Readings from Sipser

- Section 4.2, Subsection:

  "A Turing Unrecognizable Language"

From Last Time…

- So far, we've seen languages that are:
  - Decidable
  - Undecidable, but still recognizable (A_{TM})
- Can we find a language that is not even recognizable?

Some Definitions

- The complement of a language A is the set of all finite strings over the given alphabet *not* in A
  - Notation: $\overline{A}$
- A language A is co-recognizable if its complement $\overline{A}$ is recognizable

A Simple Theorem

- If a language A is decidable, then its complement $\overline{A}$ is also decidable
- Proof:

A Simple Theorem

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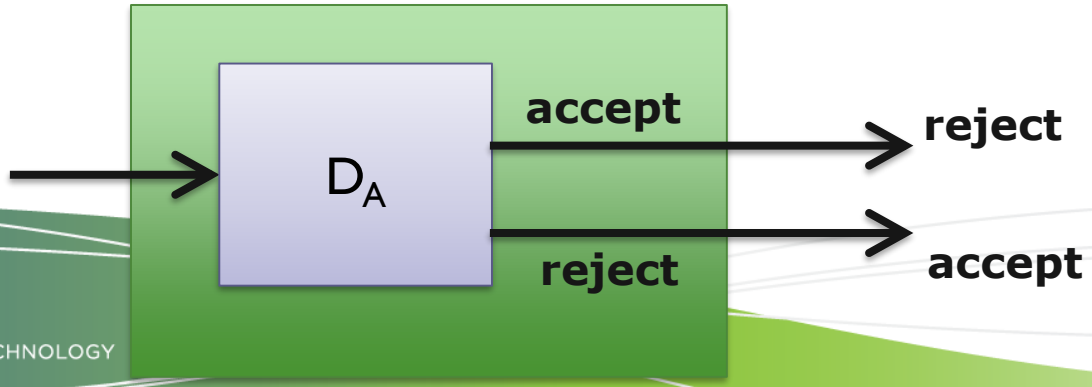
- Proof: Suppose A is decidable, and D_A is a Turing Machine that decides it. We can thus construct a decider for $\overline{A}$ as follows:



A Useful Theorem

- If a language A, and its complement $\overline{A}$ are both recognizable, then A is decidable
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A Useful Theorem

- If a language A, and its complement $\overline{A}$ are both recognizable, then A is decidable
- Proof: Suppose A and $\overline{A}$ are both recognizable, and R1 and R2 are recognizers for these languages, respectively. Construct the following Turing Machine K:

  On input w:

  1. For n = 1, 2, 3,…
     a) Run R1 on input w for n steps – if it accepts, then "accept"
     b) Run R2 on input w for n steps – if it accepts, then "reject"

A Useful Theorem (Proof Cont.)

- We know that for any input string w, it must either be in A or $\overline{A}$, and thus must be eventually accepted by either R1 or R2
- Thus K will always terminate, i.e., it is a decider

Theorem: $\overline{A_{TM}}$ is Unrecognizable

- Proof (by contradiction): We have already proven that A_{TM} is recognizable, but not decidable
- But suppose $\overline{A_{TM}}$ is recognizable
- By the previous Theorem, that would mean that A_{TM} is also decidable, which is not possible
- Thus, $\overline{A_{TM}}$ cannot be recognizable

A Typical Question

- Which language class does the given language belong to?
  - Decidable
  - Recognizable, but not decidable
  - Unrecognizable