Homework 1

Please bring your work in hard copy at the beginning of class on 29^{th} August, Wednesday.

1. Observe that the while loop of lines 4-7 of the INSERTION-SORT procedure given below uses a linear search to scan (backward) through the sorted subarray A[1..j-1]. Can we use a binary search instead to improve the overall worst-case running time of insertion sort to $\theta(nlgn)$?

Algorithm 1 INSERTION-SORT

```
1: for j=2 do A.length
2:     key = A[j]
3:     i = j − 1
4:     while i > 0 and A[i] > key do
5:         A[ i + 1] = A[ i ]
6:         i = i − 1
7:     end while
8:     A[i + 1] = key
9: end for
```

2. Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of θ-notation, prove that $max(f(n), g(n)) = \theta(f(n) + g(n))$

3. Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

4. Use the substituion method to show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.

5. Use the recursion-tree method for $T(n) = 4T(n/2) + cn$, where c is a constant, to give a tight asymptotic bound on its solution. Use the substitution method to verify your bound.