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СРАВНЕНИЕ МЕХАНИЗМОВ ПО МАНИПУЛИРУЕМОСТИ В УСЛОВИЯХ ЧАСТИЧНОЙ ИНФОРМАЦИИ

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PREFERENCE-INCLUSION COMPARISON UNDER PARTIAL INFORMATION

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CONTENTS

Abstract: This paper examines the incentive properties of constrained versions of Deferred Acceptance (DA) and Boston (BM) mechanisms under partial information. Given partial information, each student knows only some part of the preferences of other students but this information can be enough for students to analyze possible assignments. We introduce a new criterion of comparison of mechanisms which is based on comparing obvious truthfulness of the mechanisms under partial information. We call a mechanism obviously truthful when for each student partial information is enough to see that the worst possible assignment under truthful report is no worse than the best possible assignment under any manipulation. Fixing partial information, the results show that constrained DA is obviously truthful in a larger set of profiles than constrained BM and that both mechanisms are obviously truthful in a larger set of profiles when the constraint increases. Without fixing partial information, we show that both constrained DA and constrained BM are obviously truthful in a larger set of profiles when there is more information available.

Keywords: obvious truthfulness, manipulability, partial information, constrained mechanisms, Deferred Acceptance mechanism, Boston mechanism.

JEL classification: C78, D47, D82.

INTRODUCTION

There is a general agreement in the theoretical and experimental mechanism design literature on the superiority of adoption of Deferred Acceptance over the Boston mechanisms (Abdulkadiroğlu and Sönmez, 2003; Chen and Sönmez, 2006; Pathak and Sönmez, 2008). Even when DA is constrained and thus not strategy-proof it has better incentive properties than BM by multiple criteria: less manipulable (Pathak and Sönmez, 2013), more immune (Bonkoungou and Nesterov, 2020) and more truthful (Decerf and Van der Linden, 2016).

However, all the above results are based on the assumption of complete information. Namely, students are assumed to know everything about the preferences of each other. In reality, agents have only partial information about other students' preferences, for example, the most popular schools in a district or most preferred schools of neighbors.

In this paper, we intend to relax the assumption of complete information and provide the results on comparing mechanisms under partial information. The main question is when under partial information conditions agents begin to report their true preferences more often: under which mechanism and under which information.

To answer the question we use the **row information structure**. In this case, partial information stands for the number of top rows in each student's preference list which is publicly available. Consider the following illustrative *Example 1* presented where the minimum information is sufficient for truthful report. Let there be three students: s_1, s_2 and s_3 with the left panel representing students' preferences, and let there be three schools c_1, c_2 and c_3, with the right panel representing schools' priorities.

$$\begin{array}{ccc} R_{s_1} & R_{s_2} & R_{s_3} \\ \hline c_1 & c_2 & c_3 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{array} \qquad \begin{array}{ccc} R_{c_1} & R_{c_2} & R_{c_3} \\ \hline s_1 & s_2 & s_3 \\ s_2 & s_1 & s_1 \\ s_3 & s_3 & s_2 \end{array} \tag{1}$$

Students only know the most preferred school for each student (the first grey row in the *preference profile*[1]). The other rows are unknown but each student knows his own preferences and schools' priorities. That is what we call *partial information structure* $I = 1$ later in this paper. Holding the

[1] All the definitions are provided in Section 2

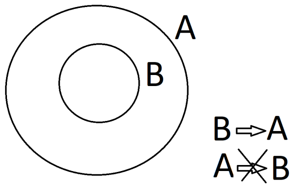

Figure 1: Preference-inclusion comparison

information about the first row, each student analyzes the possible assignments assuming various *completions* – information about the rest rows in the preferences of other students.

According to the first row in the preference profile the assignment under BM is uniquely determined for any such so-called *completion*: s_1 will be assigned to c_1, s_2 to c_2 and s_3 to c_3 because there are no two or more students applying to the same school. Furthermore, each student gets assigned to his most preferred school. Thus, each of the students cannot improve their school by misreport for any possible *completion*.

Therefore, each student in the *Example 1* has an **obviously truthful strategy** in BM: the worst assignment under truthful report about preferences for any *completion* is no worse than the best assignment under any misreport for any *completion*. This truthful strategy is *obvious* in the spirit of Li (2017): the set of outcomes under truth-telling dominates the set of outcomes under misreports and no incentive to misreport is provided.

Moreover, according to the first row in the preference profile, the assignments in DA are also uniquely determined and coincide with the assignments in BM. This is due to each student is assigned to the school where he has *top-priority*. Then, from the *Example 1*, it is seen that for such preference and priority profiles both DA and BM are **obviously truthful mechanisms**.

Nevertheless, to compare the mechanisms by truthfulness we need to know whether whenever BM is obviously truthful then DA is obviously truthful in the sense of Arribillaga and Massó (2015). Suppose that BM is obviously truthful under profiles in set B and DA is obviously truthful under profiles in set A. Consider the sets A and B from *Figure 1*. We can say that DA is **more obviously truthful** than BM because whenever BM is obviously truthful then DA is also obviously truthful but the converse is not true.

The results under *fixed information structure* lie in line with the conventional literature: constrained DA is more obviously truthful than constrained BM and the longer the constraint, the more truthful both constrained DA and constrained BM. The results on comparison under *different information structure* state that the more information is available in both constrained DA and constrained BM, the more truthful the students are.

The organization of the paper is as follows. Section 1 reviews papers on analyzing and comparing different mechanisms. Section 2 introduces the description of the school choice model and mechanisms. Section 3 provides preliminaries on obviously truthful strategies. Section 4 describes the main results on comparing mechanisms under fixed and under different information. In Conclusion we sum up the findings and discuss remaining open questions.

1 LITERATURE BACKGROUND

For decades, school choice has been a widely discussed topic in market design research (Pathak, 2011; Abdulkadiroglu, 2013; Kojima, 2017; Hakimov and Kübler, 2019). A school choice problem in literature is described as a set of students and a set of schools with fixed capacities which should be matched through a particular mechanism. The mechanism in turn describes how reported preferences of students over schools and priorities of schools over students translate into a matching outcome. The goal of the mechanism designer is to construct a mechanism that satisfies key properties - efficiency, fairness, simplicity, stability and incentives.

Despite the theoretical and experimental superiority of adoption of student-optimal stable Deferred Acceptance or the Top Trading Cycle (TTC) mechanism, in real-life school choice mechanisms are typically constrained. That means that students are allowed to report preferences on a limited number of schools only (Haeringer and Klijn, 2009). Two mentioned celebrated mechanism DA and TTC are strategy-proof in unconstrained versions. However, impeding students to submit a constrained preference list is likely to distort the desirable strategy-proofness property of the mechanisms.

Although such transformations of well-established mechanisms are prevalent in practice, the literature on the constrained versions of DA and BM is relatively thin. Haeringer and Klijn (2009), who study the efficiency and stability properties of the Nash equilibria of constrained school choice mechanisms, indicated the effects of constraints on the strategic behavior of students. They showed that the BM is robust in the sense that stability is guaranteed in equilibrium notwithstanding the length of preference lists after impending constraints. The remaining mechanisms, which have desirable properties in the unconstrained case, do not perform as well: both mechanisms allow for equilibria in undominated strategies that induce unstable outcomes. Chen and Sönmez (2004) also analyze the effect of constraints. The authors experimentally show that the presence of constraints has large negative effects on the manipulability of the mechanisms since the existence of a dominant strategy to truth-telling disappears. The introduction of constraints is also shown by Chen and Sönmez (2004) to reduce efficiency and stability of mechanisms. Calsamiglia et al. (2010) study constrained mechanisms experimentally replicating the design of Chen and Sönmez (2004) while adding a treatment where mechanisms are constrained and receive similar results.

In this paper, the focus is on the manipulability of constrained school choice mechanisms. The closest papers to mine but in the context of complete information are those by Pathak and Sönmez (2013) and Decerf and Van der Linden (2016). The former compares the manipulability of constrained school choice mechanisms from the perspective of truthful Nash equilibria. Among applications the authors show that the recent changes of adopted mechanisms (increasing constraints, moving from BM to DA) involve abandoning more manipulable mechanisms. The latter uses another methodology with a focus on dominant strategies. This comparison criterion has limitations and the analysis is viewed as complementary to the first work. An advantage of focusing on dominant strategies is that they provide students with clear incentives that are independent of any beliefs about other students' reported preferences. The cost of focusing on dominant strategies is the disregard for the effect of choosing one mechanism over another on the incentives of students who have dominant strategies in neither mechanism. Nonetheless, Pathak and Sönmez (2013) methodology disregard the difficulty for students to coordinate on equilibria. The papers provided two independent parallel results: particularly, they prove that both BM and DA are more manipulable when the constraint allows to report less schools and that constrained BM is more manipulable than constrained DA with the same constraint.

However, one more real-life shortcoming of theoretically analyzing properties of the mechanism is the fact that the majority of literature considers only the cases of complete information. This assumption means that students do not only know their own preferences but also the preferences of all the other students which is not the case in practice. Only recently researchers have turned their attention to weakening the information assumption. In an experimented study by Chen and Sönmez (2006) authors compared Boston, DA and TTC in the complete information and random information environment. Another experimented study by Pais et al. (2011) also compared the manipulability of DA, Boston and TTC in different information environments: zero information, partial and complete.

Later analytical results explaining the findings in experiments on comparing mechanisms under different information structures were obtained. Bonkoungou and Nesterov (2019) proposed a useful framework of partial information. They assume that each student is certain about some aspects of the preferences of other students and completely uncertain about the rest aspects. For example, each student knows only a few top rows of the preference profile. Their results suggest that constrained DA is less manipulable under partial information than constrained BM, and a longer list makes constrained DA less manipulable which lies in line with the complete information results. Comparing mechanisms under different information they showed that constrained DA is becoming more manipulable with more information but constrained BM is becoming less manipulable with more information. Their definition of "obvious manipulations" is the exact counterpart of the definition of "obviously strategy-proof" mechanisms (Li, 2017). Troyan and Morrill (2020) considered zero information environment and also defined obvious manipulations differently.

Compared to the previous results under partial information, this paper introduces several innovations. In contrast to formulation of the criterion as an "obviously manipulable" (Bonkoungou and Nesterov, 2019), we introduce the criterion as "obviously truthful". Thus, such a new criterion of obvious truthfulness assumes that even risky manipulations can provide an incentive for a student to manipulate whereas the criterion of obvious manipulation assumes that only safe manipulations can provide students with an incentive to manipulate. This way, the criterion of Bonkoungou and Nesterov (2019) is more conservative compared to the criterion in this paper. Furthermore, this paper will be the first to our knowledge to analyze the manipulability of a mechanism from the perspective of a particular student under partial information. Previously, the criteria were formulated to the problem as a whole: whether under the given preference profile and priority profile the mechanism is manipulable. Now we analyze whether the mechanism is manipulable from the perspective of each particular student and if the mechanism is truthful for each particular student we say that the mechanism is truthful. Thus, in this paper we intend to reinforce the results under partial information obtained so far and to fill in the gap in the literature on comparing school choice mechanisms.

2 THE SCHOOL CHOICE MODEL AND MECHANISMS

There is a finite set of m schools $C := \{c_1, ..., c_m\}$ and a finite set of n students $S := \{s_1, ..., s_n\}$. Every student $s \in S$ has a **preference relation** R_s denoting a linear ordering on $C \cup \{s\}$. The domain of all preferences for s is $\mathcal{R}_s$. A **preference profile** $R_S := (R_{s_1}, ..., R_{s_n})$ is a list containing the preference of every student $s \in S$. For a given preference profile R_S, the list containing the preferences of everyone but s is R_{-s}. A strict preference of s for school c over school c' is denoted by $c\ P_s\ c'$. A school c is **acceptable** for s if $c\ R_s\ s$. To avoid trivialities, we assume that every student has at least one acceptable school. $R_s(l)$ stands for l'th preferred school of s.

Every school $c \in C$ has a capacity q_c of natural numbers denoting schools' capacities and a **priority relation** R_c denoting a linear ordering on S. A **profile of priorities** $R_C := (R_{c_1}, ..., R_{c_m})$ is a list containing the priorities of every $c \in C$ and the domain of all priority profiles is $\mathcal{R}_C$. School c is a **top-priority** school for student s if no more than $q_c - 1$ students have a higher priority at c than s.

A **matching** is a function $\mu : C \cup S \to C \cup S$ that matches every student with a school or with himself such that:
1. $\mu(s) = c$ if and only if $s \in \mu(c)$;
2. $\mu(s) = s$ if $\mu(s) \notin C$ and then s is **unassigned** in μ;
3. $|\mu(s)| = 1$ for every student s if $\mu(s) \in C$;
4. $|\mu(c)| \leq q_c$ for every school c.

A **school choice mechanism** M associates every profile of reported preferences $R_S := (R_{s_1}, ..., R_{s_n})$ in some domain $\mathcal{R}_S := \times_{s \in S} \mathcal{R}_s$ with an assignment μ. We define **constrained mechanism** M^k for each mechanism M as follows: $M^k(R_S) = M(R_S^k)$. In a constrained version M^k the domain is $\mathcal{R}_S^k := \times_{s \in S} \mathcal{R}_s^k$ where for every $s \in S$ the set of all reported preferences is R_s^k in which s reports no more than $k \leq m$ schools.

We focus on a **row information structure** I (Bonkoungou and Nesterov, 2019). If each student knows his own preferences R_s and priority profile R_C but only the top $I \geq 0$ rows of the preference profile R_{-s}, then the information is denoted as I. The domain of preferences which are consistent with (c.w.) information structure I is $\mathcal{R}^I_{-s}$. We denote the first I rows preference profile by R^I_{-s} and refer to each $\hat{R}_{-s} c.w. R^I_{-s}$ as to **completion**. Each completion $\hat{R}_{-s} c.w. R^I_{-s}$ coincide with R^I_{-s} on the first I rows but the domain of each $\hat{R}_{-s} c.w. R^I_{-s}$ is $\mathcal{R}^k_{-s}$.

2.1 Obvious truthfulness

As a criterion of truthfulness we use obvious dominance in the spirit of Li (2017). In order to have an incentive for being truthful under given information structure an agent needs to know whether the set of outcomes under truth-telling dominates the set of outcomes for any misreport.

Definition 2.1. Given information structure I and mechanism M, for student $s \in S$ preference R_s is **obviously truthful under information structure *I*** if for any $R^{'}_s \in \mathcal{R}_s$:

$$\min_{R_{-s} c.w. R^I_{-s}} M_s(R_s, R_{-s}) \; R_s \max_{R_{-s} c.w. R^I_{-s}} M_s(R^{'}_s, R_{-s}).$$

Thus, note that we also can say that student s has an obviously truthful strategy in M^k under I if truth-telling is an **obviously dominant strategy** for s in M^k under I.

Based on *Definition 2.1* we now define how a student compares mechanisms under fixed and under different information structures employing the standard profile-inclusion comparison technique (Arribillaga and Massó, 2015).

Definition 2.2. Given information structure I, for student $s \in S$ mechanism M is **more obviously truthful** than $M^{'}$ under I if:
(i) whenever R_s is obviously truthful under information structure I for s in M' then R_s is obviously truthful under information structure I for s in M;
(ii) there is at least one completion and misreport where R_s is obviously truthful under information structure I for s in M but not in $M^{'}$.

Definition 2.3. Given information structure I and $I^{'}$, for student $s \in S$ mechanism M is **more obviously truthful under *I* than under *I*$^{'}$** if:
(i) whenever R_s is obviously truthful under information structure $I^{'}$ for s in M then R_s is obviously truthful under information structure I for s in M;
(ii) there is at least one completion and misreport where R_s is obviously truthful under information structure I for s in M but not under $I^{'}$.

Based on *Definition 2.2* and *Definition 2.3* we now define how to compare mechanisms under fixed and under different information structures from the perspective of a given set of students.

Definition 2.4. Given information structure I, mechanism M is **more obviously truthful** than $M^{'}$ under I if M is more obviously truthful than $M^{'}$ under I for every student $s \in S$.

Definition 2.5. Given information structure I and $I^{'}$, mechanism M is **more obviously truthful under *I* than under *I*$^{'}$** if it is more obviously truthful under I than under $I^{'}$ for every student $s \in S$.

2.2 School choice mechanisms

We focus on two classes of algorithms corresponding to constrained versions of well-known DA and BM. We first describe unconstrained BM (Abdulkadiroğlu et al., 2005).

Round 1: Each student applies to the school he reported as his most-preferred acceptable school (if any). Every school rejects the students in excess of its capacity according to its priority. Each student who is not rejected is *assigned* to the school he applied to and capacities are adjusted accordingly.

⋮

Round l: Each rejected student applies to the best acceptable school that did not reject him yet. Every school rejects the students in excess of its remaining capacity according to its priority. Each student who is not rejected is *assigned* to the school he applied to and capacities are adjusted accordingly.

The Boston algorithm terminates when each student is assigned to a school or is rejected from every school he applied to. We denote the constrained version of BM by BM^k such that $BM^k(R_S) = BM(R_S^k)$.

We now describe the unconstrained version of the DA algorithm (Gale and Shapley, 1962).

Round 1: Each student applies to the school he reported as his most-preferred acceptable school (if any). Every school rejects the students in excess of its capacity according to its priority. Each student who is not rejected is *temporarily assigned* to the school he applied (could be rejected on any further step).

⋮

Round l: Each rejected student applies to the best acceptable school that did not reject him yet. Every school rejects the students in excess of its remaining capacity according to its priority. Each student who is not rejected is *temporarily assigned* to the school he applied.

The DA algorithm terminates when no student is rejected. Each student who is temporarily assigned to a school gets assigned to that school. We denote the constrained version of DA by DA^k such that $DA^k(R_S) = DA(R_S^k)$.

3 PRELIMINARIES

In this section we first define the *safe set* of schools for a student and *pre-assignment* under partial information structure. We then characterize the only cases of existence of an *obviously truthful strategy* for a student under partial information in DA^k and BM^k mechanisms.

3.1 Pre-assignment under partial information

In some cases, the given information structure I is enough to determine that student s is guaranteed to get assigned to some school $c \in C$. Some schools can form a set $\hat{C}$ reporting which s is guaranteed to get assigned to one of the schools from the set. Such set $\hat{C}$ is called *safe set* in Decerf and Van der Linden (2016).

Definition 3.1. A set $\hat{C}$ forms **safe set** for student s in mechanism M if $\hat{C} \subseteq C$ protects s from being unassigned under information structure I when report of s includes $\hat{C}$.

Consider the following profile in *Example 2* with information structure $I = 1$, where the left panel represents students' preferences and the right panel represents schools' priorities. Each school has one seat (in all examples) and "⋮" indicates that the rest of the ordering is arbitrary.

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}	
c_1	c_2	c_3	c_1	s_1	s_1	s_1	s_1	
⋮	⋮	⋮	c_2	⋮	s_2	s_2	s_2	(2)
⋮	⋮	⋮	c_3	⋮	⋮	s_3	s_3	
⋮	⋮	⋮	c_4	⋮	⋮	⋮	s_4	

Under top row information structure in M^3 (either BM^3 or DA^3), student s_1 will be assigned to school c_1 (as c_1 is both top-priority and first choice school for s_1), student s_2 will be assigned to school c_2 (as the only student s_1 with higher priority will be assigned to another school) and student s_3 will be assigned to school c_3 (as the students s_1 and s_2 with higher priority will be assigned at another schools). As a consequence, if s_4 reports c_4, he is guaranteed to get assigned to c_4.

In the *Example 2* for student s_4 the set $\hat{C}$ is singleton under $I = 1$. It is obvious that when $\hat{C}$ is a singleton then the assignment for s is uniquely determined by information structure I. Thus, the assignment for s_4 is uniquely determined under $I = 1$ if he reports c_4.

A special case is when $\hat{C}$ is not a singleton but the assignment for s is also uniquely determined by information structure I. For student s_1 from the *Example 2* there are four schools at which he has top-priority and which form $\hat{C}$ for s_1 but the assignment for s_1 is also uniquely determined. All uniquely determined assignment is what we will refer to as *pre-assignments*.

Definition 3.2. A student s is **pre-assigned** to a school c in mechanism M if c is the first school to which s is (temporarily) assigned without the threat of being rejected on any step given information I.

It is also can be the case that information structure I is enough to determine that such set $\hat{C}$ does not exist for s. That can be achieved when s is rejected from every school $c \in C$ under information structure I. Consider the following *Example 3* with $I = 1$.

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{s_5}
c_1	c_2	c_3	c_4	c_4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_2	s_3	s_4
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

(3)

Under top row information structure in M^k, student s_1 will be assigned to school c_1, student s_2 to school c_2, student s_3 to school c_3 and student s_4 to school c_4. As a consequence, all places in all schools will be occupied already on the first step of M^k and s_5 will stay unassigned. Therefore, there is no $\hat{C}$ for s_4 already under $I = 1$.

Practically, in the case of DA^k the pre-assignment of student s to school c states that there are less than q_c students with higher priority at c who was not pre-assigned to any school or who was pre-assigned to school c, in other words, who were not pre-assigned to some $c' \in C \backslash \{c\}$.

In the case of BM^k student s do not get temporary assigned to c but assigned to c or rejected from c. Therefore, if s is not rejected from at least one school under information structure I, then s is pre-assigned to that school. Whenever information structure I induces pre-assignment for student s to c in BM^k, he is guaranteed to get assigned to c if he reports c in BM^k.

3.2 Obviously truthful strategies

In some cases existence of a *safe set* $\hat{C}$ prevents s from having truth-telling as an obviously dominant strategy. Thus, this provides s with no incentive to be truthful. Consider the following *Example 4* with $I = 1$ and DA^3:

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}		R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
c_1	c_1	c_1	c_4		s_1	s_1	s_1	s_4
⋮	⋮	c_2	⋮		⋮	s_2	s_3	⋮
⋮	⋮	c_3	⋮		⋮	s_3	⋮	⋮
⋮	⋮	c_4	⋮		⋮	⋮	⋮	⋮

(4)

Student s_1 is pre-assigned to c_1, student s_4 to c_4. However, s_3 is not pre-assigned to any school although it is clear that s_3 will not stay unassigned (he will be assigned to c_2 or c_3). It is not certain to which particular school s_3 will be assigned.

Minimum possible outcome under truth-telling for s_3 (for the following completion 4.1) is equal to c_3.

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}
c_1	c_1	c_1	c_4
⋮	c_2	c_2	⋮
⋮	⋮	c_3	⋮
⋮	⋮	c_4	⋮

(4.1)

Maximum possible outcome under misreport for s_3 (for the following completion 4.2) is equal to c_2.

R_{s_1}	R_{s_2}	R'_{s_3}	R_{s_4}
c_1	c_1	c_2	c_4
⋮	c_3	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮

(4.2)

Thus, the minimum possible outcome under truth-telling is worse than the maximum possible outcome under misreport $\implies$ truth-telling can be not obviously dominant strategy under information structure I when truth-telling forms a safe set. The next proposition characterizes the only cases in which student s has an obviously truthful strategy in DA^k (all the proofs can be found in the Appendix).

Proposition 1. (Obviously truthful strategy in DA^k). For any $k \in \{1, .., m\}$, truth-telling is an obviously dominant strategy in DA^k for student s if and only if under information structure $I \in \{1, ..., k\}$ in DA^k:
(i) s is pre-assigned to an acceptable school c under truth-telling or
(ii) there exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s schools.

Obviously truthful strategies are much rarer in BM^k. The reason for this is that top-ranking misreporting strategies usually provide a strong incentive for students to manipulate with their preferences. Thus, in contrast to DA^k, student s should be pre-assigned to most preferred *feasible* school c under truth-telling. *Feasibility* in this case means that under information structure I student

s can get an assignment to school c using top-ranking strategies. The next proposition characterizes the only cases in which student s has an obviously truthful strategy in BM^k.

Proposition 2. (Obviously truthful strategy in BM^k). For any $k \in \{1, ..., m\}$, truth-telling is an obviously dominant strategy in BM^k for student s if and only if under information structure $I \in \{1, .., k\}$ in BM^k:
(i) s is pre-assigned to most preferred feasible school c under truth-telling or
(ii) there exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s schools.

Therefore, the cases for which there exists an obviously truthful strategy for a student are very similar in both mechanisms. The point (i) in each proposition is the condition on truth-telling: in any mechanism student needs to know that he is assigned under truth-telling (which is sufficient for DA^k but not BM^k). The point (ii) in each proposition is the condition both on the truthful report and any misreport: in any mechanism the student needs to know that he is unassigned under truthful report and cannot improve this by any manipulation.

Nevertheless, pre-assignment in DA implies that truthful report eliminates the risk of running out of schools in constrained versions of the mechanism. As DA is not manipulable in the sense of top-ranking strategies when a student is pre-assigned, any manipulation will not result in a better assignment than the pre-assignment. However, in any constrained version of BM the student also needs to be pre-assigned to the most preferred feasible school as top-ranking misreporting strategies provide the student with a strong incentive to manipulate. This implies that obviously truthful strategies are rarer in BM^k compared to DA^k as there is one more condition on pre-assignments.

4 MAIN RESULTS

Based on the characterization results of obviously truthful strategies, it is possible to compare the incentive properties of DA^k and BM^k using the preference-inclusion technique.

4.1 Comparing mechanisms under fixed information

We first present the comparison between the constrained versions of DA and BM. From characterization results of obviously truthful strategies in DA^k and BM^k it is easy to see that students who have obviously truthful strategies in BM^k also have obviously truthful strategies in DA^k. For example, this is because DA^k is less strict than BM^k in the sense that there is no risk to run out of k schools if the student has top-priority in one of the k most preferred schools (but which is the case in BM^k as all the places at school can be fulfilled by other students before the student applies to the school). This yields the following result.

Theorem 1. For any $k \in \{1, ..., m\}$ and fixed information structure $I \in \{1, ..., k\}$, DA^k is more obviously truthful than BM^k under I.

We now present the results on comparing mechanisms with different constraints: as the constraint increases, both mechanisms become more truthful. The intuition behind these results lies in the general logic that the increase in the number of schools that can be reported reduces the risk of running out of reported schools and staying unassigned. Consequently, if a student has an obviously truthful strategy when the number of schools that are allowed to be reported is small, then the student also has an obviously truthful strategy when the number of schools that are allowed to be reported is large.

Theorem 2. For any $k \in \{1, ..., m\}$ and fixed information structure $I \in \{1, ..., k\}$, DA^{k+1} is more obviously truthful than DA^k under I.

Note that all the results are transitive in the sense that if DA^{k+1} is more obviously truthful than DA^k under I then DA^l is more obviously truthful than DA^k under I for any $l > k$.

Theorem 3. For any $k \in \{1, ..., m\}$ and fixed information structure $I \in \{1, ..., k\}$, BM^{k+1} is more obviously truthful than BM^k under I.

It does not follow directly from the obtained results how to compare DA^k and BM^{k+1} in general cases. Let us consider separately two cases to show this.

Case 1. If BM^{k+1} is obviously truthful under given I it does not imply that DA^k is obviously truthful under I.

- Suppose that student s is pre-assigned to his $k+1$'th preferred school in BM^{k+1} under $I \implies BM^{k+1}$ is obviously truthful for s under I.
- It can be the case that student s is rejected from every school under truth-telling in DA^k under I but he cannot be rejected from his $k+1$'th preferred school which provides an incentive to misreport $\implies DA^k$ is not obviously truthful for s under I.

Case 2. If DA^k is obviously truthful under given I it does not imply that BM^{k+1} is obviously truthful under I.

- Suppose that student s is pre-assigned to some school c in DA^k under $I \implies DA^k$ is obviously truthful for s under I.
- It can be the case that student s is rejected from all schools in BM^{k+1} under I because he applies too late but he has a strong incentive to misreport c as his most preferred school $\implies BM^{k+1}$ is not obviously truthful for s under I.

Therefore, it is still undecided whether DA^k or BM^{k+1} is more obviously truthful. On the one hand, as k increases both mechanisms are becoming more obviously truthful. On the other hand, with the same constraint DA^k is more obviously truthful than BM^k. It is uncertain whether increasing the constraint to $k+1$ and larger is sufficient for BM^k to become more obviously truthful than DA^k with fixed constraint k.

4.2 Comparing mechanisms under different information

In this subsection, we proceed to the results without fixing the information structure. We show that as there is more information announced, BM^k becomes more truthful and that the same result holds for DA^k.

If any mechanism M^k is obviously truthful under I, then it follows that the minimum outcome under truth-telling is better than the maximum outcome under misreport for any $\hat{R}_{-s} c.w. R^I_{-s}$. As more information becomes available the set of possible outcomes narrows: it is always the case that all completions $\hat{R}_{-s} c.w. R^{I+1}_{-s}$ under information structure $I+1$ are considered as completions $\hat{R}_{-s} c.w. R^I_{-s}$ under I but not the converse. Therefore, the next two theorems show that more information available incentivizes students to report their preferences more truthfully in both BM^k and DA^k respectively.

Theorem 4. For any $k \in \{1, ..., m\}$ and information structure $I \in \{1, ..., k\}$, BM^k is more obviously truthful under $I + 1$ than under I.

Note that all the results are transitive in the sense that if BM^k is more obviously truthful under $I + 1$ than under I then BM^k is more obviously truthful under L than under I for any $L > I$.

Theorem 5. For any $k \in \{1, ..., m\}$ and information structure $I \in \{1, ..., k\}$, DA^k is more obviously truthful under $I + 1$ than under I.

The result that BM^k is becoming more truthful with more information is consistent with the results from the literature with partial information. However, it is still controversial whether DA^k is becoming more truthful with more information. Bonkoungou and Nesterov (2019) show that DA^k is becoming more obviously manipulable with more information. This is due to the fact that with more information there may be more profiles for which DA^k is obviously truthful as well as for which DA^k is obviously manipulable. As each of the criteria focuses on profiles under which DA^k is either obviously truthful or obviously manipulable, the two criteria provide parallel results showing that DA^k is becoming both more obviously manipulable and more obviously truthful with more information provided.

CONCLUSION

We introduced a new criterion comparing mechanisms by "degree" of their truthfulness under realistic assumptions about information structure.

Preliminaries show that in order for an obviously truthful strategy to exist for a particular student in any mechanism, it is important that the student is pre-assigned under a truthful report under given information structure. Pre-assignment in DA implies that truthful report eliminates the risk of running out of schools in constrained versions of the mechanism and that is sufficient in DA for a student to have an obviously truthful strategy. However, in any constrained version of BM the student also needs to be pre-assigned to the most preferred feasible school to have an obviously truthful strategy which means that obviously truthful strategies are rarer in BM compared to DA. This can be explained by the fact that top-ranking misreporting strategies provide students with a stronger incentive to manipulate in constrained BM compared to constrained DA.

The *main results* of the comparison under fixed information structure once again confirm that DA is superior to BM in terms of incentives and that the longer the list of schools for submission is possible, the more often true preferences are reported. However, it does not follow directly from the obtained results how to compare constrained DA and constrained BM when the constraint of BM is longer than the constraint of DA as was shown. On the one hand, as constraint increases both mechanisms are becoming more obviously truthful. On the other hand, with the same constraint DA is more obviously truthful than BM. It is still uncertain whether increasing the constraint in BM is sufficient for constrained BM to become more obviously truthful than DA with fixed constraint.

The *new results* on comparison under different information structures show that the more information is available, the more truthfully students behave in both constrained DA and constrained BM. Thus, based on the obtained results it can be predicted that the announcement of more information to the public could provide a strong incentive for students to be more truthful.

Nevertheless, the properties of constrained DA under different information discussed in the literature somehow contradict our result. With more information DA becomes more obviously manipulable according to Bonkoungou and Nesterov (2019) and at the same time more obviously truthful according to our results. This is due to the formulation of criteria in the existing literature: with more information there may be more profiles for which constrained DA is obviously truthful as well as for which constrained DA is obviously manipulable. This shows that some further research is needed to compare mechanisms under different information by incentives which we leave as an open question for now.

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APPENDIX

Proof of Proposition 1. Consider any $k \in \{1, ..., m\}$, any $s_i \in S$ and fixed information structure $I \in \{1, ..., k\}$.

Sufficiency: If in DA^k under information structure I (i) s_i is pre-assigned to acceptable school c under truth-telling $R^k_{s_i}$ or (ii) there exists no subset $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools, then R^k_i is an obviously dominant strategy in DA^k for s_i.

(i) In DA^k under information structure I s_i is pre-assigned to acceptable school c under truth-telling $R^k_{s_i}$.

As a result of pre-assignment

$$\min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) = c = \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}).$$

It is obvious that the completion R'_{-s_i} that results in $\max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R'_{s_i}, \hat{R}_{-s_i})$ for any report $R'_{s_i} \in \mathcal{R}^k_{s_i}$ is such that for every $s_j \neq s_i$ $R'_{s_j} : R^I_{s_j}(1) \, ... \, R^I_{s_j}(I) \; s_j$.

As DA^m is not manipulable (Dubins and Freedman, 1981) then particularly for completion R'_{-s_i}:

$$\begin{gathered} DA^m_i(R_{s_i}, R'_{-s_i}) \; R_{s_i} \; DA^m_i(R'_{s_i}, R'_{-s_i}) \text{ for any } R'_{s_i} \in \mathcal{R}_{s_i} \\ \Longrightarrow \mathrm{DA}^k_i(R^k_{s_i}, R'_{-s_i}) \; R_{s_i} \; DA^k_i(R'_{s_i}, R'_{-s_i}) \text{ for any } R'_{s_i} \in \mathcal{R}^k_{s_i}. \end{gathered}$$

Therefore,

$$\min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) = c = \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) \geq_{s_i} \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R'_{s_i}, \hat{R}_{-s_i})$$

and $R^k_{s_i}$ is an obviously dominant strategy for s_i.

(ii) In DA^k under information structure I there exists no subset $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools.

No such subset $\hat{C}$ means that for any $R'_{s_i} \in \mathcal{R}^k_{s_i}$

$$\begin{gathered} \min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) = s_i = \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R'_{s_i}, \hat{R}_{-s_i}) \\ \Longrightarrow \min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) \geq_{s_i} \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R'_{s_i}, \hat{R}_{-s_i}) \end{gathered}$$

and $R^k_{s_i}$ is an obviously dominant strategy for s_i.

Necessity: If in DA^k under information structure I (i) s_i is not pre-assigned to any acceptable school under truth-telling $R^k_{s_i}$ and (ii) there exist a subset $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools, then truth-telling $R^k_{s_i}$ is not an obviously dominant strategy for s_i in DA^k.

The proof is by contradiction: assume that under information structure I truthful strategy $R^k_{s_i} \in \mathcal{R}^k_{s_i}$ is obviously dominant for student s_i.

Case 1: $R^k_{s_i}$ contains $\hat{C}$ for s_i.

As $R^k_{s_i}$ contains $\hat{C}$ for s_i under I, then

$$\min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) = c$$

so that s_i does not stay unassigned. Also, as s_i is not pre-assigned (which means that $\hat{C}$ is not a singleton), then there also exists some c^* such that $c^* \ P_{s_i} \ c$ and to which s_i can be assigned under some $\hat{R}_{-s_i} c.w. R^I_{-s_i}$.

We construct misreport $R'_{s_i} \ : \ c^* \ s_i$ and profiles for each $s_j \neq s_i$ such that $R'_{s_j} \ : \ R^I_{s_j}(1) \dots R^I_{s_j}(I) \ s_j$ are consistent with $R^I_{-s_i}$. Thus,

$$DA^k_i(R'_{s_i}, R'_{-s_i}) = c^*$$
$$\implies \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R'_{s_i}, \hat{R}_{-s_i}) = c^*.$$

Therefore,

$$c = \min_{\hat{R}_{-i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) <_{s_i} \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R'_{s_i}, \hat{R}_{-s_i}) = c^*$$

and $R^k_{s_i}$ is not an obviously dominant strategy for s_i.

Case 2: $R^k_{s_i}$ does not contain $\hat{C}$ for s_i.

As $R^k_{s_i}$ does not contain $\hat{C}$ for s_i then

$$\min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) = s_i.$$

As there exists some $\hat{C}$ that includes only acceptable schools, let R'_{s_i} include schools from $\hat{C}$. Thus,

$$\max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R'_{s_i}, \hat{R}_{-s_i}) = c$$

that is s_i does not stay unassigned reporting R'_{s_i}. Therefore,

$$s_i = \min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R^k_{s_i}, \hat{R}_{-s_i}) <_{s_i} \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^k_i(R'_{s_i}, \hat{R}_{-s_i}) = c$$

and $R^k_{s_i}$ is not an obviously dominant strategy for s_i.

□

Proof of Proposition 2. Consider any $k \in \{1, \dots, m\}$, any $s_i \in S$ and fixed information structure $I \in \{1, \dots, k\}$.

Sufficiency: If in BM^k under information structure I (i) s_i is pre-assigned to the most preferred feasible school c under truth-telling $R^k_{s_i}$ or (ii) there exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools, then $R^k_{s_i}$ is an obviously dominant strategy for s_i in BM^k.

(i) In BM^k under information structure I s_i is pre-assigned to the most preferred feasible school c under truth-telling $R^k_{s_i}$.

As under information structure I student s_i is pre-assigned to c under truth-telling $R^k_{s_i}$, then

$$\min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R^k_{s_i}, \hat{R}_{-s_i}) = c = \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R^k_{s_i}, \hat{R}_{-s_i}).$$

Also, as there are no more preferred than c schools which are feasible for s_i, under any $R'_{s_i} \in \mathcal{R}^k_{s_i}$ student can get assigned only to schools no better than c:

$$\max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R'_{s_i}, \hat{R}_{-s_i}) \leq_{s_i} c.$$

Therefore,

$$\min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R^k_{s_i}, \hat{R}_{-s_i}) \geq_{s_i} \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R'_{s_i}, \hat{R}_{-s_i})$$

and $R^k_{s_i}$ is an obviously dominant strategy for s_i.

(ii) In BM^k under information structure I there exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools.
It directly follows that for any $R'_{s_i} \in \mathcal{R}^k_{s_i}$

$$s_i = \min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R^k_{s_i}, \hat{R}_{-s_i}) \geq_{s_i} \max_{\hat{R}_{-s_i} c.w. Q^I_{-s_i}} BM^k_i(R'_{s_i}, \hat{R}_{-s_i}) = s_i$$

and $R^k_{s_i}$ is an obviously dominant strategy for s_i.

Necessity: If in BM^k under information structure I (i) s_i is pre-assigned to not the most preferred feasible school c^* under truth-telling $R^k_{s_i}$ and (ii) there exists a subset $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools, then truth-telling $R^k_{s_i}$ is not an obviously dominant strategy in BM^k for s_i.

There are at least 2 schools (c and c^*) to which s_i can get assigned if reports them first $\Longrightarrow$ they form set $\hat{C}$.

As student s_i is pre-assigned to c under I under truth-telling $R^k_{s_i}$, then

$$\min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R^k_{s_i}, \hat{R}_{-s_i}) = c = \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R^k_{s_i}, \hat{R}_{-s_i}).$$

There exists school c^* such that $c^* \ P_{s_i} \ c$. We construct profile R'_{s_i}: c^* ... . As school c^* is also feasible for s_i, then s_i is pre-assigned to c^* under R'_{s_i}:

$$\min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R'_{s_i}, \hat{R}_{-s_i}) = c^* = \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R'_{s_i}, \hat{R}_{-s_i}).$$

Thus,

$$c = \min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R^k_{s_i}, \hat{R}_{-s_i}) <_{s_i} \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^k_i(R'_{s_i}, \hat{R}_{-s_i}) = c^*$$

and $R^k_{s_i}$ is not obviously dominant for s_i.

□

Proof of Theorem 1. Consider any $k \in \{1, ..., m\}$, any $s_i \in S$ and fixed information structure $I \in \{1, ..., k\}$.

(i) When $R^k_{s_i}$ is obviously truthful under information structure I in BM^k then $R^k_{s_i}$ is obviously truthful under information structure I in DA^k.

By Proposition 2, s_i has an obviously truthful strategy under information structure I in BM^k in two cases.

Case 1. s_i is pre-assigned to the most preferred feasible school c under truth-telling $R^k_{s_i}$ in BM^k.

As s_i is pre-assigned to c under truth-telling in BM^k, on steps induced by information structure I s_i applies to c. When s_i applies to c there are less than adjusted capacity $q^{'}_c$ students with higher priority who apply on the same step with s_i. Therefore, under truth-telling s_i is also guaranteed at least school c as an assignment in DA^k.

Also as there are no feasible schools better than c for s_i, then in DA^k student s_i will also be rejected from all such schools. Combining this with the fact that c is guaranteed for s_i in DA^k under $I \implies s_i$ is pre-assigned to c in DA^k under I. By Proposition 1, $R^k_{s_i}$ is an obviously truthful strategy for s_i in DA^k.

Case 2. There exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools in BM^k.

The condition means that any $R^{'}_{s_i}$ reporting any acceptable school as the first choice will not result in an assignment to that school. Thus, under information structure I there are at least $\sum_{c \in C} q_c$ students with higher than s_i priorities at each school who are pre-assigned to those schools. Therefore, in DA^k s_i also will not get assigned to any school under I because he will be rejected from every school under any $R^{'}_{s_i} \in \mathcal{R}^k_{s_i}$. Therefore, there exists no subset $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools in DA^k. By Proposition 1, $R^k_{s_i}$ is an obviously truthful strategy for s_i in DA^k.

(ii) There exist some completion $\hat{R}_{-s_i} c.w. R^I_{-s_i}$, some R_c and some q such that $R^k_{s_i}$ is obviously truthful in DA^k but not in BM^k under information structure I.

Take any case when student s_i is (a) pre-assigned to some school under I in DA^k, (b) not pre-assigned under I in BM^k and (c) $R^k_{s_i}$ does not form $\hat{C}$ for s_i in BM^k but some $R^{'}_{s_i}$ forms $\hat{C}$ for s_i in BM^k. By Proposition 1, $R^k_{s_i}$ is an obviously truthful strategy for s_i in DA^k, however, by Proposition 2, $R^k_{s_i}$ is not obviously truthful for s_i in BM^k .

An illustrative example (A.1) with $I = 1$ and $k \geq 2$:

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{s_5}
c_1	c_2	c_1	c_1	c_1
⋮	⋮	c_2	⋮	⋮
⋮	⋮	c_3	⋮	⋮
⋮	⋮	c_4	⋮	⋮

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_3	⋮	⋮
⋮	s_2	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
s_3	⋮	s_3	s_3

(A.1)

Under truth-telling student s_3 is pre-assigned to school c_2 under $I = 1$ in DA^k, therefore, R_{s_3} is obviously dominant for s_3 in DA^k. However, under truth-telling s_3 is not pre-assigned to any school under $I = 1$ in BM^k.

Minimum possible outcome under truth-telling in BM^k for s_3 (for the following completion A.1.1) is when t_3 is unassigned.

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{s_5}
c_1	c_2	c_1	c_1	c_1
⋮	⋮	c_2	c_3	c_4
⋮	⋮	c_3	⋮	⋮
⋮	⋮	c_4	⋮	⋮

(A.1.1)

Maximum possible outcome under misreport containing c_2 in BM^k for s_3 (for any completion as in A.1.2) is c_2.

$$\begin{array}{ccccc}
\hline
R_{s_1} & R_{s_2} & R'_{s_3} & R_{s_4} & R_{s_5} \\
\hline
c_1 & c_2 & c_2 & c_1 & c_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots
\end{array} \tag{A.1.2}$$

Therefore, the minimum result under truth-telling is worse than the maximum result under misreport in $BM^k \implies R_{s_3}$ is not obviously truthful for s_3 in BM^k. Thus, in this example composed by conditions (a), (b) and (c), s_3 has an obviously truthful strategy under I in DA^k but not in BM^k.

It directly follows that when for each $s \in S$ DA^k is more obviously truthful than BM^k under I then DA^k is more obviously truthful than BM^k under I.

□

Proof of Theorem 2. Consider any $k \in \{1, ..., m\}$, any $s_i \in S$ and fixed information structure $I \in \{1, ..., k\}$.

(i) When $R^k_{s_i}$ is obviously truthful under information structure I in DA^k then $R^{k+1}_{s_i}$ is obviously truthful under information structure I in DA^{k+1}.

By Proposition 1, s_i has an obviously truthful strategy under information structure I in DA^k in two cases.

<u>Case 1</u>. s_i is pre-assigned to an acceptable school c under truth-telling $R^k_{s_i}$ in DA^k.

If s_i is pre-assigned under I in DA^k to c, then first I rows predetermine that pre-assignment under $R^k_{s_i}$ as, by definition of pre-assignment, it is already known that s_i cannot be rejected from c in favor of other students because there are less than q_c students with higher priority at school c who was not pre-assigned to any school under I or who was pre-assigned to c.

Therefore, as information is fixed and $R^k_{s_i} \subseteq R^{k+1}_{s_i}$, s_i is also pre-assigned under I in DA^{k+1} under truth-telling $\implies$ by Proposition 1, $R^{k+1}_{s_i}$ is obviously truthful under information structure I in DA^{k+1} for s_i.

<u>Case 2</u>. There exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools in DA^k.

This condition implies that for any $R'_{s_i} \in \mathcal{R}^k_{s_i}$ student s_i is unassigned in DA^k. From Gale and Sotomayor (1985) it follows that for any $R_{s_i} \in \mathcal{R}_{s_i}$ and any $R_{-s_i} \in \mathcal{R}_{-s_i}$

$$DA^k_i(R_{s_i}, R_{-s_i}) \; R_{s_i} \; DA^{k+1}_i(R_{s_i}, R_{-s_i}).$$

Therefore, if s_i is unassigned in DA^k for any $R'_{s_i} \in \mathcal{R}^k_{s_i}$, then s_i is also unassigned in DA^{k+1} for any $R'_{s_i} \in \mathcal{R}^{k+1}_{s_i}$. Thus, for any $R'_{s_i} \in \mathcal{R}^{k+1}_{s_i}$

$$s_i = \min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^{k+1}_i(R^{k+1}_{s_i}, \hat{R}_{-s_i}) \leq_{s_i} \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} DA^{k+1}_i(R'_{s_i}, \hat{R}_{-s_i}) = s_i$$

and $R^{k+1}_{s_i}$ is obviously truthful for s_i in DA^{k+1}.

(ii) There exist some completion $\hat{R}_{-s_i} c.w. R^I_{-s_i}$, some R_c and some q such that R_{s_i} is obviously truthful under information structure I in DA^{k+1} but not in DA^k.

Take any case when student s_i is pre-assigned to its $k+1$'th choice school under I in DA^{k+1}. By Proposition 1, $R^{k+1}_{s_i}$ is an obviously truthful strategy in DA^{k+1} for s_i. It follows that:

(1) under information structure I s_i is rejected from all its choices form 1'st to k'th in favour of students with higher priorities at these schools;
(2) at $k+1$'s choice school there is a place for s_i because s_i has top-priority there or because students with higher than s_i priority at this school are pre-assigned to other schools.

Therefore, s_i is not pre-assigned to any school under I in DA^k but there exists some $\hat{C}$ containing $k+1$'th choice school. Any $R^{'}_{s_i} \in \mathcal{R}^k_{s_i}$ reporting $\hat{C}$ will result in assignment to $k+1$'th choice school. By Proposition , $R^k_{s_i}$ is not an obviously truthful strategy for s_i in DA^k.

An illustrative example (A.2) with $I = 2$:

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{s_5}
c_1	c_1	c_1	c_1	c_1
$\vdots$	c_2	c_2	c_3	$\vdots$
$\vdots$	$\vdots$	c_3	c_4	$\vdots$
$\vdots$	$\vdots$	c_4	c_3	$\vdots$

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_1	s_1	s_1
$\vdots$	s_2	s_4	s_2
$\vdots$	$\vdots$	$\vdots$	s_4
$\vdots$	$\vdots$	$\vdots$	s_3
$\vdots$	$\vdots$	$\vdots$	$\vdots$

(A.2)

Student s_3 is pre-assigned to c_4 under $I = 2$ in DA^4 (4'th choice of s_3) but s_3 is not pre-assigned under $I = 2$ in DA^3. Nevertheless, there exist $\hat{C}$ ($c_4 \subseteq \hat{C}$) in DA^3 such that $R^{'}_{s_3} \in \mathcal{R}^3_{s_3}$ reporting c_4 will result in pre-assignment to c_4 under I in DA^3. By Proposition 1, R_{s_3} is not obviously truthful under $I = 2$ in DA^3 but is obviously truthful under $I = 2$ in DA^4.

It directly follows that when for each $s \in S$ DA^{k+1} is more obviously truthful than DA^k under I then DA^{k+1} is more obviously truthful than DA^k under I.

□

Proof of Theorem 3. Consider any $k \in \{1, ..., m\}$, any $s_i \in S$ and fixed information structure $I \in \{1, ..., k\}$.

(i) When $R^k_{s_i}$ is obviously truthful under information structure I in BM^k then $R^{k+1}_{s_i}$ is obviously truthful under information structure I in BM^{k+1}.

By Proposition 2, s_i has an obviously truthful strategy under information structure I in BM^k in two cases.

<u>Case 1</u>. s_i is pre-assigned to the most preferred feasible school c under truth-telling in BM^k.

As s_i is pre-assigned to c under truth-telling in BM^k, on steps induced by information structure I s_i applies to c. When s_i applies to c there are less than adjusted capacity $q^{'}_c$ students with higher priority who apply on the same step with s_i. Therefore, under truth-telling s_i is also guaranteed at least school c as an assignment in BM^{k+1}.

Also as there are no feasible schools better than c for s_i, then in BM^{k+1} student s_i will also be rejected from all such schools. Combining this with the fact that c is guaranteed for s_i in BM^{k+1} $\Longrightarrow$ under truth-telling s_i is pre-assigned to c in BM^{k+1} under I. By Proposition 2, $R^{k+1}_{s_i}$ is an obviously truthful strategy for s_i in BM^{k+1}.

<u>Case 2</u>. There exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools in BM^k.

From this condition it follows that for any $R^{'}_{s_i} \in \mathcal{R}^k_{s_i}$ student s_i is unassigned in BM^k. $R^{'}_{s_i} \in \mathcal{R}^k_{s_i}$ include strategies reporting school c as first choice for any $c \in C$. Therefore, student s_i is rejected from any school $c \in C$ $\Longrightarrow$ s_i will also be rejected from all $c \in C$ in BM^{k+1} reporting $R^{k+1}_{s_i}$ or any $R^{'}_{s_i} \in \mathcal{R}^{k+1}_{s_i}$. Therefore, for any $R^{'}_{s_i} \in \mathcal{R}^{k+1}_{s_i}$

$$s_i = \min_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^{k+1}_i(R^{k+1}_{s_i}, \hat{R}_{-s_i}) \leq_{s_i} \max_{\hat{R}_{-s_i} c.w. R^I_{-s_i}} BM^{k+1}_i(R^{'}_{s_i}, \hat{R}_{-s_i}) = s_i$$

and $R^{k+1}_{s_i}$ is obviously truthful for s_i in BM^{k+1}.

(ii) There exist some completion $\hat{R}_{-s_i} c.w. R^I_{-s_i}$, some R_c and some q such that R_{s_i} is obviously truthful in BM^{k+1} but not in BM^k.

Take any case when student s_i is pre-assigned to its $k+1$'th choice school under I in BM^{k+1}. By Proposition 2, $R^{k+1}_{s_i}$ is an obviously truthful strategy for s_i in BM^{k+1}. It follows that:
(1) under information structure I s_i is rejected from all its choices form 1'st to k'th in favor of students with higher priorities at these schools;
(2) at $k+1$'s choice school there is a place for s_i because s_i has top-priority there or because students with higher than s_i priority at this school are pre-assigned to other schools.

Therefore, s_i is not pre-assigned to any school under I in BM^k but there exists some $\hat{C}$ containing $k+1$'th choice school. Any $R^{'}_{s_i} \in \mathcal{R}^{k+1}_{s_i}$ reporting $\hat{C}$ will result in assignment to $k+1$'th choice school. By Proposition 2, $R^k_{s_i}$ is not an obviously truthful strategy for s_i in BM^k.

An illustrative example (A.3) with $I = 2$:

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{s_5}
c_1	c_1	c_1	c_1	c_1
$\vdots$	c_2	c_2	c_3	$\vdots$
$\vdots$	$\vdots$	c_3	$\vdots$	$\vdots$
$\vdots$	$\vdots$	c_4	$\vdots$	$\vdots$

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_1	s_1	s_1
$\vdots$	s_2	s_4	s_2
$\vdots$	$\vdots$	$\vdots$	s_4
$\vdots$	$\vdots$	$\vdots$	s_3
$\vdots$	$\vdots$	$\vdots$	$\vdots$

(A.3)

Student s_3 is pre-assigned to c_4 under $I = 2$ in BM^4 (4'th choice of s_3) but s_3 is not pre-assigned under $I = 2$ in BM^3. Nevertheless, there exist $\hat{S}$ in BM^3 such that $R^{'}_{s_3} \in \mathcal{R}^3_{s_3}$ reporting s_4 will result in pre-assignment to s_4 under I in BM^3. By Proposition 2, R_{s_3} is not obviously truthful under $I = 2$ in BM^3 but is obviously truthful under $I = 2$ in BM^4.

It directly follows that when for each $s \in S$ BM^{k+1} is more obviously truthful than BM^k under I then BM^{k+1} is more obviously truthful than BM^k under I.

□

Proof of Theorem 4. Consider any $k \in \{1, ..., m\}$, any $s_i \in S$ and not fixed information structure $I \in \{1, ..., k\}$.

(i) When $R^k_{s_i}$ is obviously truthful under information structure I in BM^k then $R^k_{s_i}$ is obviously truthful under information structure $I+1$ in BM^k.

Case 1. s_i is pre-assigned to the most preferred feasible school c under truth-telling $R^k_{s_i}$ in BM^k under I.

From pre-assignment under truth-telling $R^k_{s_i}$ under I in BM^k it follows that for any completion $\hat{R}_{-s_i} c.w. R^I_{-s_i}$ s_i is pre-assigned to school c under truth-telling $R^k_{s_i}$ in BM^k. As $\hat{R}_{-s_i} c.w. R^{I+1}_{-s_i} \subseteq \hat{R}_{-s_i} c.w. R^I_{-s_i}$, s_i is also pre-assigned to school c under truth-telling $R^k_{s_i}$ under $I+1$ in BM^k. Thus, by Proposition 2, $R^k_{s_i}$ is obviously truthful for s_i under information structure $I+1$ in BM^k.

Case 2. There exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools in BM^k under I.

From this condition it follows that for any $R^{'}_{s_i} \in \mathcal{R}^k_{s_i}$ and any $\hat{R}_{-s_i} c.w. R^I_{-s_i}$ student s_i is unassigned under I in BM^k. As $\hat{R}_{-s_i} c.w. R^{I+1}_{-s_i} \subseteq \hat{R}_{-s_i} c.w. R^I_{-s_i}$, s_i is also unassigned for any $R^{'}_{s_i} \in \mathcal{R}^k_{s_i}$ and any $\hat{R}_{-s_i} c.w. R^{I+1}_{-s_i}$ in BM^k. Thus, by Proposition 2, $R^k_{s_i}$ is obviously truthful for s_i under information structure $I+1$ in BM^k.

(ii) There exist some completion $\hat{R}_{-s_i} c.w.R^I_{-s_i}$, some R_c and some q such that $R^k_{s_i}$ is obviously truthful for s_i under information structure $I+1$ in BM^k but not obviously truthful for s_i under information structure I in BM^k.

Take any case when s_i is (a) pre-assigned under $I+1$ in BM^k, (b) not pre-assigned under I in BM^k and (c) there exists $\hat{C} \subseteq C$ under I. By Proposition 2, $R^k_{s_i}$ is obviously truthful for s_i under information structure $I+1$ in BM^k, but $R^k_{s_i}$ is not obviously truthful for s_i under information structure I in BM^k.

An illustrative example (A.4) with $I = 1$ and $I = 2$:

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{s_5}
c_1	c_1	c_1	c_1	c_1
$\vdots$	c_4	c_2	c_3	c_4
$\vdots$	$\vdots$	c_3	$\vdots$	$\vdots$
$\vdots$	$\vdots$	c_4	$\vdots$	$\vdots$

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_1	s_1	s_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_3	s_3	s_3	s_3

(A.4)

Under truth-telling under $I = 2$ student s_3 is pre-assigned to c_2 in BM^4, therefore, R_{s_3} is obviously truthful under $I = 2$ in BM^4 for s_3.

Minimum possible outcome under truth-telling under $I = 1$ in BM^4 for s_3 (for the following completion A.4.1) is when s_3 is unassigned.

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{s_5}
c_1	c_1	c_1	c_1	c_1
$\vdots$	c_2	c_2	c_3	c_4
$\vdots$	$\vdots$	c_3	$\vdots$	$\vdots$
$\vdots$	$\vdots$	c_4	$\vdots$	$\vdots$

(A.4.1)

Maximum possible outcome under misreport reporting c_2 as his first choice under $I = 1$ in BM^4 for s_3 (for any completion as in A.4.2) is c_2.

R_{s_1}	R_{s_2}	R'_{s_3}	R_{s_4}	R_{s_5}
c_1	c_1	c_2	c_1	c_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

(A.4.2)

Thus, minimum possible outcome for s_3 under truth-telling under $I = 1$ is worse than maximum possible outcome for s_3 under misreport under $I = 1$, therefore, R_{s_3} is not obviously truthful under $I = 1$ in BM^4 for s_3.

It directly follows that when for each $s \in S$ BM^k is more obviously truthful under $I + 1$ than under I then BM^k is more obviously truthful under $I + 1$ than under I.

$\square$

Proof of Theorem 5. Consider any $k \in \{1, ..., m\}$, any $s_i \in S$ and not fixed information structure $I \in \{1, ..., k\}$.

(i) When $R^k_{s_i}$ is obviously truthful under information structure I in DA^k then $R^k_{s_i}$ is obviously truthful under information structure $I+1$ in DA^k.

Case 1. s_i is pre-assigned to acceptable school c under truth-telling $R^k_{s_i}$ in DA^k under I.

From pre-assignment under truth-telling $R^k_{s_i}$ under I in DA^k it follows that for any completion $\hat{R}_{-i}c.w.R^I_{-s_i}$ s_i is pre-assigned to school c under truth-telling $R^k_{s_i}$ in DA^k. As $\hat{R}_{-s_i}c.w.R^{I+1}_{-s_i} \subseteq \hat{R}_{-s_i}c.w.R^I_{-s_i}$, s_i is also pre-assigned to school c under truth-telling $R^k_{s_i}$ under $I+1$ in DA^k. Thus, by Proposition 1, $R^k_{s_i}$ is obviously truthful for s_i under information structure $I+1$ in DA^k.

Case 2. There exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s_i schools in DA^k under I.

From this condition it follows that for any $R'_{s_i} \in \mathcal{R}^k_{s_i}$ and any $\hat{R}_{-s_i}c.w.R^I_{-s_i}$ student s_i is unassigned under I in DA^k. As $\hat{R}_{-s_i}c.w.R^{I+1}_{-s_i} \subseteq \hat{R}_{-s_i}c.w.R^I_{-s_i}$, s_i is also unassigned for any $R'_{s_i} \in \mathcal{R}^k_{s_i}$ and any $\hat{R}_{-s_i}c.w.R^{I+1}_{-s_i}$ in DA^k. Thus, by Proposition 1, $R^k_{s_i}$ is obviously truthful for s_i under information structure $I+1$ in DA^k.

(ii) There exist some completion $\hat{R}_{-s_i}c.w.R^I_{-s_i}$, some R_c and some q such that $R^k_{s_i}$ is obviously truthful for s_i under information structure $I+1$ in DA^k but not obviously truthful for s_i under information structure I in DA^k.

Take any case when s_i is (a) pre-assigned under $I+1$ in DA^k, (b) not pre-assigned under I in DA^k and (c) there exists $\hat{C} \subseteq C$ under I. By Proposition 1, $R^k_{s_i}$ is obviously truthful for s_i under information structure $I+1$ in DA^k, but $R^k_{s_i}$ is not obviously truthful for s_i under information structure I in DA^k.

An illustrative example (A.5) with DA^3 under $I = 1$ and $I = 2$:

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}
c_1	c_1	c_1	c_1
⋮	c_4	c_2	c_3
⋮	⋮	c_3	⋮
⋮	⋮	c_4	⋮

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
c_1	c_1	c_1	c_1
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
c_3	c_3	c_3	c_3

(A.5)

Under $I = 2$ s_3 pre-assigned to c_2, therefore, R_{s_3} is obviously dominant under $I = 2$ in DA^3 for s_3.

Minimum possible outcome under truth-telling under $I = 1$ for s_3 (for the following completion A.5.1) is when s_3 is unassigned.

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}
c_1	c_1	c_1	c_1
⋮	c_2	c_2	c_2
⋮	c_3	c_3	c_3
⋮	⋮	c_4	⋮

(A.5.1)

Maximum possible outcome under misreport under $I = 1$ s_3 (for the following completion A.5.2) is c_4

R_{s_1}	R_{s_2}	R'_{s_3}	R_{s_4}
c_1	c_1	c_4	c_1
$\vdots$	c_2	$\vdots$	c_2
$\vdots$	c_3	$\vdots$	c_3
$\vdots$	$\vdots$	$\vdots$	$\vdots$

(A.5.2)

Therefore, minimum result under truth-telling is worse than maximum result under misreport under $I = 1 \implies R_{s_3}$ is not obviously dominant under $I = 1$ in DA^3 for s_3.

It directly follows that when for each $s \in S$ DA^k is more obviously truthful under $I + 1$ than under I then DA^k is more obviously truthful under $I + 1$ than under I.

□