For this homework assignment, you may work <u>in pairs</u>. However, you may not consult other pairs, or online sources – to do so constitutes academic misconduct. Submit a hard copy of your solutions at the beginning of class (first 10 minutes) on Monday, October 29, with both of the partners name at the top of the first page. Make sure that your answers are legible – if I can't read your solutions, I will just assume it is wrong.

Also, if you simply put "I don't know" as an answer to a problem, or part of a problem, you automatically get 25% of the points.

1. **(6 pts.)** Suppose we have the languages:

    L_1 = { $w \in$ {a, b}* | w starts with the character b, and ends with the character a }

    L_2 = { $w \in$ {a, b}* | w contains the substring ab }.

    Create DFAs that recognize the languages:

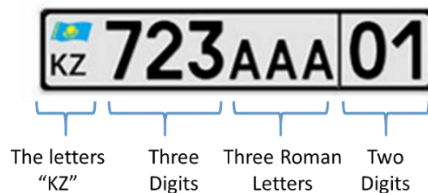   a) $\overline{L_2}$

    b) $L_1 \cap L_2$

    c) $L_1 \cup L_2$

2. **(4 pts.)** Create NFAs that recognize exactly the same sequences of characters as the following regular expressions:
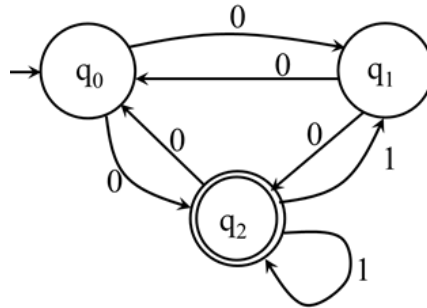
    a) a* (b | c) d*

    b) v (x* | (y z))* w

3. **(2 pts.)** Write a regular expression for representing legal license plate codes for private vehicles in Kazakhstan following the given criteria:

| The letters
"KZ" | Three
Digits | Three Roman
Letters | Two
Digits |

4. **(2 pts.)** Using the process described in class, convert the following NFA into a DFA:

5. **(12 pts.)** Determine whether the following languages are regular or non-regular. If regular, provide a DFA, NFA, or a regular expression for the language. If non-regular, prove it using the pumping lemma or closure properties for regular languages.

a) $\{a^{3n} \mid n \geq 1\}$

b) $\{a^n b^{3n} \mid n \geq 1\}$

c) $\{a^i b^j c^k \mid 1 \leq i \leq j \leq k \leq \min(99, j^2)\}$

d) $\{a^n \mid n \text{ is a prime number}\}$