Reading for Today

- Pumping Lemma
  - Sipser Section 1.4

Regular Languages – Quick Quiz

- True or false?
  - All finite languages are regular
  - All infinite languages are not regular
  - The language {a, b}* is regular
  - The language of palindromes over {a, b} is regular
  - The language $\{ a^n b^n \mid \text{for some } n \in N\}$ is regular

Non-Regular Languages

- Some languages would require a DFA with unbounded memory to recognize them, which is not possible
- DFAs (and NFAs) can only keep track of their state, and do not have any other "memory" – and they can only have a finite number of states

Key Insights

- Suppose we have a DFA D with p states
- If w is a string accepted by D with length $\geq$ p, then one of the states on its acceptance path must be visited more than once -- hence, a cycle
- Thus, we can generate an infinite number of strings that would be accepted by D, by repeating that same cycle any number of times

Pumping Lemma (for Regular Languages)

- If L is a regular language, then there exists a p for the language such that for any $w \in L$ where $|w| \geq p$, we can divide w into parts xyz where:

1. $| xy | \leq p$
2. y is not the empty string
3. $xy^iz \in L$ for every i = 0, 1, 2, 3,…

Uses of the Pumping Lemma

- The Pumping Lemma is a tool that can be used to show that some languages are <u>not regular</u>
- However, it *cannot* be used to show that a language is regular

Uses of the Pumping Lemma

- Note that the Pumping Lemma says that

$$\text{Regular} \Rightarrow \text{Pumpable}$$

and so

$$\boxed{\text{Not Pumpable} \Rightarrow \text{Non-Regular}}$$

but this does *not* mean

This is how we use the lemma in practice

$$\text{Pumpable} \Rightarrow \text{Regular}$$

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$$\cancel{\text{Pumpable} \Rightarrow \text{Regular}}$$

Steps to prove L is <u>not</u> regular BWOC:

1. Assume L is pumpable with pumping length p
   - You don't get to choose p, you only know that such a p exists
2. Find some string $w \in L$ longer than p
3. Show that no matter how we break up w into xyz (with restrictions $|xy| \leq p$, and y nonempty), that there is some i where $xy^iz \notin L$
   - You don't get to choose how to break up w
4. Thus, L cannot be pumpable, and thus cannot be regular

Example Proof: $a^n b^n$

- Suppose $L = \{a^n b^n \mid n \in N\}$ is pumpable with pumping length p
- Let $w = a^p b^p$, which is in L
- However, if we divide up $a^p b^p$ into xyz with $|xy| \leq p$ and y nonempty, then:
  - x and y only contains a's
  - z may contain some a's, but contains all the b's
- Thus $xy^{42}z$ should also be in L, but it cannot be since it would contain more a's than b's
- Thus, L cannot be pumpable, and thus cannot be regular

In-Class Exercises

- Use the Pumping Lemma to show that the following languages are not regular:
  - Words over {a, b} than contain more a's than b's
  - Palindromes over {a, b}
  - $\{ a^k \mid k = 2^n \text{ for some } n \in N\}$