Regular Languages

- Recall that a language that is recognized by a DFA (or NFA, or RegExp) is called a regular language

Closure Properties

- If L_1 and L_2 are regular languages, then the following are also regular languages:
  - $L_1 \cup L_2$
  - $L_1 \cap L_2$
  - $\Sigma^* - L_1$ (the complement, $\overline{L_1}$)
  - $L_1 \circ L_2$ (concatenation)

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How would you go about proving these?

Constructing DFAs for $\cup$ and $\cap$

- Suppose we have DFAs D_1 and D_2 for regular languages R_1 and R_2
- Construct DFAs for the languages $R_1 \cup R_2$ and $R_1 \cap R_2$
- General idea: Construct a new DFA with states that are labeled with the Cartesian product of the states from D_1 and D_2
- Transitions now define where we would go next in both D_1 and D_2 for a given character
- Final states: For $\cup$, $<q_k, q_j>$ is a final state if either q_k or q_j are final; for $\cap$, $<q_k, q_j>$ is final if both are final

In-Class Exercise

- Construct DFAs for $R_1 \cup R_2$ and $R_1 \cap R_2$ where:
- R_1 = strings in $\{a, b\}^*$ that contain 1 mod 2 b's
- R_2 = strings in $\{a, b\}^*$ that end with ab

Non-Regular Languages

- There are some languages that are not regular
- Example: $\{ a^n b^n \mid n \in N\}$
- Try creating an NFA for this language to intuitively see why it is not regular (we will show how to prove it next time)

Quick Quiz: True or False?

- If R is a regular language, then $R \cup \{a^n b^n\}$ must be non-regular
- If R is a regular language, then $R \cap \{a^n b^n\}$ must be non-regular
- If F is a finite language, then it must be regular
- If B is an infinite language, then it must be non-regular
- If N is a non-regular language, then N must be non-regular
- If N_1 and N_2 are non-regular languages, then $N_1 \cup N_2$ must be non-regular
- If N_1 and N_2 are non-regular languages, then $N_1 \cap N_2$ must be non-regular