$$x_1 \cdot \ldots \cdot x_n = 1$$

$$x_1 + \ldots + x_n \geq n$$

База $n=2$: $x_1 \cdot x_2 = 1$ $\to$ $x_1 + \dfrac{1}{x_1} \geq 2$

$$x_1 + x_2 \geq 2$$

Шаг $n=k$: $x_1 \cdot \ldots \cdot x_k = 1$

$$x_1 + \ldots + x_k$$

$n=k+1$: $x_1 \cdot x_2 \cdot \ldots x_{k+1} = 1$

новый x

$$x_1 + \ldots + x_{k+1} \overset{?}{\geq} n+1$$

$$x_1 + \ldots + x_k \geq n$$

$$x_1 \cdot x_2 = 1$$

$$(x_1 - 1)(x_2 - 1) = x_1 x_2 - (x_1 + x_2) + 1$$

$$x_1 + x_2 = x_1 \cdot x_2 + 1 + (1 - x_1)(x_2 - 1) \geq 2$$

$\sup X = \alpha$

а) $x \in X \quad x \leq \alpha$

б) $\forall \varepsilon > 0, \ \exists x(\varepsilon) \in X, \ \alpha - \varepsilon < x_\varepsilon \leq \alpha$

$\{x_n\}$ – сх-ся к a

$\exists a: \forall \varepsilon > 0, \ \exists N(\varepsilon), \ \forall n \geq N(\varepsilon)$

$|x_n - a| < \varepsilon$

$$\left\{ \frac{n^2+4n+8}{(n+1)^2} \right\}$$

$$\frac{n^2+4n+8}{n^2+2n+1} = \frac{1 + \frac{4}{n} + \frac{8}{n^2}}{1 + \frac{2}{n^2} + 1} = 1$$

$$\lim_{n \to \infty} \left\{ \frac{n^2+4n+8}{n^2+2n+1} \right\} = 1$$

$$1 + \frac{2n}{(n+1)^2} + \frac{7}{(n+1)^2} =$$

$$= 1 + \frac{2}{4} + \frac{7}{4} = 1 + \frac{9}{4} = \frac{13}{4}$$

Опр. $\forall c > 0 \ \exists n, \ |x_n| \geq c$

276(б).

1) $X_{2k} = (2k)^{(-1)^{(2k)}} = 2k$

2) $X_{2k+1} = (2k+1)^{-1} = \frac{1}{2k+1}$

$X_{2k} \nearrow$

$$X_n = \frac{n}{n+1} \qquad \left| 1 - \frac{n}{n+1} \right| < \varepsilon$$

$N(\varepsilon)$: $\frac{1}{n+1} < \varepsilon$; $n+1 > \frac{1}{\varepsilon}$; $N(\varepsilon) =$

$= \left[\frac{1}{\varepsilon} - 1 \right] + 1$

279(2)

$$x_n = \sum_{k=1}^{n} \frac{1}{4k^2-1}$$

$\sup\{x_n\}$

$\inf$

реш. у доски

$$\frac{1}{2} > \frac{n}{2n+1} \geq \frac{1}{3}$$

$$x_n = \sqrt[3]{n^3-1} - n = \sqrt[3]{n^3-1} - \sqrt[3]{n^3} =$$

$$= \frac{n^3-1-n^3}{\sqrt[3]{n^3-1}^{\,2} + n\sqrt[3]{n^3-1} + n^2} = \frac{-1}{\dots} =$$

$$= -\frac{1}{y_n} \quad y_n \uparrow$$

а) $x_n = \frac{1}{n!} < \frac{1}{n}$

б) $x_n = \frac{2n}{n^2+1}$

в) $x_n = (-1)^n \cdot 0{,}999^n$

б) $\frac{2n}{n^3+1} < \varepsilon$ $\quad \forall \varepsilon > 0 : \exists N(\varepsilon),$
$\forall n \geq N(\varepsilon)$
$|x_n| < \varepsilon$

$\frac{2n}{n^3} < \varepsilon, \quad \frac{2}{n^2} < \varepsilon \ ; \ n > \sqrt{\frac{2}{\varepsilon}}$

$N(\varepsilon) = \left\lceil \sqrt{\frac{2}{\varepsilon}} \right\rceil$

$x_n = (-1)^n \cdot 0{,}999^n$

$\log_{0{,}999} \varepsilon < n$

$q^n \to 0$

$\forall \varepsilon > 0 \ \exists N(\varepsilon) \ [\log_q \varepsilon] + 1$

$x_n = 2^{\sqrt{n}} \geq c \qquad \sqrt{n} \geq \log_2 c$

C	10	100	1000
N	16	49	100

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$\{x_n\} \to a$ $\quad \begin{cases} z_{2k} = x_k \\ z_{2k-1} = y_k \end{cases}$ $\quad \{z_k\} \to a$

$\{y_n\} \to a$

$|x_n - a| < \varepsilon$ $\quad \forall \varepsilon > 0$ $\quad \exists N_1(\varepsilon)\ \forall n \geq N_1(\varepsilon)$

$|y_n - a| < \varepsilon$ $\quad \exists N_2(\varepsilon)\ \forall n \geq N_2(\varepsilon)$

$\forall \varepsilon > 0\ \exists N_3(\varepsilon)\ \forall n \geq N_3(\varepsilon)\ |z_n - a| < \varepsilon$

$N_3(\varepsilon) = 2 \cdot \max(N_1(\varepsilon), N_2(\varepsilon))$

$\{x_n\} \to a \quad \{\sqrt{x_n}\} \to \sqrt{a}$

$\forall \varepsilon' > 0\ \exists N_1(\varepsilon')\ \forall n \geq N_1(\varepsilon')\ |x_n - a| < \varepsilon'$

$\forall \varepsilon > 0 \ \exists N_2(\varepsilon) \ \forall n \geq N_2(\varepsilon) \ |\sqrt{x_n} - \sqrt{a}|$

$|(\sqrt{x_n} - \sqrt{a})(\sqrt{x_n} + \sqrt{a})| < \varepsilon'$

$|\sqrt{x_n} - \sqrt{a}| < \dfrac{\varepsilon'}{\sqrt{x_n} + \sqrt{a}}$ $\quad \varepsilon'$ - число > 0

$N_2(\varepsilon) = N_1(\sqrt{a} \cdot \varepsilon)$

$N_2(\varepsilon) = N_1(\varepsilon^2)$

$\{x_n\} \to a$

$\{x_n^2\} \overset{?}{\to} a^2$

1)

$\exists N \in \mathbb{N} : \forall \varepsilon > 0 \ \forall n \geq N \hookrightarrow |x_n - a| < \varepsilon$

$\{x_n\} \overset{?}{\to} a$

2)

$\forall \varepsilon > 0 \ \exists N \in \mathbb{N}, \ \exists n \geq N : |x_n - a| < \varepsilon$