??????????

October 22, 2016

1 鶴の翼の面積を求めよ

In [1]:
```
%matplotlib inline

from sympy import *
from sympy.geometry import *
import matplotlib.pyplot as plt
import matplotlib.lines as mpl_lines
from matplotlib.path import Path
import matplotlib.patches as patches

init_printing()

import math
import numpy

xlim = (-0.25,1.25)
ylim = (-0.25,1.25)

def slope_from_points(point1, point2):
    return (point2.y - point1.y)/(point2.x - point1.x)

def get_xlim_value(point1, point2):
    # plot the secant
    slope = slope_from_points(point1, point2)
    intercept = point1.y - slope*point1.x
    # update the points to be on the axes limits
    if point1.x == point2.x:
        data_x = (point1.x, point1.x)
        data_y = ylim
    else:
        data_x = xlim
        data_y = (xlim[0]*slope+intercept, ylim[1]*slope+intercept)
    return data_x, data_y

currentobjs = []
```

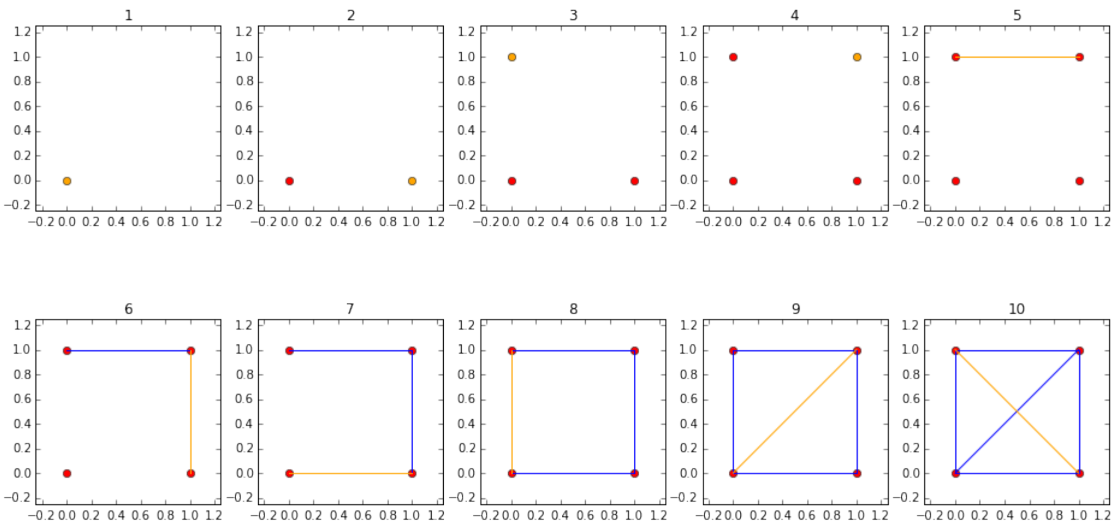
```python
def plot(obj, selected):
    if type(obj) == Point2D:
        if selected == False:
            plt.plot(obj.x, obj.y, 'ro')
        else:
            plt.plot(obj.x, obj.y, 'o', color="orange")
    if type(obj) == Segment:
        # plt.axes().add_line(mpl_lines.Line2D((obj.p1.x, obj.p2.x), (obj.p
        if selected == False:
            plt.plot([obj.p1.x, obj.p2.x], [obj.p1.y, obj.p2.y], 'b-')
        else:
            plt.plot([obj.p1.x, obj.p2.x], [obj.p1.y, obj.p2.y], '-', color
    if type(obj) == Line:
        coor = get_xlim_value(obj.p1,obj.p2)
        if selected == False:
            plt.plot(coor[0], coor[1], 'b-')
        else:
            plt.plot(coor[0], coor[1], '-', color="orange")
    if type(obj) == Polygon:
        verts = []
        codes = []

        for k,v in enumerate(obj.vertices):
            verts.append((v.x, v.y))
            if k == 0:
                codes.append(Path.MOVETO)
            else:
                codes.append(Path.LINETO)
        verts.append((obj.vertices[0].x, obj.vertices[0].y))
        codes.append(Path.CLOSEPOLY)

        path = Path(verts, codes)
        if selected == False:
            patch = patches.PathPatch(path, facecolor=(0,1,0.5,0.5), lw=0.5
        else:
            patch = patches.PathPatch(path, facecolor=(1,0.5,0,0.5), lw=0.5
        plt.gca().add_patch(patch)

def draw(*objs):
    rowcount = 2
    colcount = math.ceil(len(objs) / rowcount)
    plt.figure(figsize=(16,8))
    for index, obj in enumerate(objs):
        plt.subplot(rowcount, colcount, index+1)

        plt.gca().set_xlim(xlim)
        plt.gca().set_ylim(ylim)
        plt.gca().set_aspect('equal')
```

```
            for currentobj in currentobjs:
                plot(currentobj, False)
            plot(obj, True)

            currentobjs.append(obj)
            plt.title(str(len(currentobjs)))
        plt.show()
```

1.1 作図手順

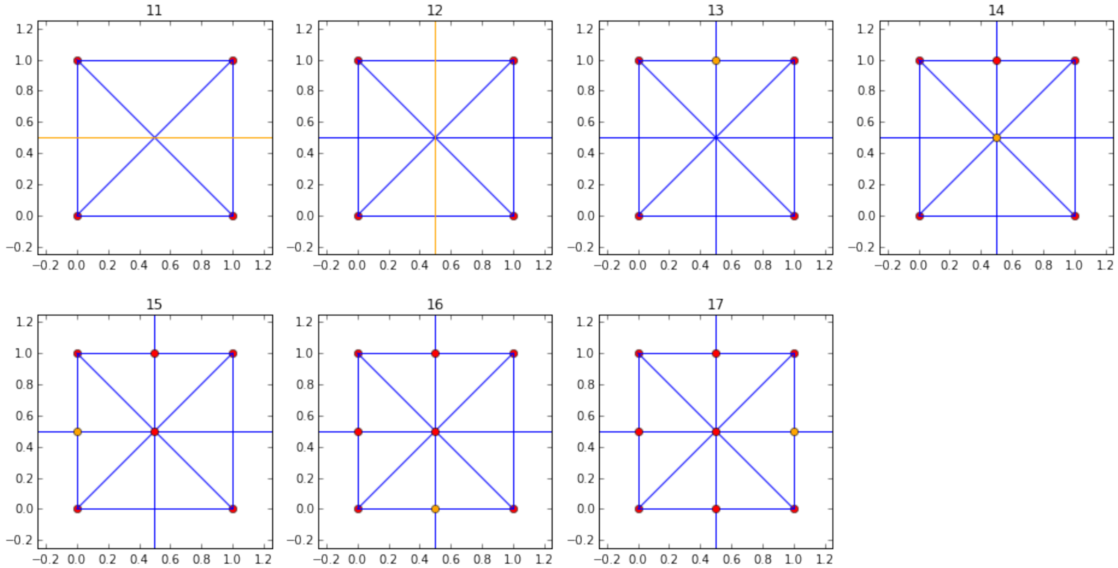
1 - A(0,0)をとる
2 - B(1,0)をとる
3 - C(0,1)をとる
4 - D(0,1)をとる
5 - CとDを端点とする線分fを引く
6 - DとBを端点とする線分gを引く
7 - BとAを端点とする線分hを引く
8 - AとCを端点とする線分iを引く
9 - AとDを端点とする線分pを引く
10 - CとBを端点とする線分lを引く

```
In [2]: A = Point(0, 0) # 1
        B = Point(1, 0) # 2
        C = Point(0, 1) # 3
        D = Point(1, 1) # 4

        f = Segment(C, D) # 5
        g = Segment(D, B) # 6
        h = Segment(B, A) # 7
        i = Segment(A, C) # 8
        p = Segment(A, D) # 9
        l = Segment(C, B) # 10

        draw(A,B,C,D,f,g,h,i,p,l)
```



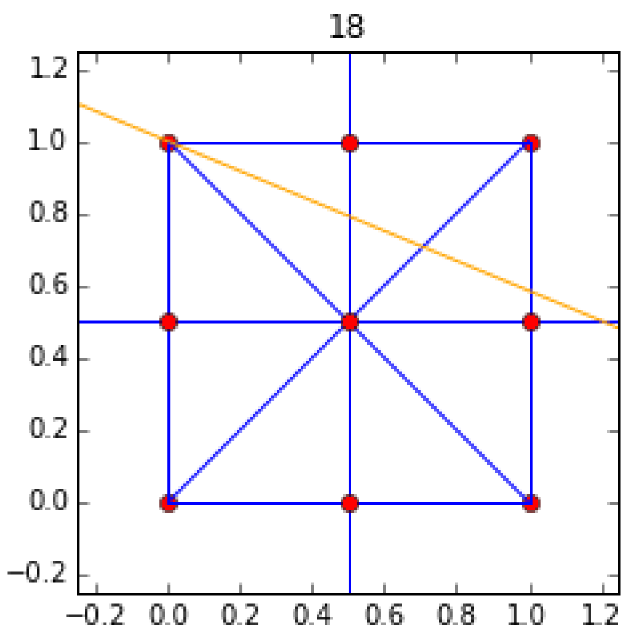
11 - CAを垂直に二等分する直線jを引く
12 - CDを垂直に二等分する直線kを引く
13 - kとfの交点Eをとる
14 - jとkの交点Fをとる
15 - jとiの交点Gをとる
16 - kとhの交点Hをとる
17 - jとgの交点Iをとる

In [3]:
```
j = Segment(C, A).perpendicular_bisector() # 11
k = Segment(C, D).perpendicular_bisector() # 12

E = intersection(k,f)[0] # 13
F = intersection(j,k)[0] # 14
G = intersection(j,i)[0] # 15
H = intersection(k,h)[0] # 16
I = intersection(j,g)[0] # 17

draw(j,k,E,F,G,H,I)
```



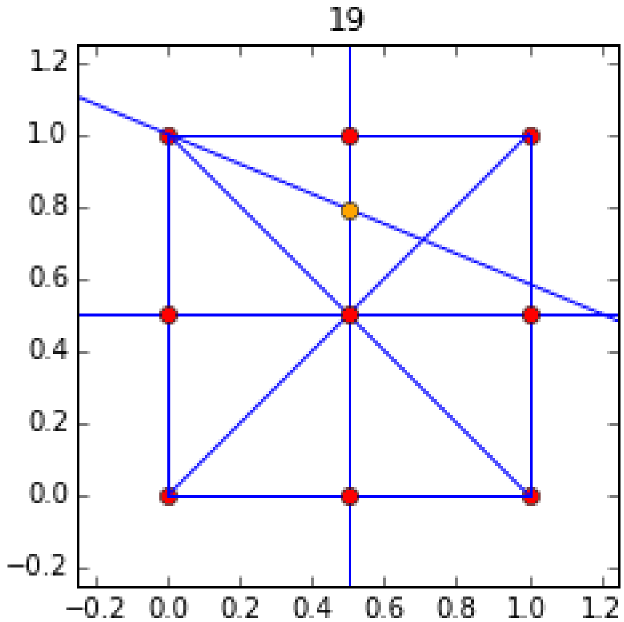
18 - ∠ECFを2等分する直線mを引く

```
In [4]: m = Line(Triangle(E, C, F).bisectors()[C]) # 18
        draw(m)
```



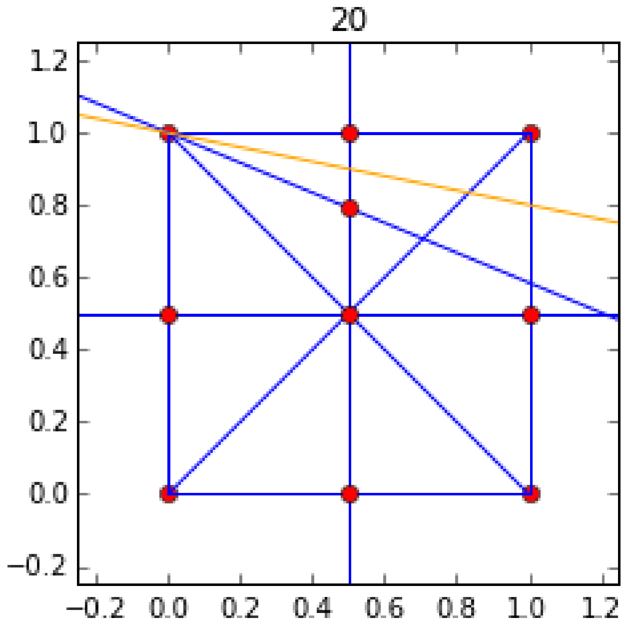
19 - mとkの交点Jをとる

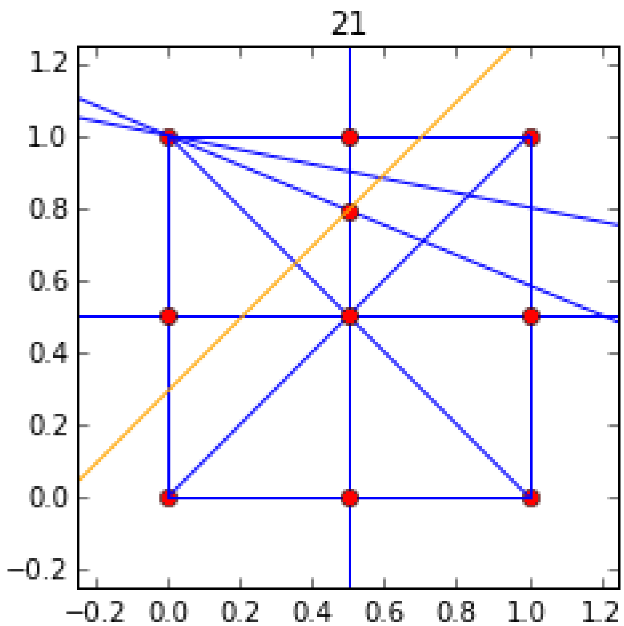
```
In [5]: J = intersection(m,k)[0] # 19
        draw(J)
```



20 - ∠ECJを2等分する直線nを引く
21 - pと平行でJを通る直線qを引く

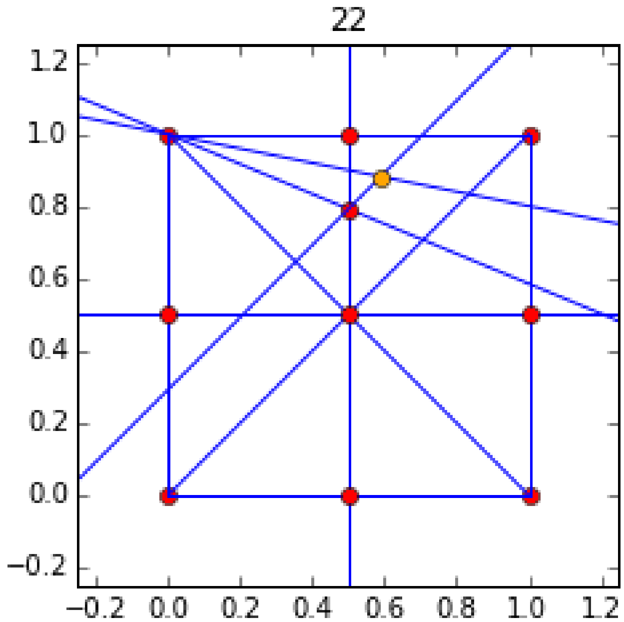
```
In [6]: n = Line(Triangle(E, C, J).bisectors()[C]) # 20
        q = p.parallel_line(J) # 21
        draw(n,q)
```





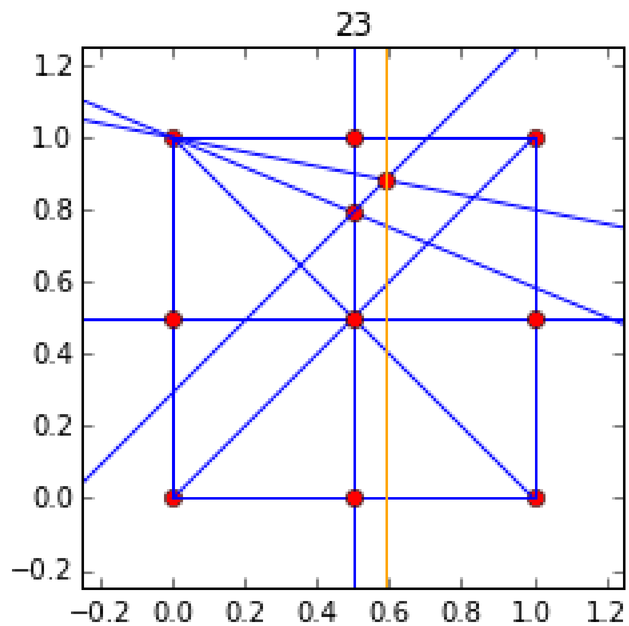
22 - nとqの交点Kをとる

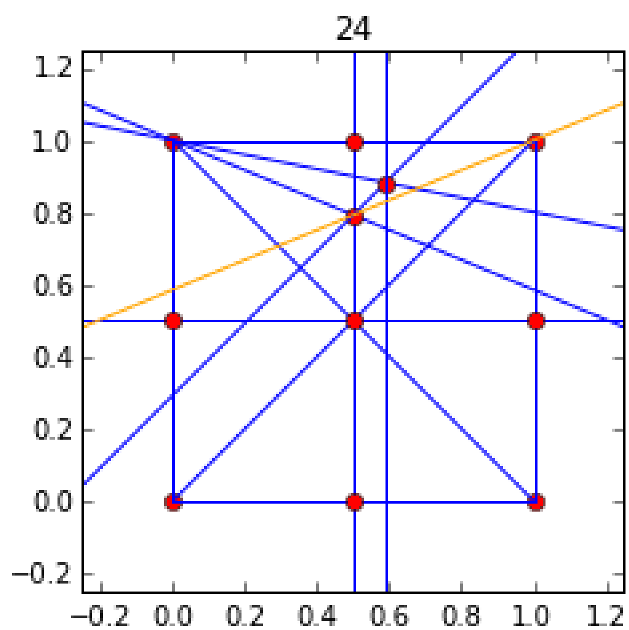
```
In [7]: K = intersection(n, q)[0] # 22
        draw(K)
```



23 - kと平行でKを通る直線rを引く
24 - ∠EDFを2等分する直線sを引く

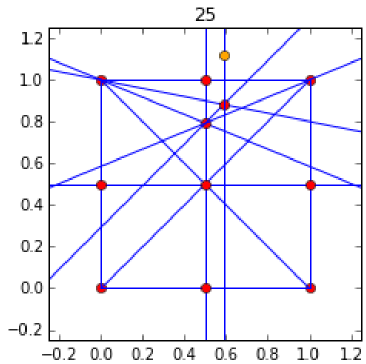
```
In [8]: r = k.parallel_line(K) # 23
        s = Line(Triangle(E, D, F).bisectors()[D]) # 24
        draw(r,s)
```

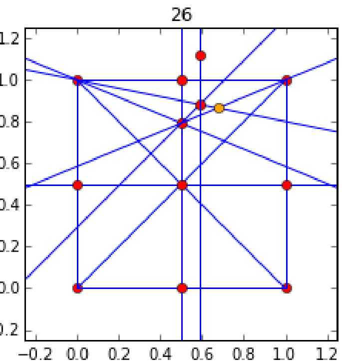


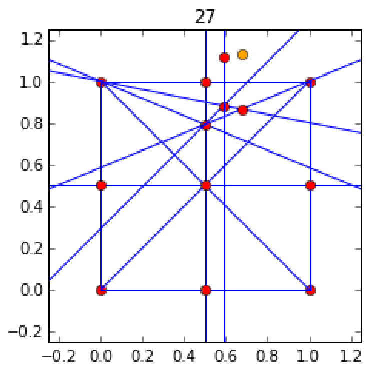


25 - fに関してKと対象な点K’をとる
26 - nとsの交点Lをとる
27 - fに関してLと対象な点L’をとる

```
In [9]: K_ = K.reflect(f) # 25
        L = intersection(n,s)[0] # 26
        L_ = L.reflect(f) # 27
        draw(K_,L,L_)
```

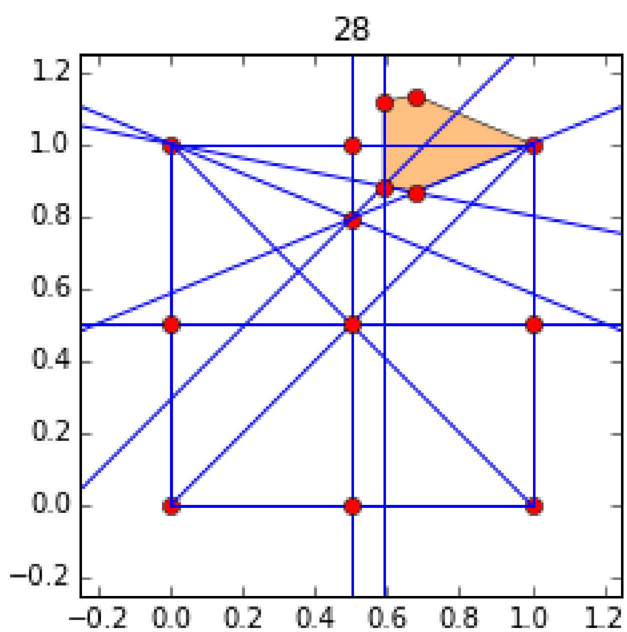






28 - K’, K, L, D, L’を結んだ5角形が鶴の翼の片側の部分である。

```
In [10]: tsubasa = Polygon(K_, K, L, D, L_)
         draw(tsubasa)
```



では、次に、この部分の面積を求めてみよう。上記の手順で作図すると、色のついた部分の面積は、

```
In [11]: summation = tsubasa.area
```

```
In [12]: summation
```

Out[12]:

$$\frac{-9\sqrt{2} - \frac{11}{2}\sqrt{-\sqrt{2}+2} + \frac{9}{2}\sqrt{-2\sqrt{2}+4} + 12}{-4\sqrt{2} - 2\sqrt{-\sqrt{2}+2} + 2\sqrt{2}\sqrt{-\sqrt{2}+2} + 4}$$

と表せる。

2倍して、

```
In [13]: 2*(summation)
```

Out[13]:

$$\frac{-9\sqrt{2} - \frac{11}{2}\sqrt{-\sqrt{2}+2} + \frac{9}{2}\sqrt{-2\sqrt{2}+4} + 12}{-2\sqrt{2} - \sqrt{-\sqrt{2}+2} + \sqrt{2}\sqrt{-\sqrt{2}+2} + 2}$$

これが鶴の翼の面積となる。

```
In [ ]:
```