Efficient Portfolio Optimization via Quadratic Programming and Monte Carlo Simulations

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Abstract

This paper explores efficient portfolio optimization techniques using quadratic programming and Monte Carlo simulations. We apply Markowitz's Modern Portfolio Theory (MPT) to minimize portfolio risk under given constraints. The results are visualized through the Efficient Frontier, demonstrating optimal asset allocation strategies.

1 Introduction

Portfolio optimization is a fundamental problem in financial mathematics, aiming to allocate capital among assets to maximize return while minimizing risk. This study employs Markowitz's MPT to formulate the optimization problem as a quadratic program and compares results with randomly generated portfolios via Monte Carlo simulations.

2 Mathematical Formulation

The optimization problem is defined as follows:

$$\text{Minimize} \quad \frac{1}{2} x^T \Sigma x, \tag{1}$$

$$\text{subject to} \quad \mu^T x = r_{\text{target}}, \quad \sum x_i = 1, \quad x_i \geq 0. \tag{2}$$

where x represents asset weights, Σ is the covariance matrix of returns, and μ denotes expected returns.

3 Computational Methods

Quadratic programming is implemented in MATLAB using `quadprog`. Monte Carlo simulations generate random portfolios to construct the Efficient Frontier. We evaluate the performance of optimized portfolios against random allocations.

4 Results and Analysis

Figure 1 shows the Efficient Frontier, where the optimal portfolio is marked in red. The optimization successfully minimizes risk compared to random portfolios.

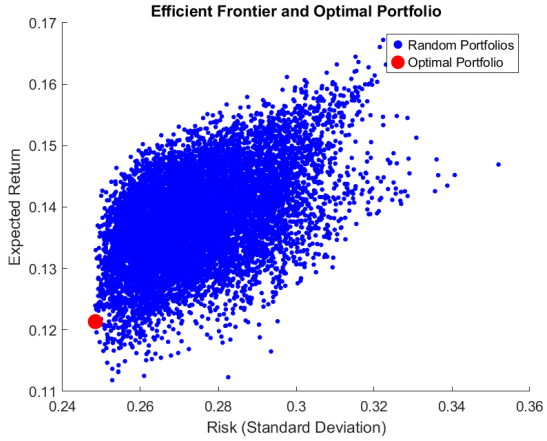

Figure 1: Efficient Frontier with Monte Carlo Simulations

5 Conclusion and Future Work

This study demonstrates that quadratic programming provides an efficient approach to portfolio optimization. Future work could explore dynamic asset allocation models, transaction costs, and machine learning-based optimization techniques.

6 References