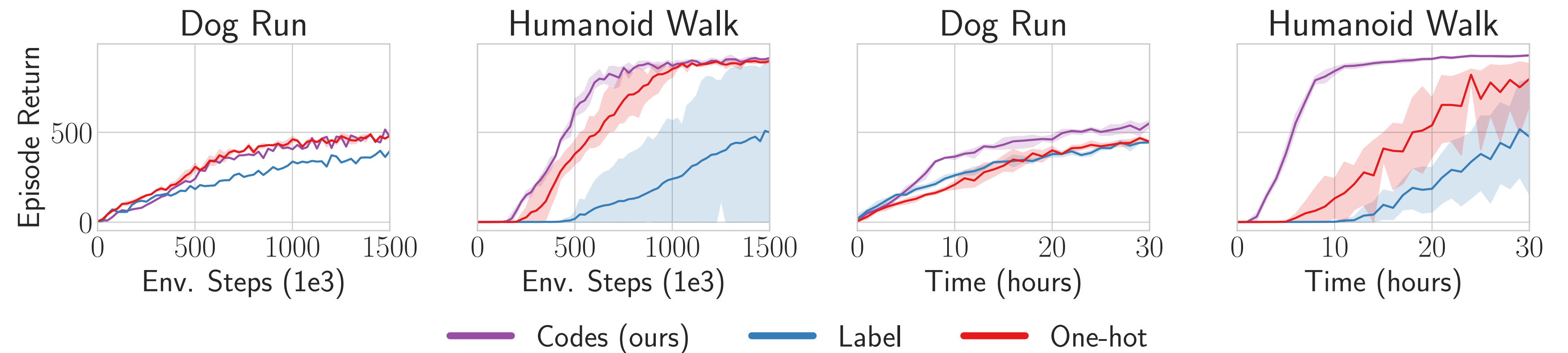
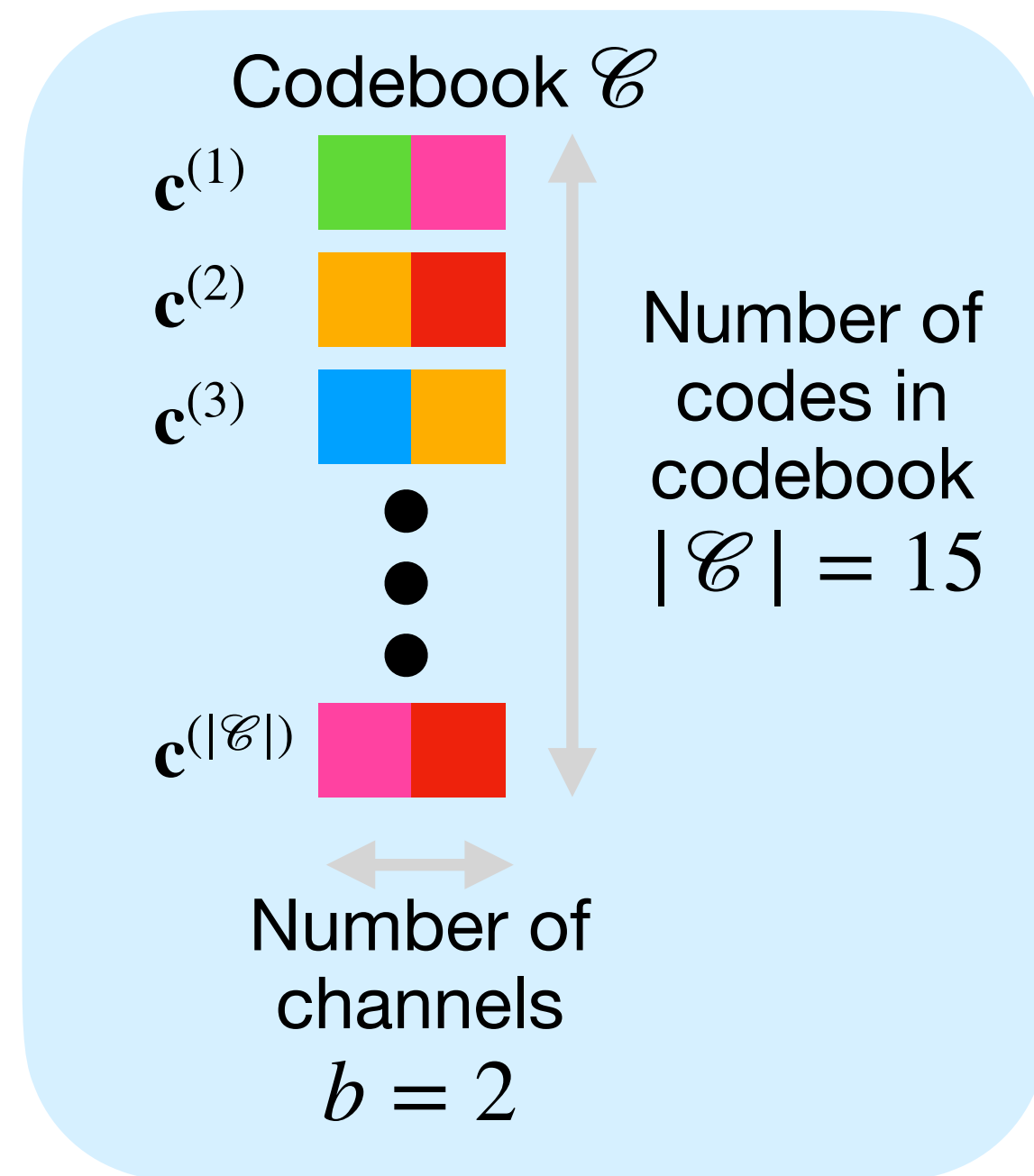
## Combination of Discrete Codebook and Stochastic Dynamics





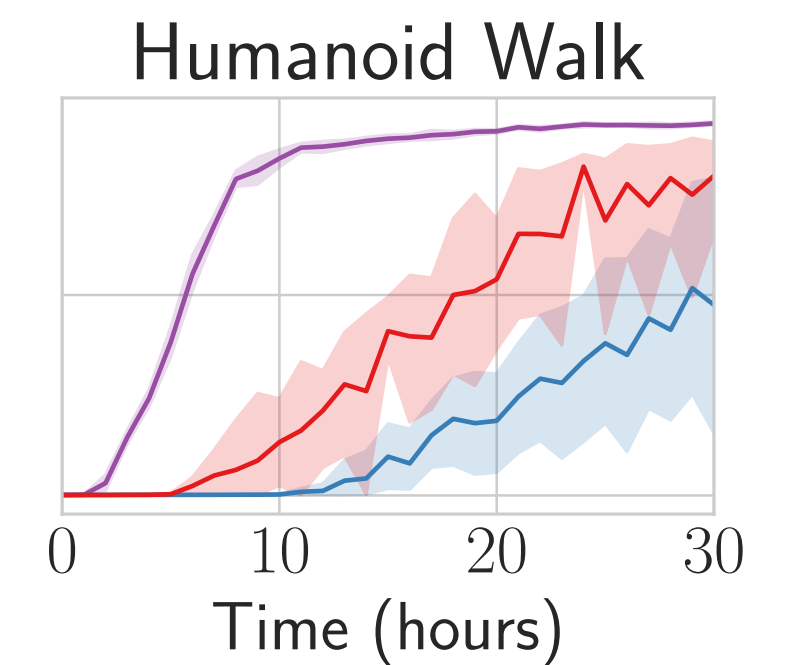
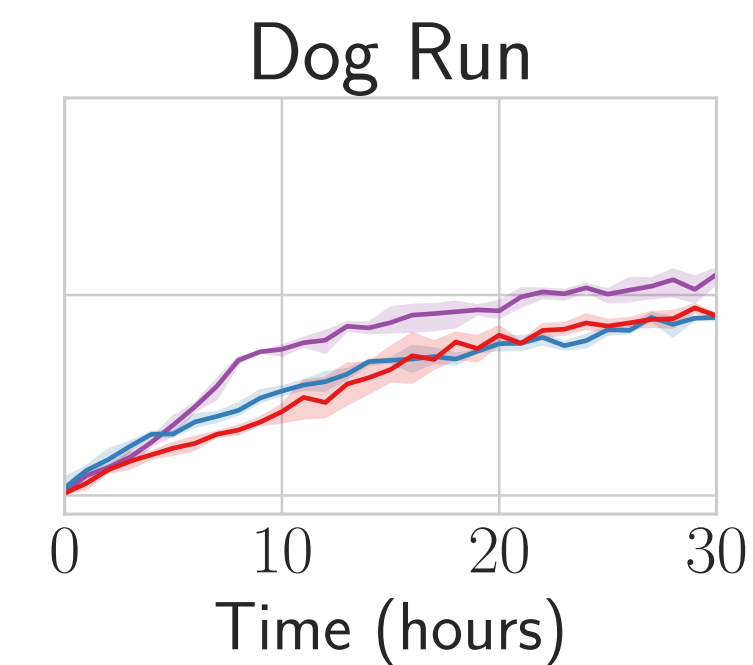
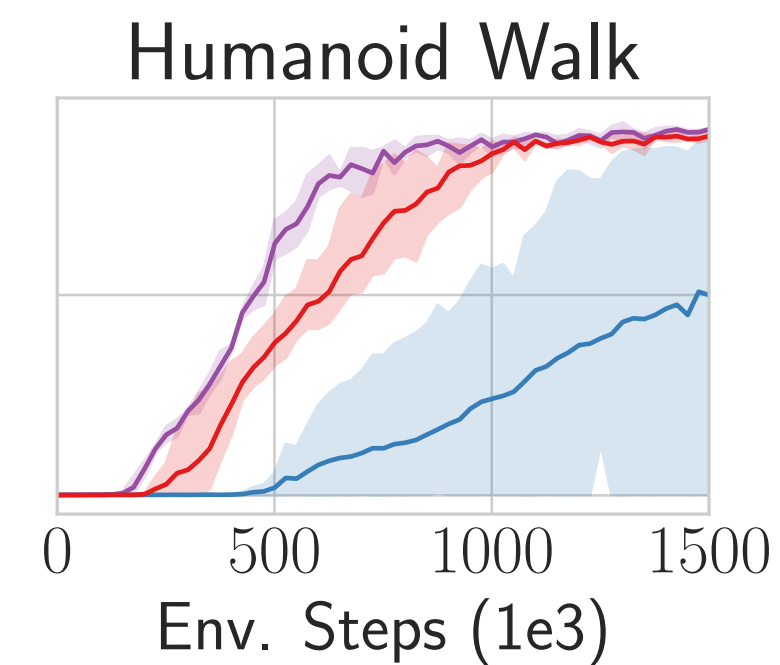
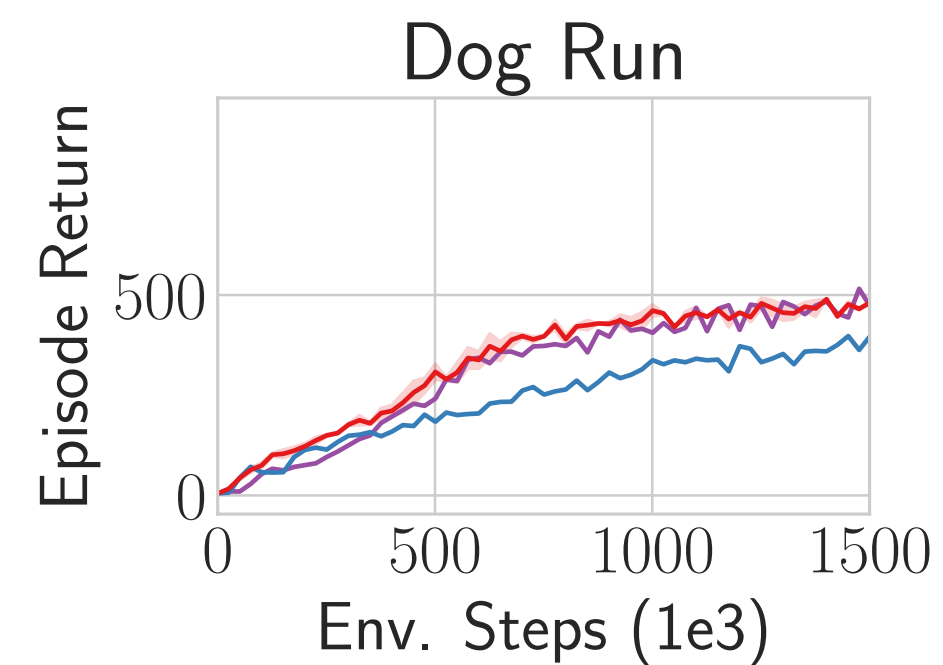
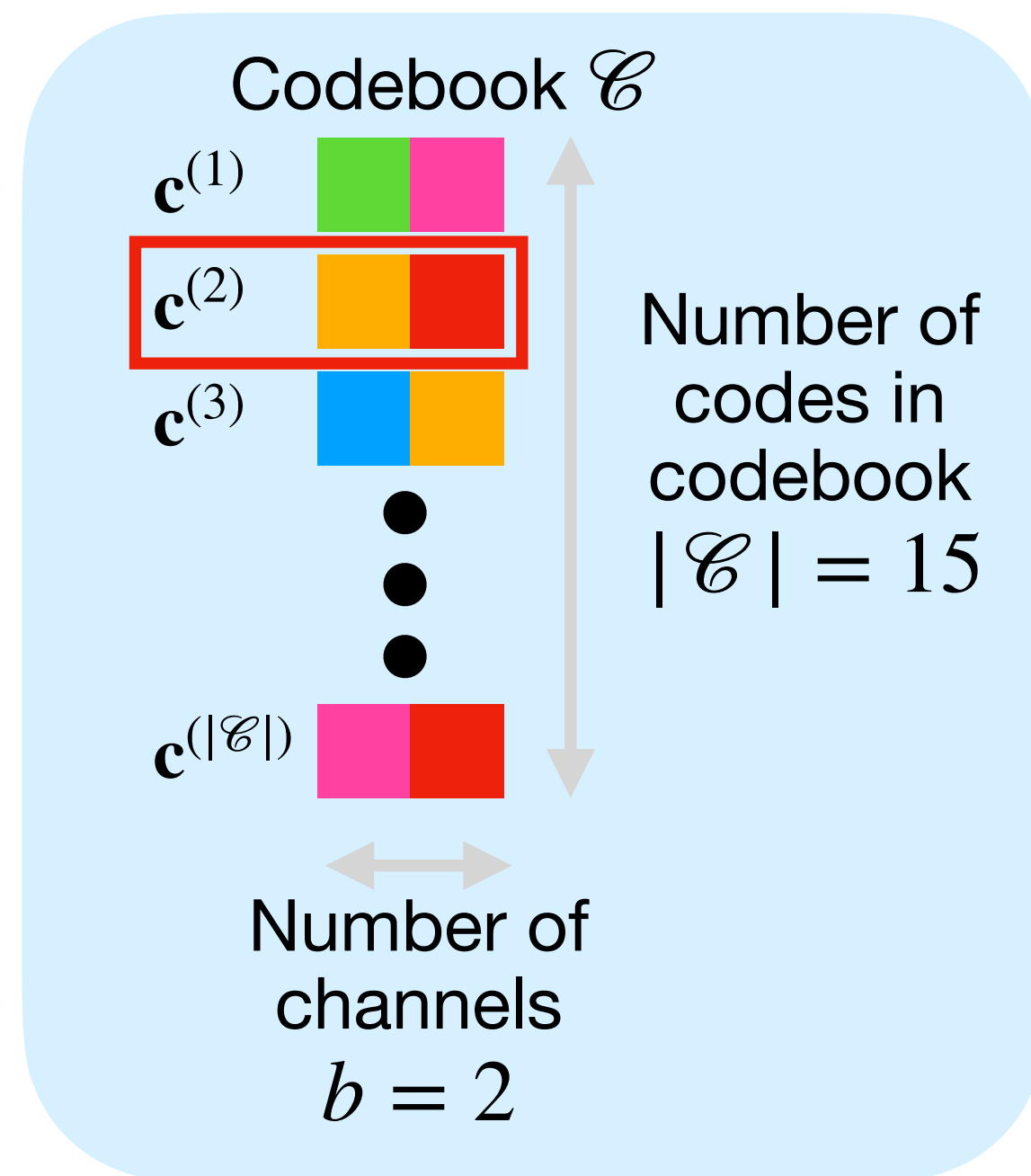
# Comparison of Different Discrete Encodings

Codebook > One-hot > Label



# Comparison of Different Discrete Encodings

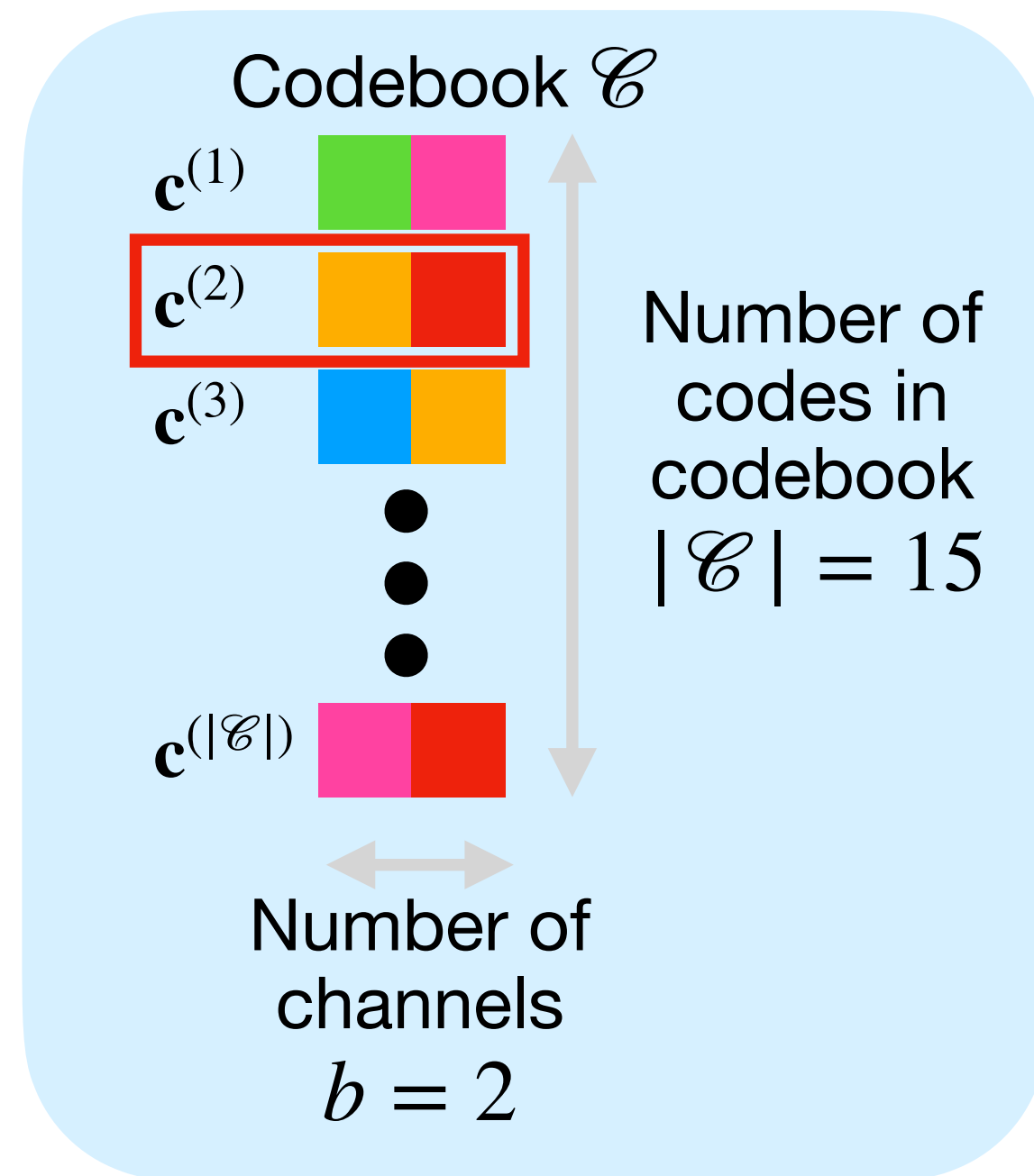
Codebook > One-hot > Label



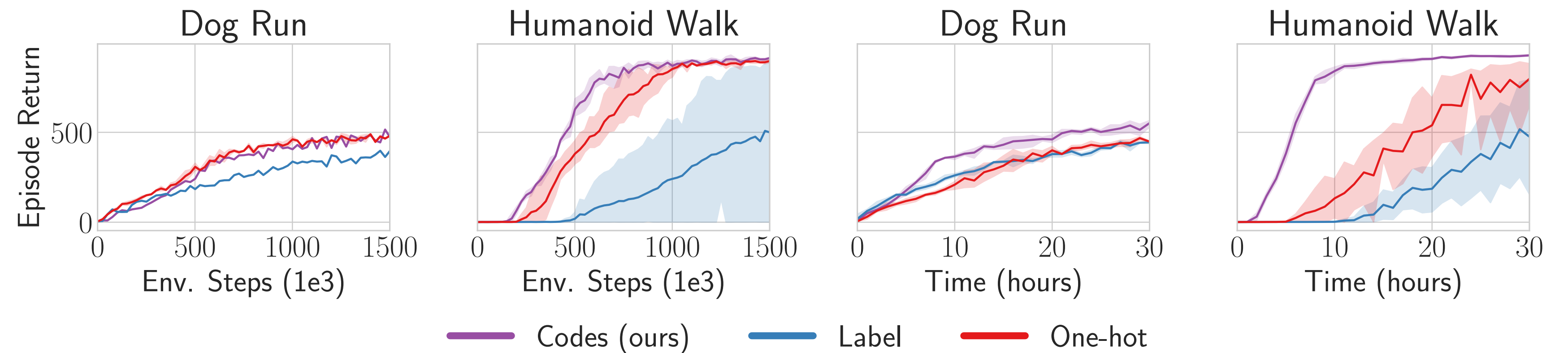
— Codes (ours) — Label — One-hot

# Comparison of Different Discrete Encodings

Codebook > One-hot > Label

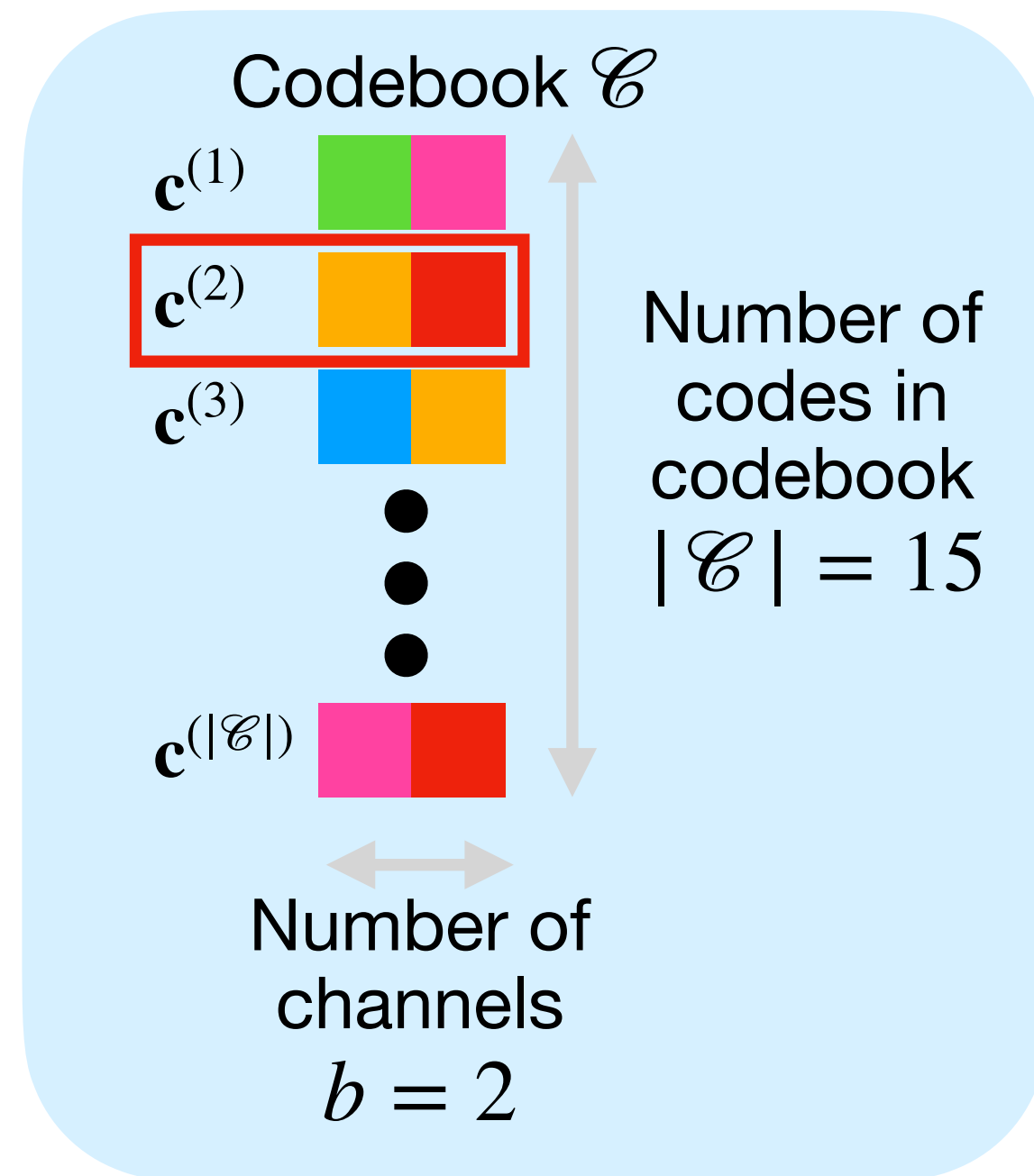


$$\mathbf{e}_{\text{code}} = \mathbf{c}^{(2)} = \{-0.5, 1\}$$



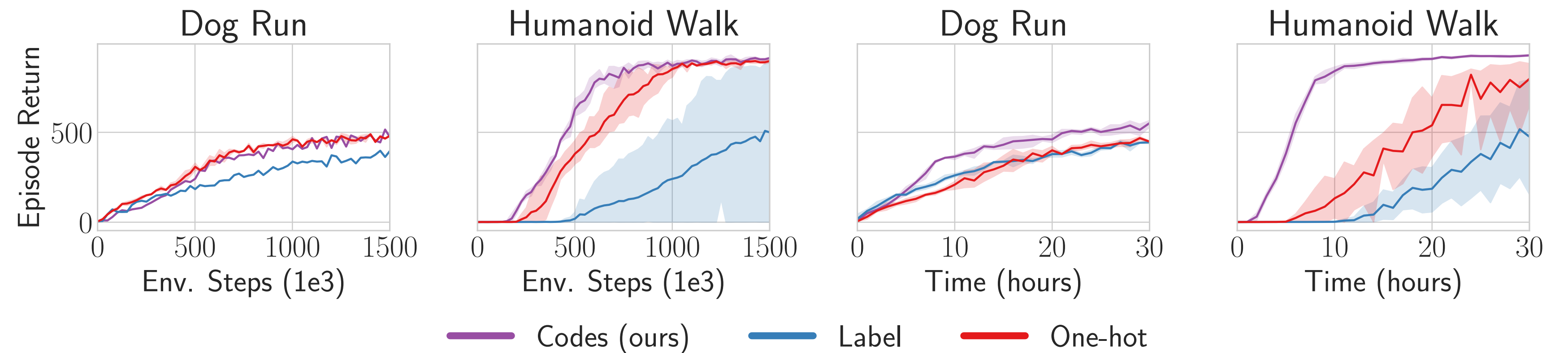
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Codebook > One-hot > Label



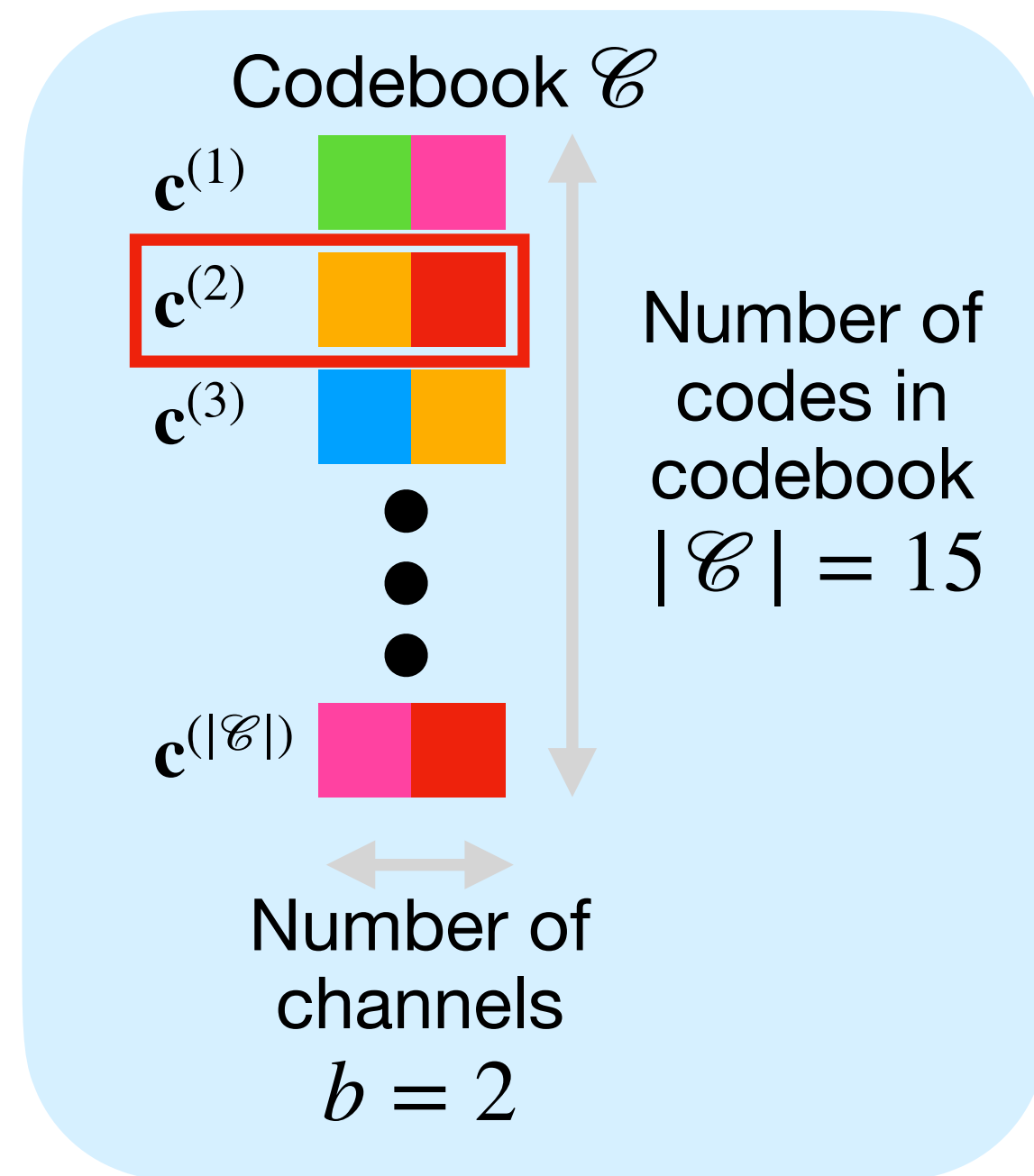
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$$\mathbf{e}_{\text{label}} = 2$$



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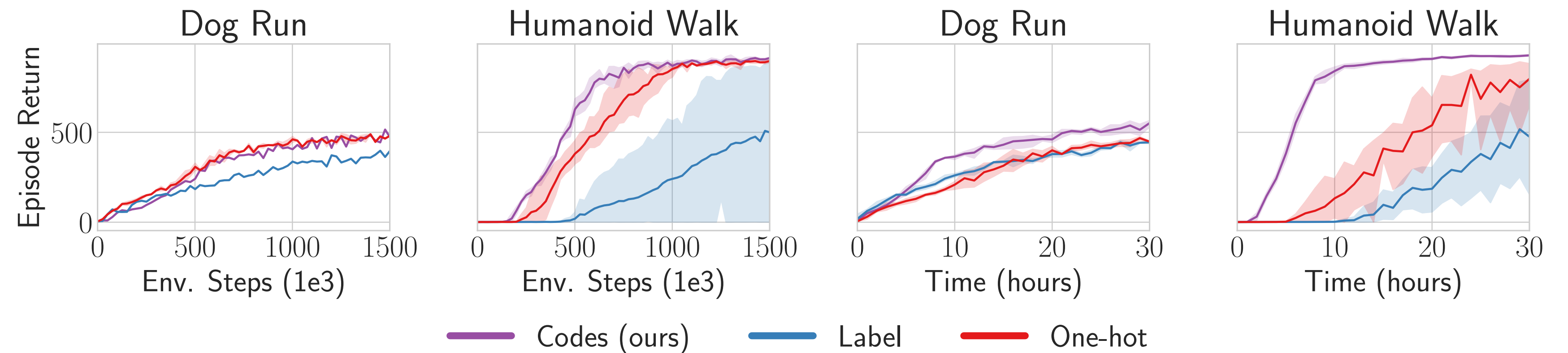
Codebook > One-hot > Label



$$\mathbf{e}_{\text{code}} = \mathbf{c}^{(2)} = \{-0.5, 1\}$$

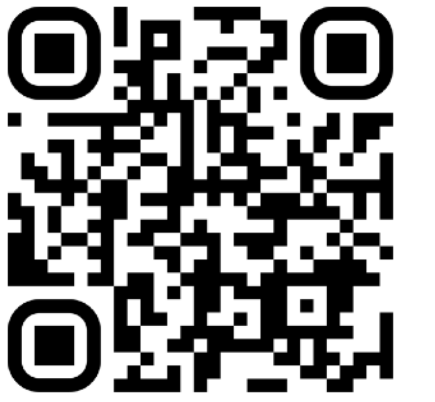
$$\mathbf{e}_{\text{label}} = 2$$

$$\mathbf{e}_{\text{one-hot}} = \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$



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## Poster 28506:

Wednesday 23rd April 10 am - 12.30 pm (GMT+8)