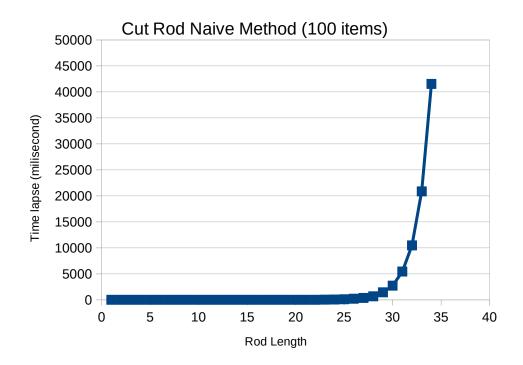
HW4 Aida Sharif Rohani

Array for the 100 elements part:

[4, 14, 20, 15, 6, 2, 11, 6, 11, 9, 17, 16, 13, 11, 1, 17, 20, 18, 11, 14, 8, 14, 18, 13, 9, 13, 10, 9, 4, 20, 12, 3, 10, 9, 1, 13, 9, 19, 11, 5, 1, 7, 16, 13, 20, 17, 8, 11, 5, 3, 14, 16, 8, 10, 3, 4, 3, 8, 15, 13, 9, 9, 18, 3, 19, 18, 6, 16, 4, 15, 15, 4, 3, 9, 4, 17, 8, 5, 2, 10, 4, 12, 11, 10, 10, 5, 11, 12, 18, 20, 19, 14, 14, 9, 20, 2, 8, 1, 16, 9]

## Result:

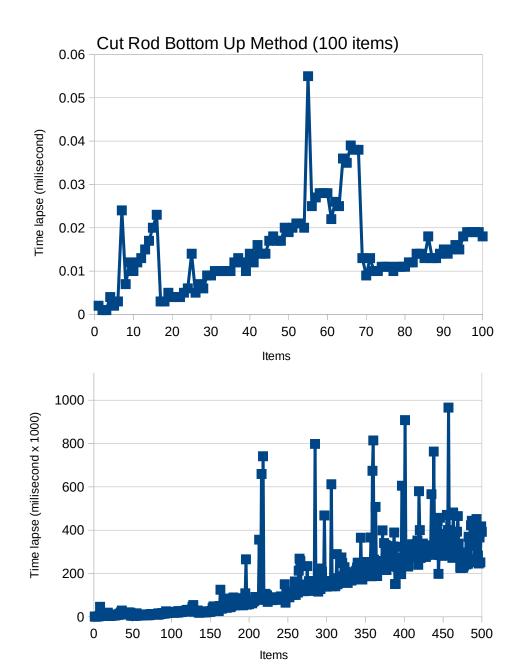
1- The maximum length calculated was 34. As the length increases the number of calculations exponentially increases. The shape of the graph is exponential, it means we can't calculate large array of elements with this method and it's very expensive and lengthy time wise.

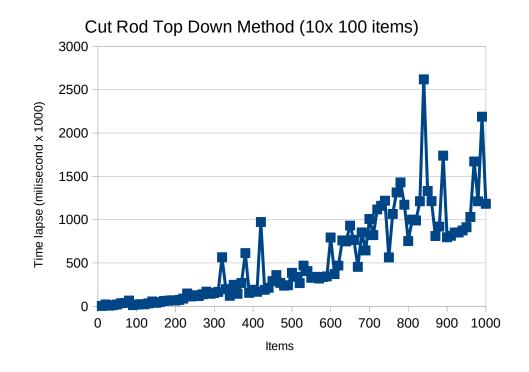


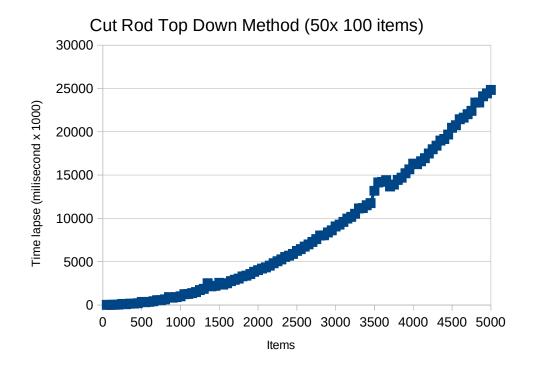
2- In top down in contrast to naive method all the numbers are calculated in less than 0.06 milliseconds for 100 elements. It shows that memoization makes it much faster to do the calculations as each number is only calculated once and after that it's saved in an array. The plot is not very smooth but the higher than average points are just noises and it's normal in order of milliseconds.

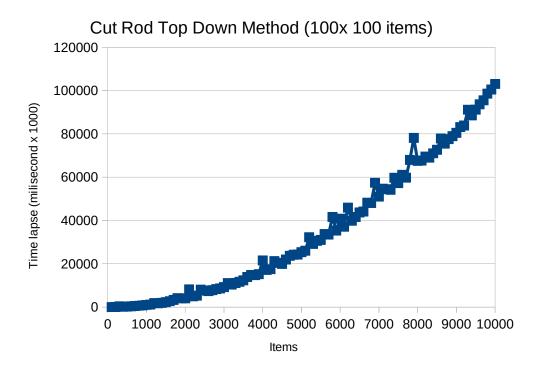
The higher number of elements makes the graph smoother and the time is increasing in  $n^2$ . For example when doing for 100x times the time increases by 10000 times.  $N^2$  behaviour compared to  $2^n$  in naive method implies that we can calculate much larger arrays compared to naive method. It takes more time for every increase in multiplication factor to finish the array. And the integral of the time is proportional to  $n^2$ . In  $n^2$  the shape of the plot is more like a line but when it gets to  $n^2$  and  $n^2$  we'll more clearly see the  $n^2$  behaviour. For  $n^2$  and it took several hours in my computer to calculate a few numbers. So I could not finish for  $n^2$  it shows that even bottom up and top down has its own limitations when it gets to greater array sizes.

One observation was that top down has more noises than bottom up which could be due to the different cashing techniques used by each method.









3-In bottom up method we get the same results as top down; again all the numbers are calculated in less than 60 milliseconds. It shows that bottom up methods makes it much faster to do the calculations as each number is only calculated once and after that it's saved in an array. The plot is not very smooth but the higher than average points are just noises and it's normal in order of milliseconds.

The higher number of elements makes the graph smoother and the time is increasing in  $n^2$ . For example when doing for 100x times the time increases by 10000 times.  $N^2$  behaviour compared to  $2^n$  in naive method implies that we can calculate much larger arrays compared to naive method. It takes more time for every increase in multiplication factor to finish the array. And the integral of the time is proportional to  $n^2$ . In  $n^2$  the shape of the plot is more like a line but when it gets to  $n^2$  and  $n^2$  we'll more clearly see the  $n^2$  behaviour. For  $n^2$  and it took several hours in my computer to calculate a few numbers. So I could not finish for  $n^2$  it shows that even bottom up and top down has its own limitations when it gets to greater array sizes.

