Efficient Portfolio Optimization via Quadratic Programming and Monte Carlo Simulations

Aiden Black

2025

Abstract

This paper explores efficient portfolio optimization techniques using quadratic programming, Monte Carlo simulations, and artificial intelligence (AI). Monte Carlo simulations are employed to generate a large set of random portfolios, approximating the risk-return trade-offs within a given set of assets. The optimization process is further enhanced using quadratic programming, which minimizes portfolio variance while ensuring the total asset allocation sums to one. Additionally, machine learning techniques, including Support Vector Regression (SVR) and polynomial regression, are integrated to approximate the efficient frontier, reducing computational complexity while maintaining accuracy. The mathematical framework incorporates Markowitz's mean-variance model, where portfolio risk is computed using a covariance matrix, and returns are optimized under linear constraints. The results demonstrate that AI-driven techniques can effectively enhance portfolio allocation strategies, improving computational efficiency while maintaining adherence to established financial optimization principles.

1 Introduction

Portfolio optimization is a fundamental problem in financial mathematics, aiming to allocate capital among assets to maximize return while minimizing risk. This study extends traditional Markowitz's Modern Portfolio Theory (MPT) by incorporating AI-based methods to improve simulation efficiency and frontier approximation.

Monte Carlo simulations, a key component of this study, have a rich history and broad applications in computational finance. Originally developed by physicists Stanislaw Ulam and John von Neumann in the 1940s while working on nuclear weapon research at Los Alamos National Laboratory, the Monte Carlo method provides a means of solving complex problems through stochastic sampling. The method gained prominence in finance through its application to risk analysis, option pricing, and portfolio optimization, where it allows for the simulation of multiple asset allocation scenarios to evaluate the potential risk-return trade-offs. Unlike deterministic approaches, Monte Carlo simulations help model the uncertainty inherent in financial markets by randomly generating portfolio weights and computing corresponding expected returns and risks.

This study employs Monte Carlo simulations to generate thousands of portfolio samples, leveraging statistical inference to estimate the efficient frontier. However, due to the computational cost of large-scale simulations, we integrate machine learning techniques to approximate these results with greater efficiency and accuracy. By combining traditional financial optimization with AI-driven approaches, this work enhances portfolio construction while preserving the core principles of MPT.

2 Mathematical Formulation

The portfolio optimization problem follows Markowitz's mean-variance framework, where we aim to minimize portfolio risk for a given expected return. Given a portfolio consisting of n assets, we define:

- $w = (w_1, w_2, ..., w_n)^T$ as the vector of portfolio weights, where w_i represents the proportion of capital allocated to asset i.
- $\mu = (\mu_1, \mu_2, ..., \mu_n)^T$ as the expected return vector.
- Σ as the covariance matrix of asset returns.

The optimization problem is formulated as:

$$\min_{w} \quad w^T \Sigma w \text{ s.t.} \qquad \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i. \tag{1}$$

This quadratic programming problem is solved using MATLAB's built-in `quadprog` function. Additionally, Monte Carlo simulations generate random portfolio allocations to approximate the risk-return distribution, while machine learning techniques refine the results by predicting efficient portfolios.

3 Methodology

We employ the following steps for portfolio optimization:

- **Monte Carlo Simulations**: Generate 10,000 random portfolio allocations to estimate risk-return distributions.
- **Quadratic Programming**: Solve for the optimal portfolio using Markowitz's mean-variance framework.
- **Machine Learning Approximation**: Train a Support Vector Regression (SVR) model to predict portfolio risk-return pairs, reducing the need for large Monte Carlo runs.
- **Efficient Frontier Approximation**: Use polynomial regression (4th-degree) to estimate the efficient frontier from simulated data.

4 Results and Analysis

The figure illustrates the relationship between Monte Carlo-simmulated portfolios, AI-predicted portfolios, and the efficient frontier in a portfolio optimization framework. The blue dots represnt the portfolios generated through Monte Carlo simulations, where asset weights were randomly assigned to estimate the distribution of possible risk-return combinations. To improve efficiency, machine learning (ML) techniques were used to predict portfolio risk-return pairs without the need for additional Monte Carlo runs. The green dots represent AI-predicted portfolios, obtained using a Support Vector Regression (SVR) model trained on Monte Carlo data. The close alignment between the green and blue dots suggests that the ML model successfully approximates the risk-return distribution, reducing the computational burden while maintaining accuracy.

The red curve represents the efficent frontier, estimated using 4th-degreee polynomial regression. This curve defines the set of portfolios that provide the optiomal trade-off between risk and return. Portfolios lying below this curve are inefficient, meaing they have a higher risk for a given level of return compared to those on the frontier. The red circles marks the optimal portfolio, determined using quadratic programing to minimize risk while achiving optimal return. Its placement near the efficient frontier confirms the validity of the optimization approach.

overall the results demonstrate that AI can effectively enhance portfolio optimization strategies by reducing computational complexity and improving accuracy. By combining Monte Carlo simulations, quadratic programming, and machine learning techniques, investors can achieve efficient portfolio allocations that balance risk and return effectively.

Figure 1

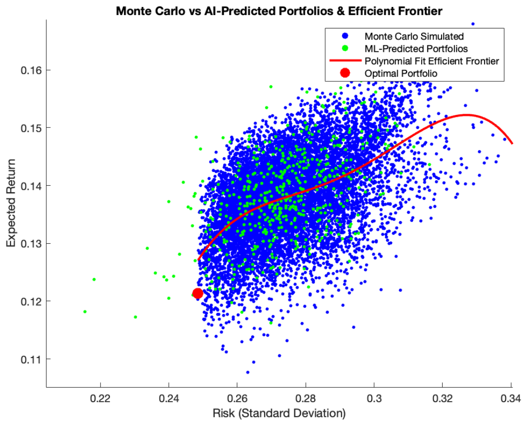

Figure 1: Efficient Frontier with Monte Carlo Simulations

5 Conclusion and Future Work

This study demonstrates that integrating machine learning techniques with traditional portfolio optimization methods can significantly enhance computational efficiency and improve portfolio selection accuracy. The combination of Monte Carlo simulations, quadratic programming, and AI-driven approximations enables a more efficient estimation of the efficient frontier while maintaining theoretical correctness.

Future work could explore the application of reinforcement learning to dynamically adjust asset allocations in response to changing market conditions. Additionally, deep learning methods could be investigated for more complex portfolio optimization scenarios, incorporating factors such as transaction costs, liquidity constraints, and real-time market data. Expanding the dataset to include a wider range of assets and macroeconomic indicators could further refine the predictive accuracy of AI-driven portfolio optimization models.

6 References

References

[1] Markowitz, H. (1952). "Portfolio Selection." *Journal of Finance*, vol. 7, no. 1, pp. 77-91.

[2] Juneja, S. (2024). "Monte Carlo Methods in Finance: An Introductory Tutorial." *ResearchGate*.

[3] Simsek, B. (2024). "Practical Applications of Monte Carlo Simulation in Financial Modeling." *PM Research*.

[4] Shaw, W. T. (2010). "Monte Carlo Portfolio Optimization for General Investor Risk-Return Objectives and Arbitrary Return Distributions." *arXiv*.

[5] Glasserman, P. (2004). "Monte Carlo Methods in Financial Engineering." *Springer*.