Preference-inclusion comparison under partial information

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Outline

REFORMS[1]

- Seattle Public Schools in 1999
- Boston Public Schools in 2005
- Ghanaian Secondary Public Schools in 2007
- Chicago Selective High Schools in 2009 and 2010
- Primary Public Schools in more than 50 cities and provinces in England and Wales in 2005-2011
- Denver Public Schools in 2012

change of mechanism:

- change of algorithm – Deferred Acceptance (DA) $\Rightarrow$ Boston (BM)
- change of constraint – $k \uparrow$ in DA^k or BM^k

Problem - moving from one manipulable mechanism to another manipulable mechanism or stable mechanism?

[1] Abdulkadiroğlu and Sönmez (2003); Abdulkadiroğlu et al. (2005); Pathak and Sönmez (2008); Pathak and Sönmez (2013)

LITERATURE

(parallel results)
– superiority of *DA* to *BM*
– superiority of M^{k+1} to M^k

comparison under complete information

- less manipulable (Pathak and Sönmez, 2013)
- more truthful (Decerf and Van der Linden, 2016)
- more immune (Bonkoungou and Nesterov, 2020)

comparison under partial information

- obviously manipulable (Bonkoungou and Nesterov, 2019)

– *BM* is less obviously manipulable with more information
– *DA* is more obviously manipulable with more information

- obviously manipulable (Troyan and Morrill, 2020)

– *BM* is the only obviously manipulable mechanism

RESEARCH QUESTION

- In reality, agents have **partial information**: something is known for sure (or fixed), something is unknown
- **When agents begin to report their *truthful* preferences more often:**

– under which mechanism?
– under which information?

- Criterion of truthfulness - **obvious dominance** (Li, 2017)

MODEL

- set of students $S := \{s_1, s_2, ..., s_n\}$
- preference profile $R_S := (R_{s_1}, R_{s_2}, ..., R_{s_n})$

- set of schools $C := \{c_1, c_2, ..., c_m\}$
- priority profile $R_C := (R_{c_1}, R_{c_2}, ..., R_{c_m})$
- capacity vector $q = (q_1, q_2, ..., q_m)$

- information structure $I = 0, 1, ..., m$

If each student knows his own preferences R_s and priority profile R_C but only the top $I \geq 0$ rows of the preference profile R_{-s}, then the information is denoted as I (Bonkoungou and Nesterov, 2019)

COMPLETE VS PARTIAL INFORMATION

3 students, 3 schools

- complete information $I = 3$

R_{s_1}	R_{s_2}	R_{s_3}
c_1	c_2	c_3
c_2	c_1	c_2
c_3	c_3	c_1

R_{c_1}	R_{c_2}	R_{c_3}
s_1	s_2	s_3
s_2	s_1	s_1
s_3	s_3	s_2

- partial information $I = 1$

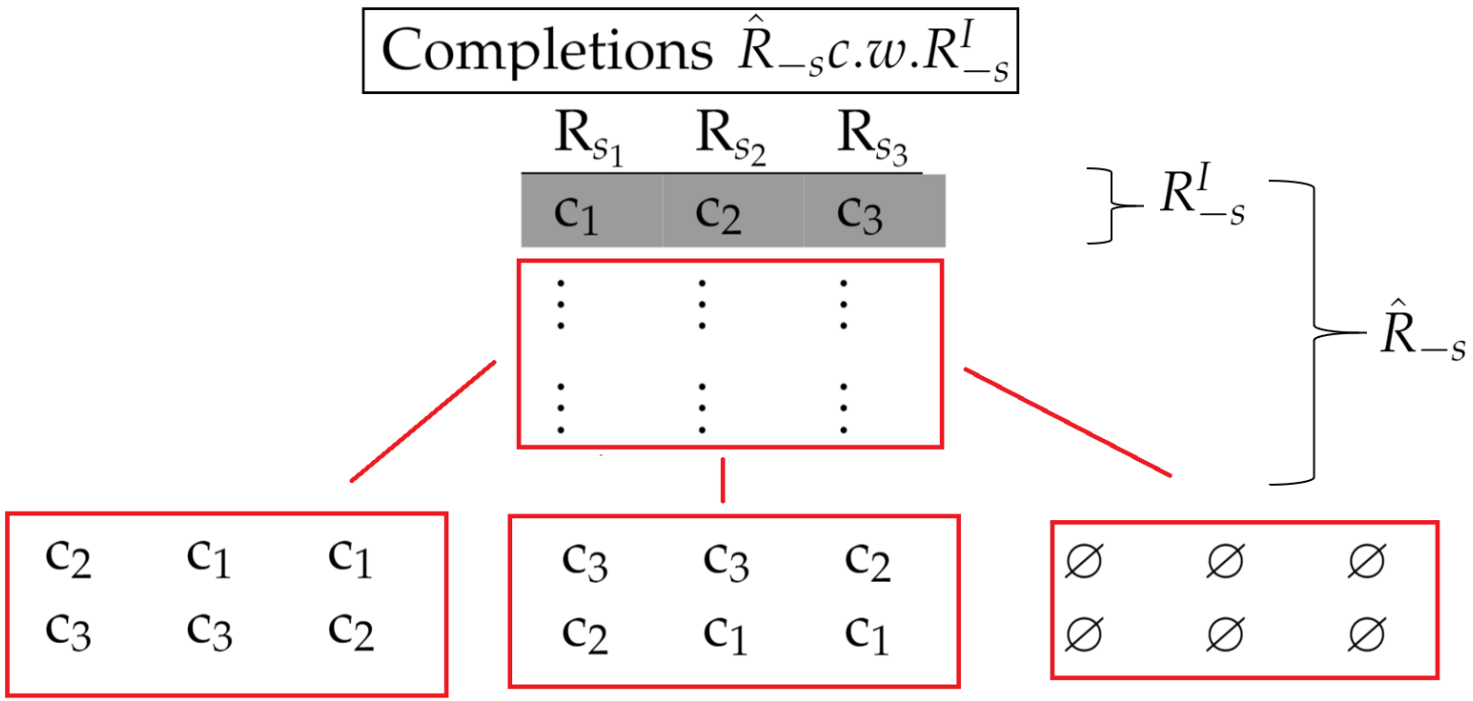
R_{s_1}	R_{s_2}	R_{s_3}
c_1	c_2	c_3
$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$

R_{c_1}	R_{c_2}	R_{c_3}
s_1	s_2	s_3
s_2	s_1	s_1
s_3	s_3	s_2

(1)

PARTIAL INFORMATION STRUCTURE

- examples fixing $I = 1$



BOSTON (IMMEDIATE ACCEPTANCE) MECHANISM

ABDULKADIROĞLU ET AL. (2005)

Round 1:
1) Each *student* applies to the school he reported as his most preferred acceptable school (if any)
2) Every *school* rejects the students in excess of its capacity according to its priority
3) Each *student* who is not rejected is **assigned** to the *school* he applied to and *capacities are adjusted* accordingly

⋮

Round l:
1) Each rejected *student* applies to the best acceptable school that did not reject him yet
2) and 3) as in Round 1

BOSTON (IMMEDIATE ACCEPTANCE) MECHANISM

R_{s_1}	R_{s_2}	R_{s_3}
c_1	c_2	c_3
⋮	⋮	⋮
⋮	⋮	⋮

R_{c_1}	R_{c_2}	R_{c_3}
s_1	s_2	s_3
s_2	s_1	s_1
s_3	s_3	s_2

(1)

Step 1

c_1	c_2	c_3
s_1	s_2	s_3

assignment is uniquely determined given $I = 1$

- no competition
- assigned to the most preferred school

$\Longrightarrow$ no improvement by misreport is possible

DEFERRED ACCEPTANCE MECHANISM

GALE AND SHAPLEY (1962)

Round 1:
1) Each *student* applies to the school he reported as his most preferred acceptable school (if any)
2) Every *school* rejects the students in excess of its capacity according to its priority
3) Each *student* who is not rejected is **TEMPORARILY assigned** to the *school* he applied (could be rejected on any further step)

⋮

Round l:
1) Each rejected *student* applies to the best acceptable school that did not reject him yet
2) and 3) as in Round 1

Termination: temporarily assigned $\Longrightarrow$ assigned

DEFERRED ACCEPTANCE MECHANISM

R_{s_1}	R_{s_2}	R_{s_3}
c_1	c_2	c_3
⋮	⋮	⋮
⋮	⋮	⋮

R_{c_1}	R_{c_2}	R_{c_3}
s_1	s_2	s_3
s_2	s_1	s_1
s_3	s_3	s_2

(1)

Step 1

c_1	c_2	c_3
s_1	s_2	s_3

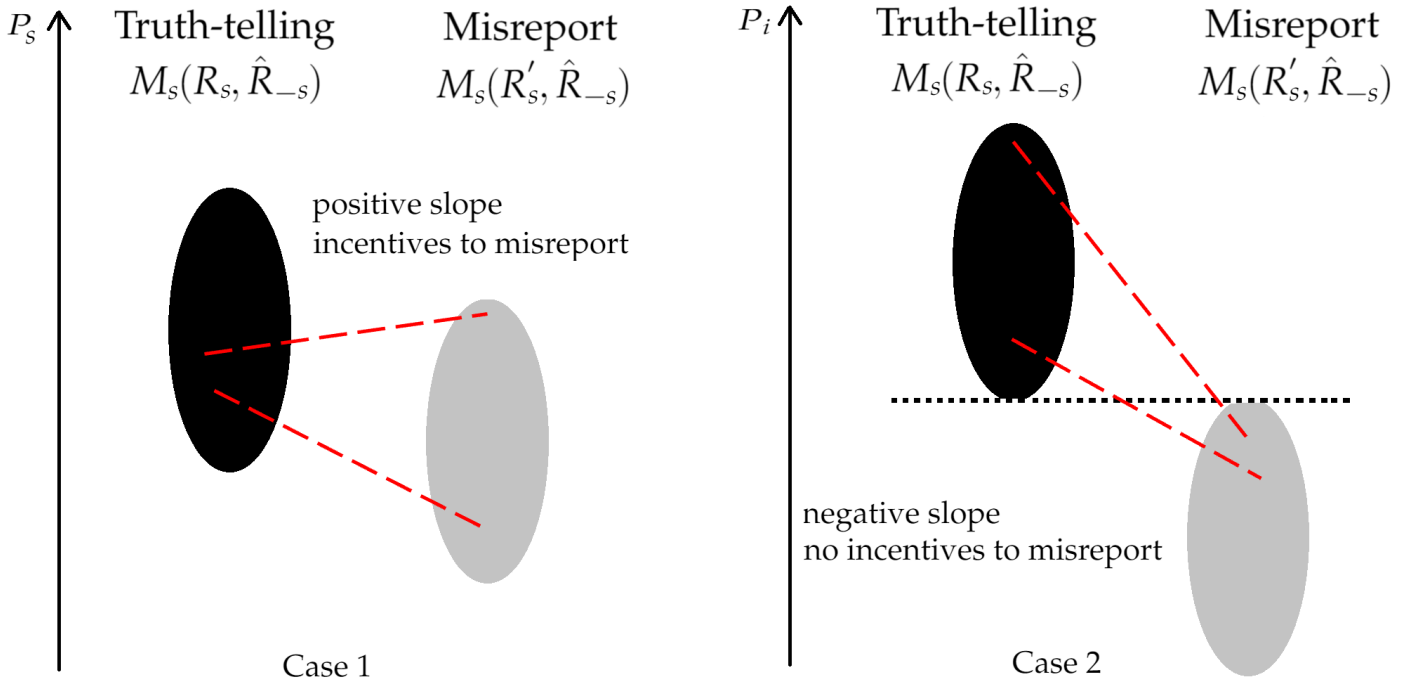
assignment is uniquely determined given $I = 1$

- assigned to both top priority and most preferred school

$\Longrightarrow$ no improvement by misreport is possible

OBVIOUS TRUTHFULNESS

- obvious dominance (Li, 2017)

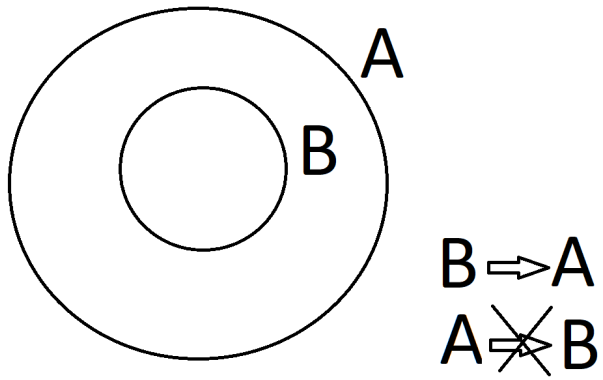


Given mechanism M, preference R_i is **obviously truthful** under information structure I for student t_i if for any R'_i:

$$\min_{\hat{R}_{-i} c.w. R^I_{-i}} M_i(R_i, \hat{R}_{-i}) \; R_i \; \max_{\hat{R}_{-i} c.w. R^I_{-i}} M_i(R'_i, \hat{R}_{-i})$$

MORE OBVIOUSLY TRUTHFUL

- preference-inclusion (Arribillaga and Massó, 2015)



Mechanism M_A is **more obviously truthful** than mechanism M_B under information structure I for student s if:

(i) R_s is o. t. under I for s in $\mathbf{M_B} \Rightarrow R_s$ is o. t. under I for s in $\mathbf{M_A}$

(ii) R_s is o. t. under I for s in $\mathbf{M_A} \not\Rightarrow R_s$ is o. t. under I for s in $\mathbf{M_B}$

PRE-ASSIGNMENT UNDER PARTIAL INFORMATION

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
c_1	c_2	c_3	c_1	s_1	s_1	s_1	s_1
⋮	⋮	⋮	c_2	⋮	s_2	s_2	s_2
⋮	⋮	⋮	c_3	⋮	⋮	s_3	s_3
⋮	⋮	⋮	c_4	⋮	⋮	⋮	s_4

(2)

- s_1 - c_1
- s_2 - c_2
- s_3 - c_3
- s_4 - c_4

A student s is **pre-assigned** to a school c in mechanism M if c is the first school to which s is (temporarily) assigned without the threat of being rejected on any step given information I

OBVIOUSLY TRUTHFUL STRATEGIES UNDER I

DEFERRED ACCEPTANCE MECHANISM

Proposition 1
For any $k \in \{1, .., m\}$, **truth-telling is an obviously dominant strategy in DA^k** for student s if and only if under information structure $I \in \{1, ..., k\}$ in DA^k:
(i) s is pre-assigned to an *acceptable* school c under truth-telling
or
(ii) there exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s schools

Suppose a student

- not pre-assigned $\implies$ min(true report)=unassigned
- set of guaranteed schools $\implies$ max(misreport)=assigned

$\implies$ improvement by misreport

OBVIOUSLY TRUTHFUL STRATEGIES UNDER I

BOSTON MECHANISM

Proposition 2
For any $k \in \{1, ..., m\}$, **truth-telling is an obviously dominant strategy in $\mathbf{BM^k}$** for student s if and only if under information structure $I \in \{1, .., k\}$ in BM^k:
(i) s is pre-assigned to *most preferred feasible* school c under truth-telling or
(ii) there exists no $\hat{C} \subseteq C$ of acceptable and guaranteed for s schools

- top-ranking misreport
- *feasibility* in terms of top-ranking strategies

COMPARING MECHANISMS UNDER FIXED I

DEFERRED ACCEPTANCE VS BOSTON

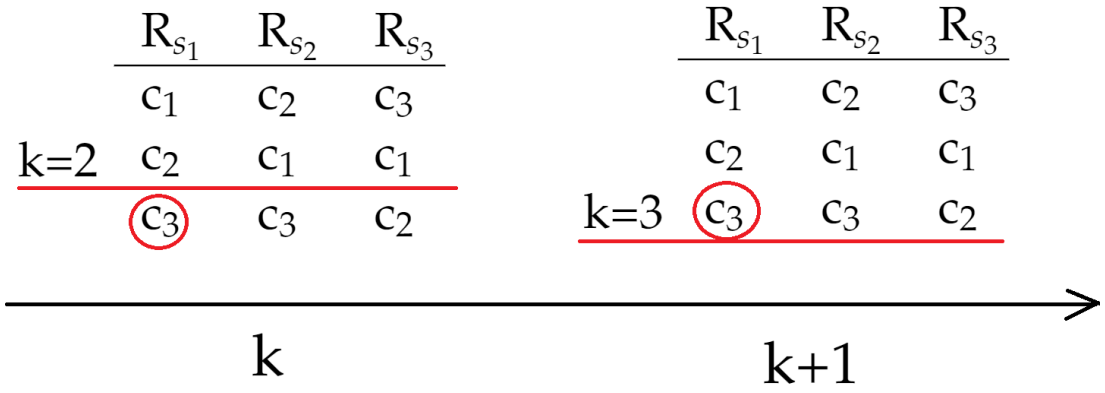
Proposition 1 and 2:
student has an obviously truthful strategy in BM^k
$\implies$ student has an obviously truthful strategy in DA^k

Theorem 1
For any $k \in \{1, ..., m\}$ and fixed information structure $I \in \{1, ..., k\}$,
DA^k is more obviously truthful than BM^k under I

COMPARING MECHANISMS UNDER FIXED I

DIFFERENT CONSTRAINTS

constraint $\uparrow \implies$ risk of running out of schools $\downarrow$



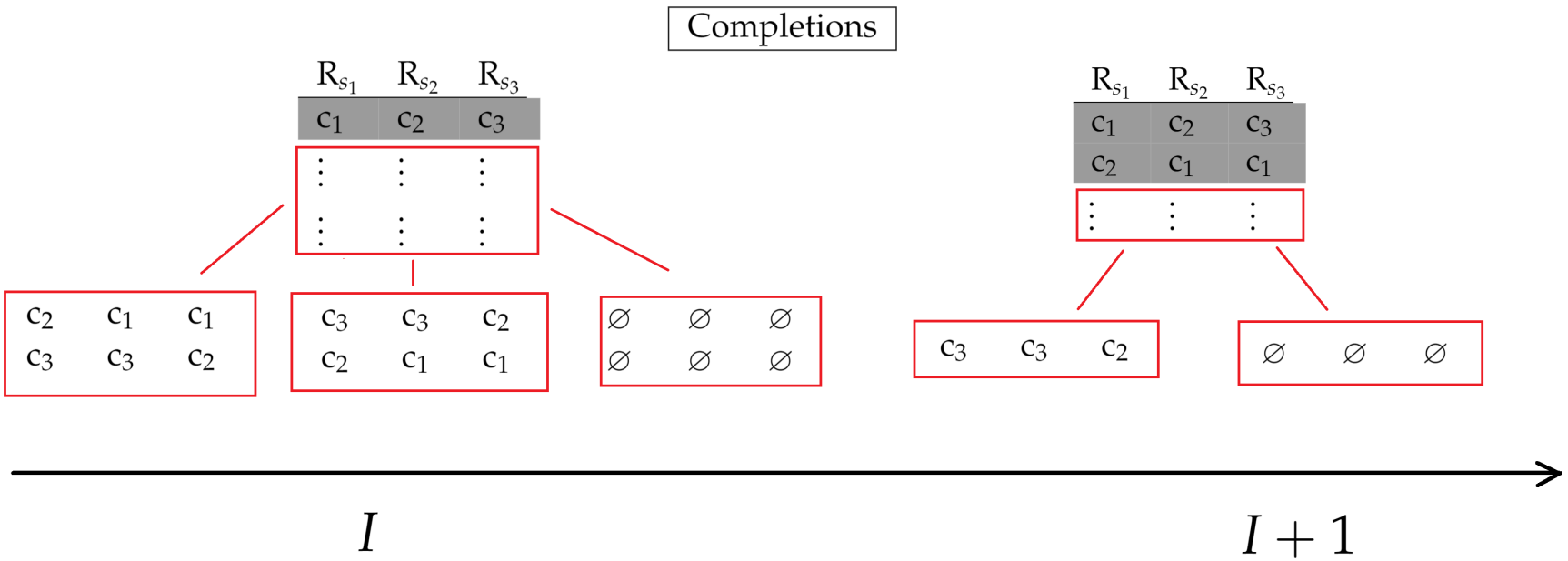
Theorems 2 and 3

For any $k \in \{1, ..., m\}$ and fixed information structure $I \in \{1, ..., k\}$,

- **DA^{k+1} is more obviously truthful than DA^{k}** under I
- **BM^{k+1} is more obviously truthful than BM^{k}** under I

COMPARING MECHANISMS UNDER DIFFERENT I

DEFERRED ACCEPTANCE AND BOSTON



Theorems 4 and 5

For any $k \in \{1, ..., m\}$ and information structure $I \in \{1, ..., k\}$

- **DA^k is more obviously truthful under $I+1$ than under I**
- **BM^k is more obviously truthful under $I+1$ than under I**

CONSISTENCY WITH EXISTING LITERATURE

COMPARISON UNDER FIXED INFORMATION

1. DA^k is more obviously truthful than BM^k
2. DA^{k+1} is more obviously truthful than DA^k
3. BM^{k+1} is more obviously truthful than BM^k

- reinforced the main conclusion regarding the Boston and constrained DA comparison
- reinforced conclusions under *fixed* partial information

CONSISTENCY WITH EXISTING LITERATURE

COMPARISON UNDER DIFFERENT INFORMATION

4. BM^k is more obviously truthful under $I+1$ than under I
5. DA^k is more obviously truthful under $I+1$ than under I

- DA^k is more **obviously manipulable** (Bonkoungou and Nesterov, 2019) and **more obviously truthful** under $I+1$ than under I

$\Longrightarrow$ new results under *not fixed* partial information

POLICY CONCLUSIONS AND REFORMS EVALUATION

- Deferred Acceptance mechanism incentivize students to be more truthful compared to Boston mechanism
- longer **constraint** incentivize students to be more truthful under both mechanisms
- more **information** announced/available incentivize student to be more truthful under Boston mechanism
- more **information** announced/available provides no clear incentive to students under Deferred Acceptance mechanism (accordingly to current research)

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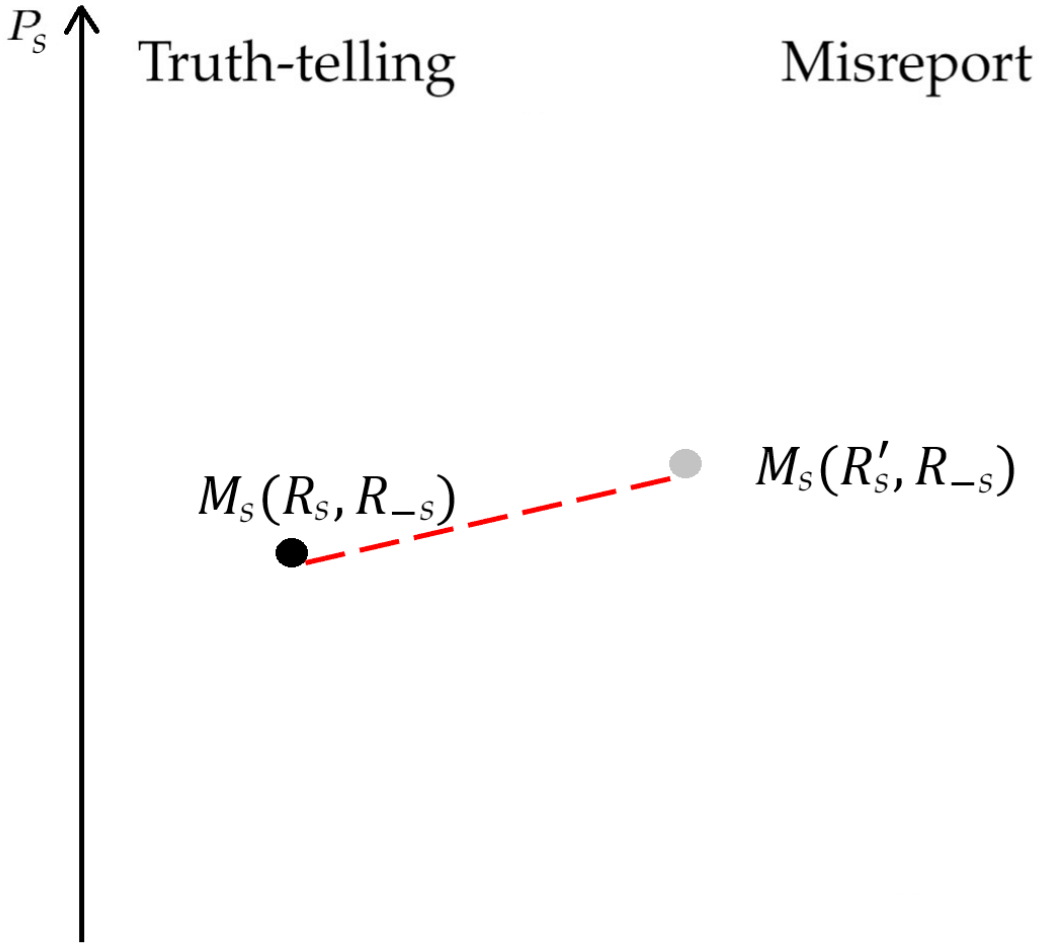
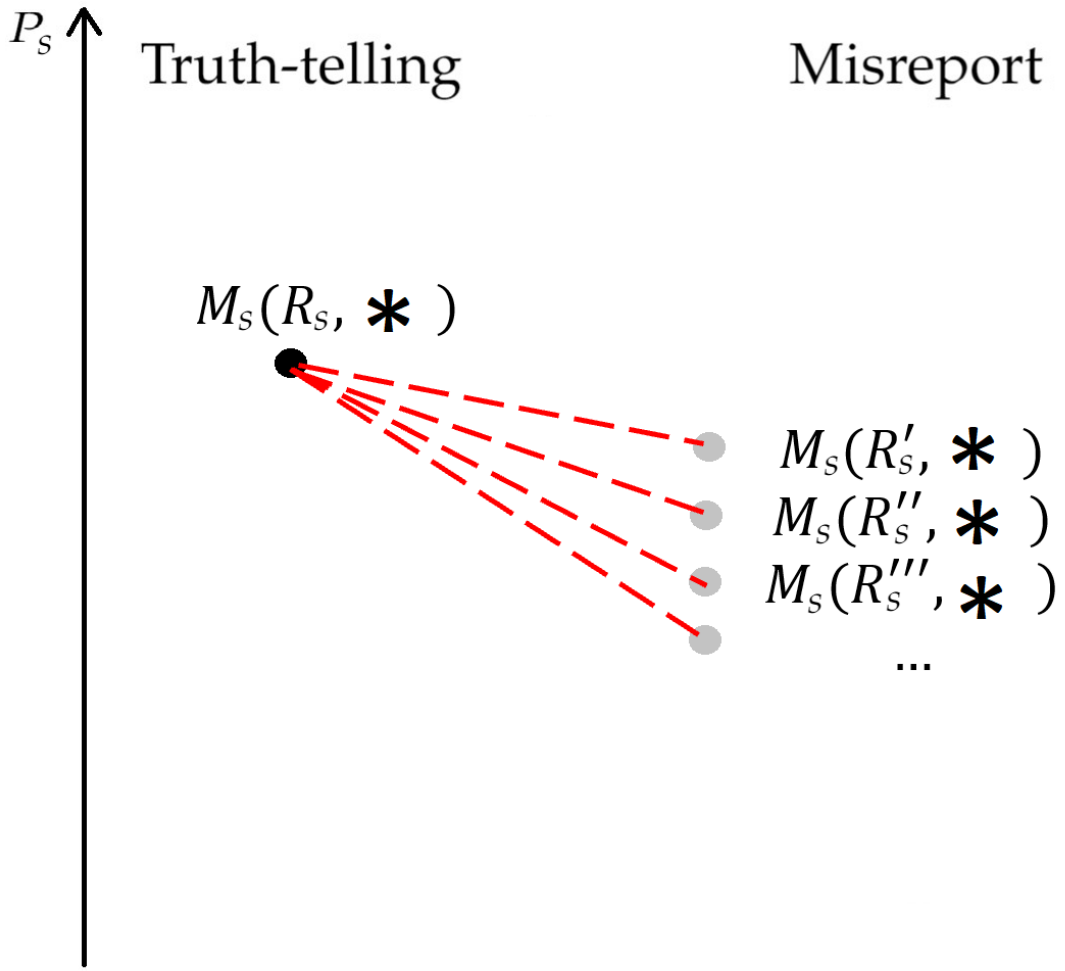
PATHAK AND SÖNMEZ (2013)

MANIPULABLE



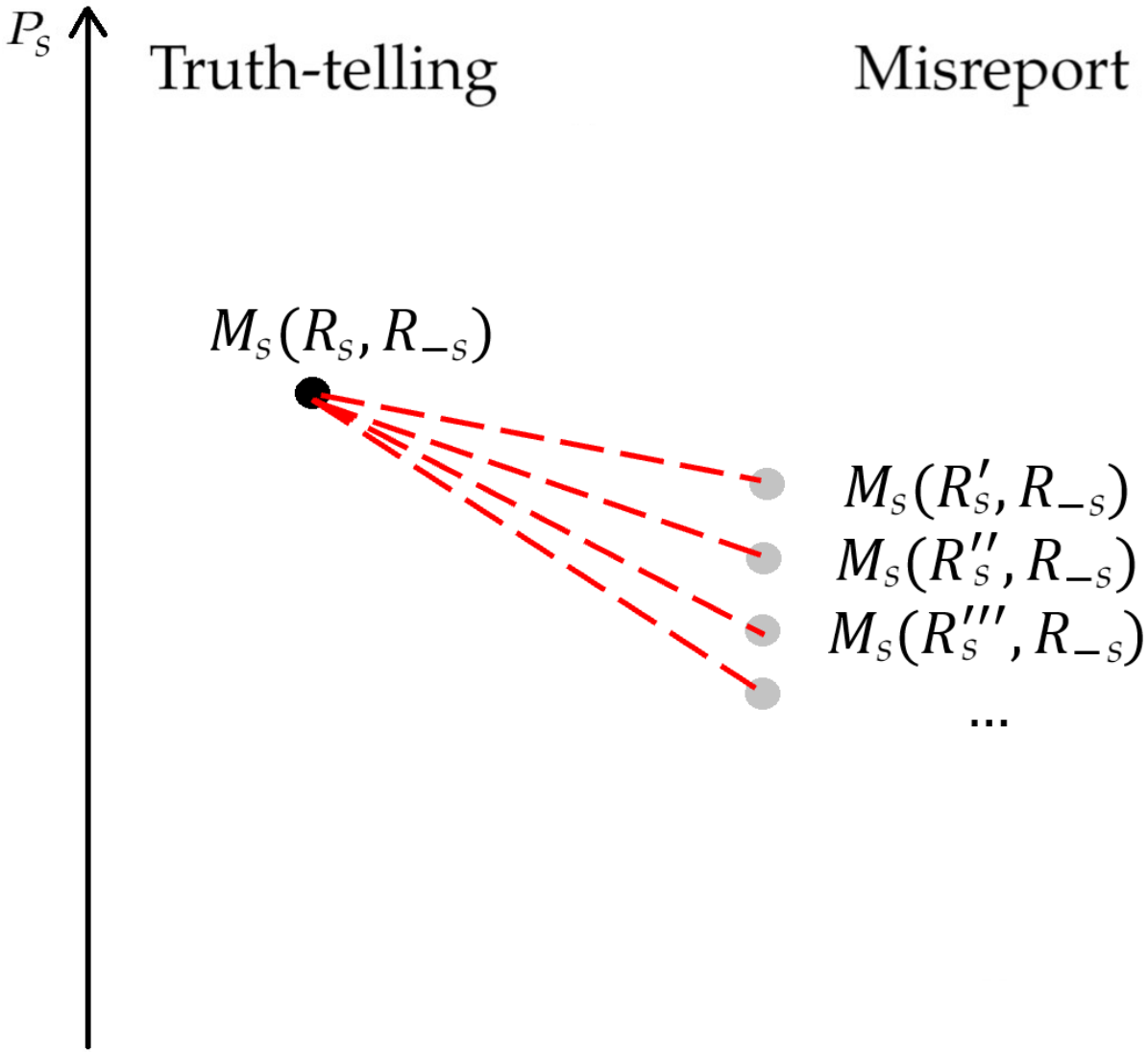
Decerf and Van der Linden (2016)

truthful



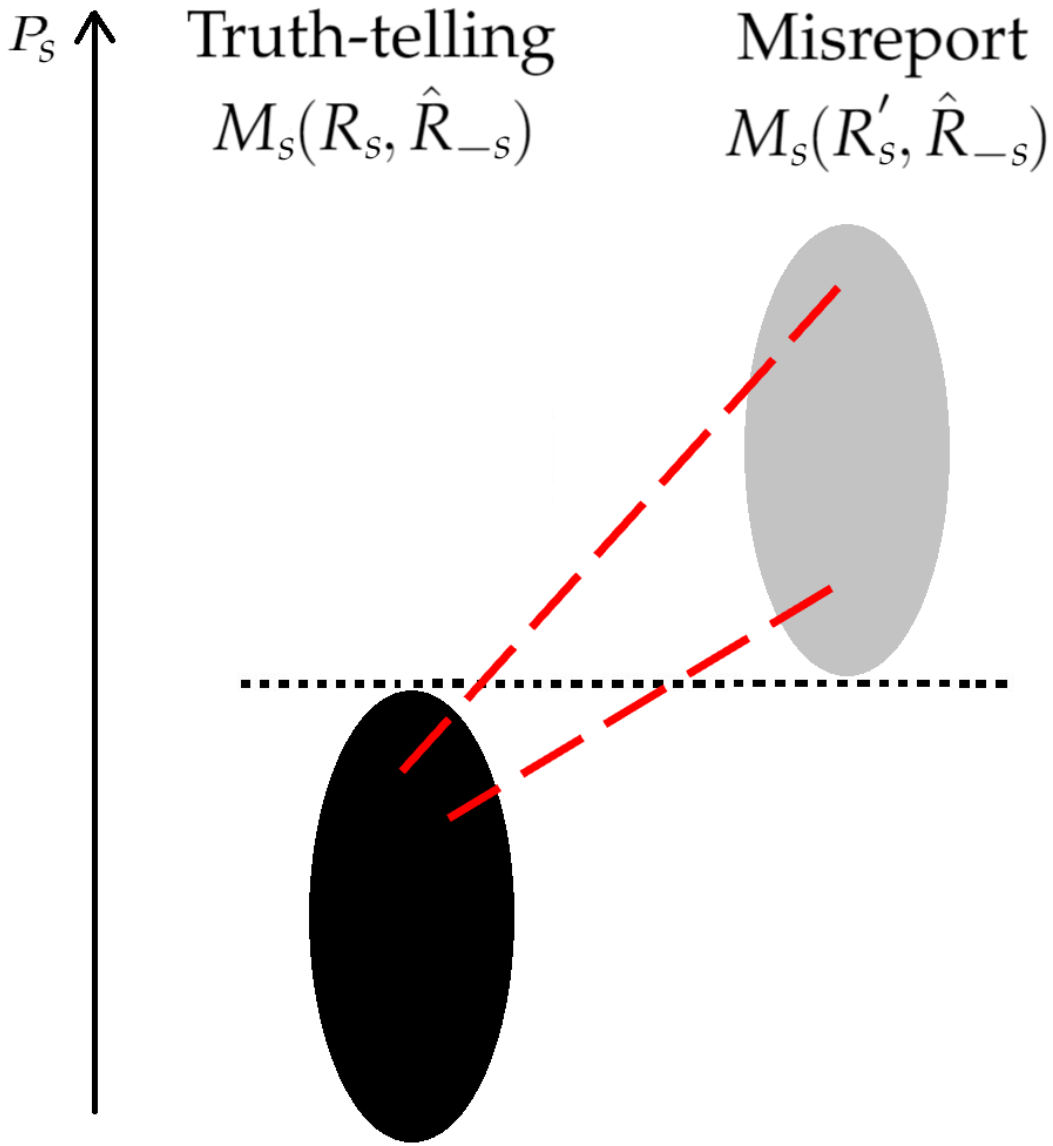
Bonkoungou and Nesterov (2020)

immune



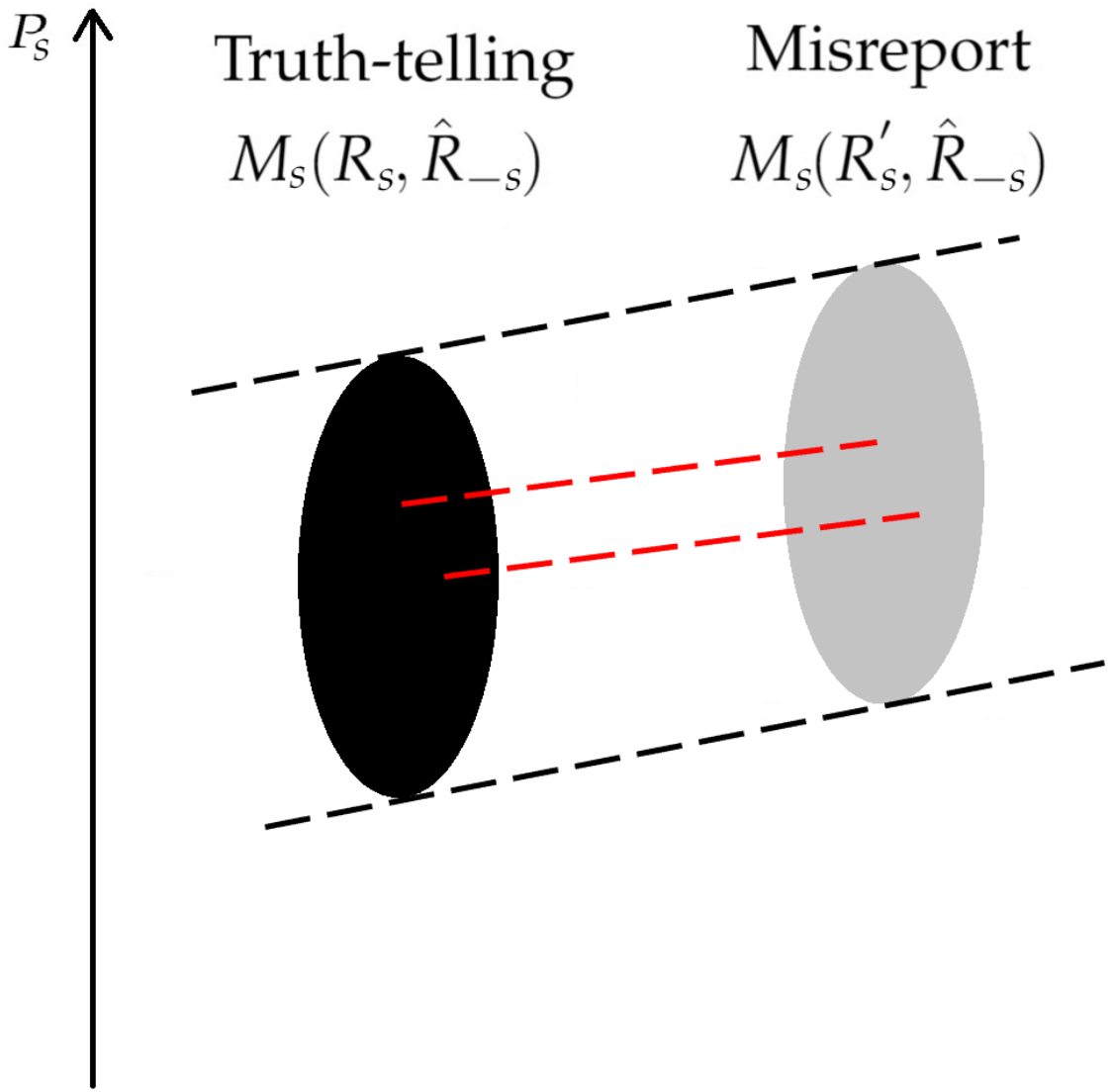
Bonkoungou and Nesterov (2019)

obviously manipulable



Troyan and Morrill (2020)

obviously manipulable



DA^k vs BM^{k+1}

Case 1. If BM^{k+1} is obviously truthful under given I it does not imply that DA^k is obviously truthful under I.

- Suppose that student s is pre-assigned to his $k+1$'th preferred school in BM^{k+1} under $I \implies BM^{k+1}$ is obviously truthful for s under I.
- It can be the case that student s is rejected from every school under truth-telling in DA^k under I but he cannot be rejected from his $k+1$'th preferred school which provides an incentive to misreport $\implies DA^k$ is not obviously truthful for s under I.

DA^k vs BM^{k+1}

Case 2. If DA^k is obviously truthful under given I it does not imply that BM^{k+1} is obviously truthful under I.

- Suppose that student s is pre-assigned to some school c in DA^k under $I \implies DA^k$ is obviously truthful for s under I.
- It can be the case that student s is rejected from all schools in BM^{k+1} under I because he applies too late but he has a strong incentive to misreport c as his most preferred school $\implies BM^{k+1}$ is not obviously truthful for s under I.

SAFE SET

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
c_1	c_2	c_3	c_1	s_1	s_1	s_1	s_1
⋮	⋮	⋮	c_2	⋮	s_2	s_2	s_2
⋮	⋮	⋮	c_3	⋮	⋮	s_3	s_3
⋮	⋮	⋮	c_4	⋮	⋮	⋮	s_4

(2)

- DA^k or BM^k

	c_1	c_2	c_3	c_4
Step 1	s_1, s_4	s_2	s_3	
Step 2	s_1	s_2, s_4	s_3	
Step 3	s_1	s_2	s_3, s_4	
Step 4	s_1	s_2	s_3	s_4

Safe set

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}	R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
c_1	c_2	c_3	c_1	s_1	s_1	s_1	s_1
⋮	⋮	⋮	c_2	⋮	s_2	s_2	s_2
⋮	⋮	⋮	c_3	⋮	⋮	s_3	s_3
⋮	⋮	⋮	c_4	⋮	⋮	⋮	s_4

(2)

- s_1 - c_1
- s_2 - c_2
- s_3 - c_3
- s_4 - c_4

*A set $\hat{C}$ forms **safe set** for student s in mechanism M if $\hat{C} \subseteq C$ protects s from being unassigned under information structure I when report of s includes $\hat{C}$* (Decerf and Van der Linden, 2016)

Obviously truthful strategies under I

Deferred Acceptance Mechanism

DA^3

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}
c_1	c_1	c_1	c_4
⋮	⋮	c_2	⋮
⋮	⋮	c_3	⋮
⋮	⋮	c_4	⋮

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_1	s_1	s_4
⋮	s_2	s_3	⋮
⋮	s_3	⋮	⋮
⋮	⋮	⋮	⋮

(3)

Completion 1			
R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}
c_1	c_1	c_1	c_4
⋮	c_2	c_2	⋮
⋮	⋮	c_3	⋮
.	.		.

	c_1	c_2	c_3	c_4
Step 1	s_1, s_2, s_3			s_4
Step 2	s_1	s_2, s_3		s_4
Step 3	s_1	s_2	s_3	s_4

Obviously truthful strategies under I

Deferred Acceptance Mechanism

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}
c_1	c_1	c_1	c_4
⋮	⋮	c_2	⋮
⋮	⋮	c_3	⋮
⋮	⋮	c_4	⋮

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_1	s_1	s_4
⋮	s_2	s_3	⋮
⋮	s_3	⋮	⋮
⋮	⋮	⋮	⋮

(3)

Completion 2

R_{s_1}	R_{s_2}	R'_{s_3}	R_{s_4}
c_1	c_1	c_2	c_4
⋮	c_3	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮

	c_1	c_2	c_3	c_4
Step 1	s_1, s_2	s_3		s_4
Step 2	s_1	s_3	s_2	s_4

Obviously truthful strategies under I

Deferred Acceptance Mechanism

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}
c_1	c_1	c_1	c_4
⋮	⋮	c_2	⋮
⋮	⋮	c_3	⋮
⋮	⋮	c_4	⋮

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_1	s_1	s_4
⋮	s_2	s_3	⋮
⋮	s_3	⋮	⋮
⋮	⋮	⋮	⋮

(3)

- Completion 1 - s_3 to c_3
- Completion 2 - s_3 to c_2

$\Longrightarrow$ trut-telling is not obviously truthful for s_3

Obviously truthful strategies under I

Boston Mechanism

as in Completion 1

R_{s_1}	R_{s_2}	R_{s_3}	R_{s_4}
c_1	c_1	c_1	c_4
⋮	c_2	c_2	⋮
⋮	⋮	c_3	⋮
⋮	⋮	c_4	⋮

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}
s_1	s_1	s_1	s_4
⋮	s_2	s_3	⋮
⋮	s_3	⋮	⋮
⋮	⋮	⋮	⋮

R_{s_1}	R_{s_2}	R'_{s_3}	R_{s_4}
c_1	c_1	c_2	c_4
⋮	c_2	⋮	⋮
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮

	c_1	c_2	c_3	c_4
Step 1	s_1, s_2	s_3		s_4
Step 2	s_1	s_3, s_2		