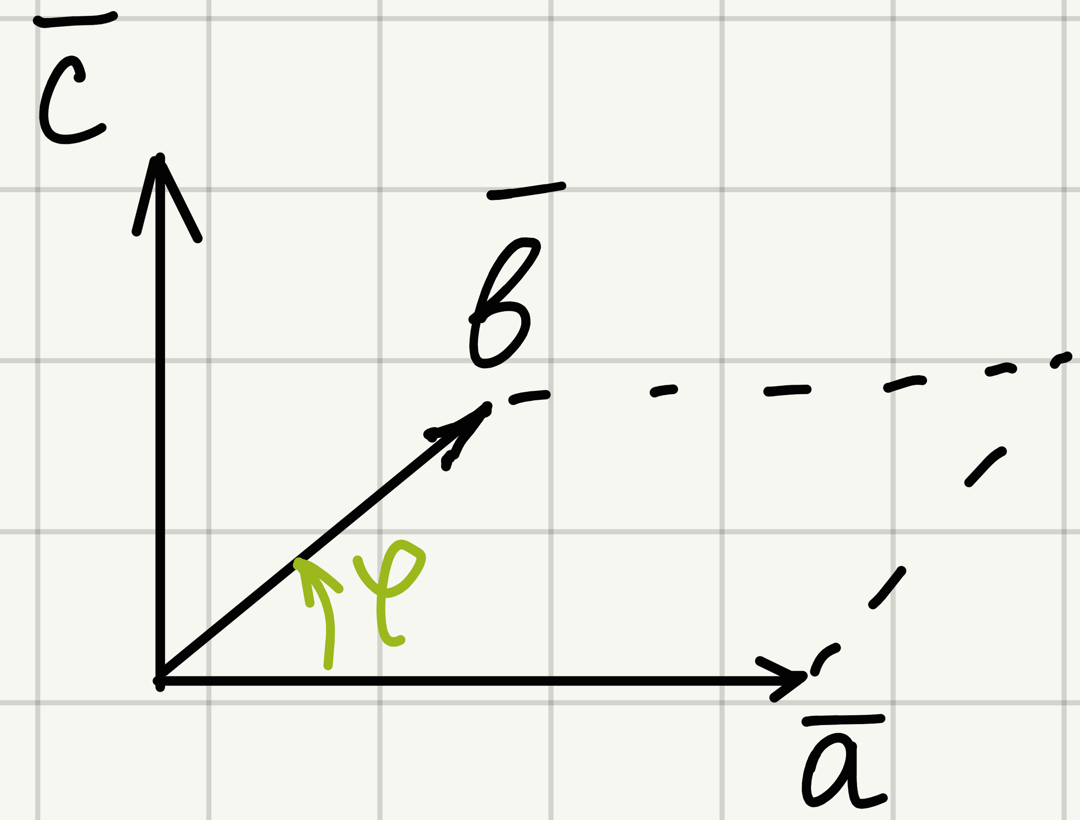
1. Векторное пространство

2. Смешанное произведение.

I

Опр. Векторным пр-ем $\bar{a}$ и $\bar{b}$ наз.

такой вектор $\bar{c}$, что:



1. $|\bar{c}| = |\bar{a}| \cdot |\bar{b}| \cdot \sin\varphi$

2. $\bar{c} \perp \bar{a}$, $\bar{c} \perp \bar{b}$

3. $\bar{a}, \bar{b}, \bar{c}$ – образуют правую тройку

$[\bar{a}, \bar{b}] = -[\bar{a}, \bar{b}]$

1. $[\bar{a}_1 + \bar{a}_2, \bar{b}] = [\bar{a}_1, \bar{b}] + [\bar{a}_2, \bar{b}]$

2. $[\lambda \bar{a}, \bar{b}] = \lambda [\bar{a}, \bar{b}]$

3. $[\bar{a}, \bar{b}] = -[\bar{b}, \bar{a}]$

$\Downarrow$

$[\bar{a}, \bar{b}] = 0 \Leftrightarrow \bar{a} \parallel \bar{b}$

$\bar{e}_1, \bar{e}_2, \bar{e}_3$ – базис в E^3

$\bar{i}, \bar{j}, \bar{k}$ – декартов базис $\equiv$ правый ортон. базис.

$$\bar a = a^1 \bar e_1 + a^2 \bar e_2 + a^3 \bar e_3 = \sum_{i=1}^{3} a^i \bar e_i$$

$$\bar b = b^1 \bar e_1 + b^2 \bar e_2 + b^3 \bar e_3 = \sum_{i=1}^{3} b^i \bar e_i$$

$$[\bar a, \bar b] = \left[\sum_{i=1}^{3} a^i \bar e_i , \sum_{j=1}^{3} b^j \bar e_j \right] = \sum_{i=1}^{3} \sum_{j=1}^{3} a^i b^j [\bar e_1, \bar e_2]$$

$$(a^1 b^2 - a^2 b^1)[\bar e_1, \bar e_2] + (a^1 b^3 - a^3 b^1)[\bar e_1, \bar e_3] +$$

$$+ (a^2 b^3 - a^3 b^2)[\bar e_2, \bar e_3]$$

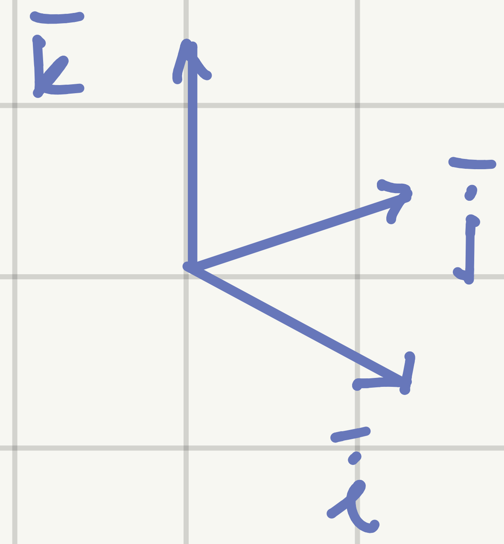
определитель

$$[\bar a, \bar b] := \begin{vmatrix} [e_2, e_3] & [e_3, e_1] & [e_1, e_2] \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

базис $\bar e_1 \quad \bar e_2 \quad \bar e_3$

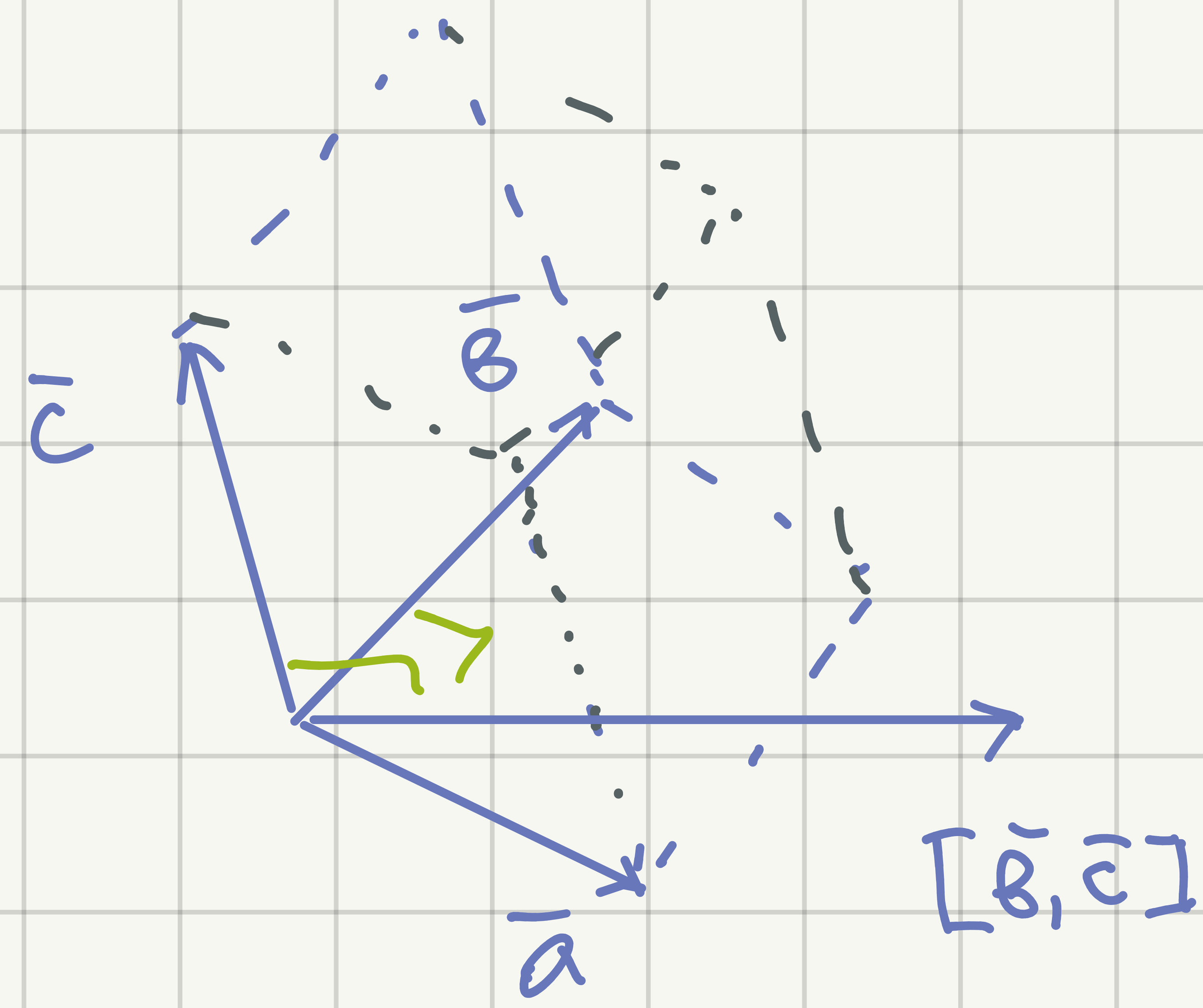
биортогональный базис $[\bar e_2, \bar e_3], [\bar e_3, \bar e_1], [\bar e_1, \bar e_2]$

$$\begin{array}{c} \begin{matrix} \bar i & \bar j & \bar k \\ \| & \| & \| \end{matrix} \\ \begin{vmatrix} [\bar j, \bar k] & [\bar k, \bar i] & [\bar i, \bar j] \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \end{array}$$



II

$$\boxed{(\bar a, \bar b, \bar c) = (\bar a, [\bar b, \bar c])}$$



$$(\bar b, [\bar c, \bar a]) =$$

$$(\bar c, [\bar a, \bar b]) =$$

$$(\bar a_1 + \bar a_2, \bar b, \bar c) = (\bar a_1, \bar b, \bar c) + (\bar a_2, \bar b, \bar c)$$

$$(\bar a, \bar b_1 + \bar b_2, \bar c) = (\bar a, \bar b_1, \bar c) + (\bar a, \bar b_2, c)$$

$$(a, [\bar b_1 + \bar b_2, c]) = (\bar a, [\bar b_1, \bar c]) + (\bar a, [\bar b_2, \bar c])$$

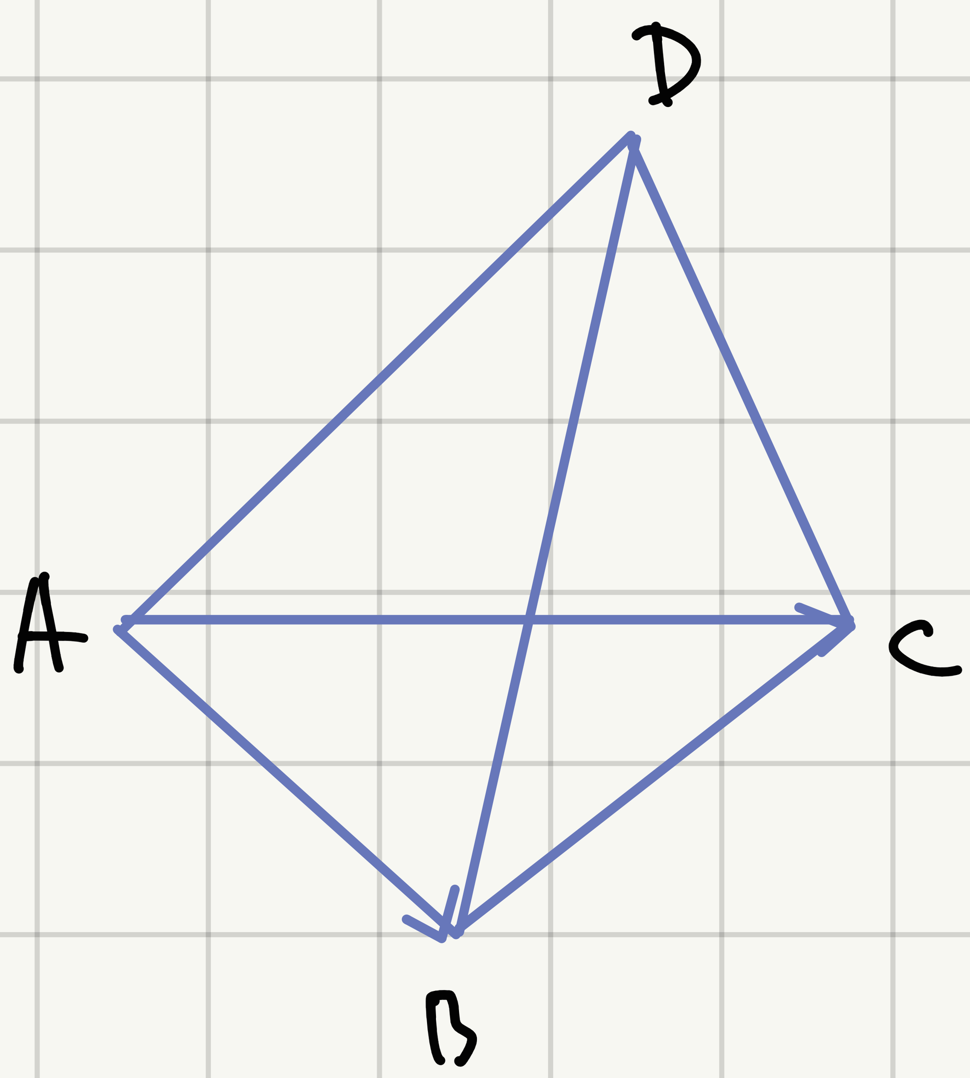
$$(\bar c, [\bar a, \bar b]) = \Big(c^1 \bar e_1 + c^2 \bar e_2 + c^3 \bar e_3, \begin{vmatrix} a^2 & a^3 \\ b^2 & b^3 \end{vmatrix} [\bar e_2, \bar e_3] -$$

$$- \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} [\bar e_3, \bar e_1] + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} [\bar e_1, \bar e_2] =$$

$$= c^1 \begin{vmatrix} a^2 & a^3 \\ b^2 & b^3 \end{vmatrix} (\bar e_1, [\bar e_2, \bar e_3]) - c^2 \begin{vmatrix} a^1 & a^3 \\ b^1 & b^3 \end{vmatrix} (\bar e_2, [\bar e_3, \bar e_1]) +$$

$$+ c^3 \begin{vmatrix} a^1 & a^2 \\ b^1 & b^2 \end{vmatrix} (\bar e_3, [\bar e_1, \bar e_2])$$

$$\begin{vmatrix} c^1 & c^2 & c^3 \\ a^1 & a^2 & a^3 \\ b^1 & b^2 & b^3 \end{vmatrix} (\bar{e}_1, \bar{e}_2, \bar{e}_3) = \boxed{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} (\bar{e}_1, \bar{e}_2, \bar{e}_3)}$$



$$S_\Delta = \frac{1}{6} \left| (\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}) \right|$$

$$H = \frac{\left| (\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AA}) \right|}{\left| [\overrightarrow{AB}, \overrightarrow{AC}] \right|}$$

$$\bar{a} = (3, -1, 2)$$

$$\bar{b} = (2, -3, -5)$$

$$[\bar{a}, \bar{b}] = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 2 & -3 & -5 \end{vmatrix} = i \begin{vmatrix} -1 & 2 \\ -3 & -5 \end{vmatrix} - j \begin{vmatrix} 3 & 2 \\ 2 & -5 \end{vmatrix} + k \begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix} =$$

$$= 11 \cdot i + 19 j + 7k$$

$$[\bar{a}, [\bar{b}, \bar{c}]] = \bar{b}(\bar{a}, \bar{c}) - \bar{c}(\bar{a}, \bar{b})$$

$$[[\bar{a}, \bar{b}], [\bar{c}, \bar{d}]] = \bar{c}(\bar{a}, \bar{b}, \bar{d}) - \bar{d}(\bar{a}, \bar{b}, \bar{c})$$